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On Semiopen Sets and Semicontinuous Functions in Intuitionistic Fuzzy Topological Spaces

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Abstract: *The purpose of this paper is to introduce "semiopen sets" in intuitionistic fuzzy topological spaces. After giving the fundamental definitions and necessary examples we introduce the definitions of intuitionistic fuzzy semicontinuity, intuitionistic fuzzy semicompactness, intuitionistic fuzzy semiconnectedness and studied several preservations properties and some characterizations theorems. We see that every intuitionistic fuzzy open set is intuitionistic fuzzy semiopen and every intuitionistic fuzzy continuous function is intuitionistic fuzzy semicontinuous.*
Key words*: Intuitionistic fuzzy topology, intuitionistic fuzzy semiopen sets, intuitionistic fuzzy semicontinuous functions, intuitionistic fuzzy semi C_5 −connectedness, intuitionistic fuzzy semicompactness.*

I. Introduction

After the introduction of the concept of fuzzy sets by Zadeh [10] several researches were conducted on the generalizations of the notion of fuzzy set. The idea of intuitionistic fuzzy set was first published by Atanassov [1] and D.Coker [6], [7] introduced the notion of intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuity, compactness, connectedness and some other related concepts. I.M.Hanafy [9] introduced the concept of completely continuous functions in intuitionistic fuzzy topological spaces. In this paper we introduce and study the concept of semiopen sets and semicontinuous functions in intuitionistic fuzzy topological spaces.

II. Preliminaries:

Throughout this section, we shall present the fundamental definitions and results of intuitionistic fuzzy sets and intuitionistic fuzzy topology as given by Atanassov[2] and Coker[7].

Definition 2.1[2] Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form, $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the functions $\mu_A : X \to I$ *and* $\gamma_A : X \to I$ denote the degree of membership(namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element x$\in X$ to the set A, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$ for the IFS $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Obviously, every fuzzy set A on a nonempty set X is an IFS having the form, $A = \{\langle x, \mu_A(x), 1 - \mu Ax: x \in X\}$.

Definition 2.2[2] Let X be a nonempty set, and let the IFSs A and B be in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$ and let $\{A_j : j \in J\}$ be an arbitrary family of IFSs in X. Then

(a) $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$;
(b) $\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
(c) $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$ and $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$;
(d) $\cap A_j = \{\langle x, \wedge\mu_{A_j}(x), \vee\gamma_{A_j}(x)\rangle : x \in X\}$;
(e) $\cup A_j = \{\langle x, \vee\mu_{A_j}(x), \wedge\gamma_{A_j}(x)\rangle : x \in X\}$;
(f) $[\ ]A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$;
(g) $< > A = \{\langle x, 1 - \gamma_A(x), \gamma_A(x)\rangle : x \in X\}$;
(h) $\bar{\bar{A}} = A$, $\bar{1}_\sim = 0_\sim$ *and* $\bar{0}_\sim = 1_\sim$;
(i) $\overline{A \cup B} = \bar{A} \cap \bar{B}$, $\overline{A \cap B} = \bar{A} \cup \bar{B}$;

Definition 2.3[7] Let X and Y be two nonempty sets and $f : X \to Y$ be a function.

(i) If $B = \{\langle y, \mu_B(y), \gamma_B(y)\rangle : y \in Y\}$ is an IFS in Y, then the preimage of B under f is denoted and defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x)\rangle : x \in X\}$.

(ii) If $A = \{\langle x, \lambda_A(x), \nu_A(x)\rangle : x \in X\}$ is an IFS in X, then the image of A under f is denoted and defined by $f(A) = \{\langle y, f(\lambda_A)(y), f_-(\nu_A)(y)\rangle : y \in Y\}$,
where $f_-(\nu_A) = 1 - f(1 - \nu_A)$.

In (i),(ii), since $\mu_B, \gamma_B, \lambda_A, \nu_A$ are fuzzy sets, we explain that

$$f^{-1}(\mu_B)(x) = \mu_B(f(x)),$$

and

$$f(\lambda_A)(y) = \begin{cases} \sup \lambda_A(x) & if\ f^{-1}(y) \neq \emptyset \\ 0 & otherwise \end{cases}$$

Definition 2.4[7] Let $A, A_i (i \in J)$ be IFSs in X and $B, B_j (j \in K)$ IFSs in Y and $f: X \to Y$ be a function. Then

(i) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$;

(ii) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$;

(iii) $f^{-1}(1_\sim) = 1_\sim$; $f^{-1}(0_\sim) = 0_\sim$;

(iv) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$;

(v) $f(\cup A_i) = \cup f(A_i)$;

Definition 2.5[7] An intuitionistic fuzzy topology (IFT, for short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$;

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$;

(iii) $\cup A_j \in \tau$ for any $\{A_j : j \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS, for short) and each IFS in τ is known as an intuitionistic fuzzy open set (IFOS, for short) in X. The complement $\bar{A}$ of IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.6[7] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in X. The the intuitionistic fuzzy closure and fuzzy interior of A are defined by

cl $(A) = \cap \{K: K$ is an IFCS in X and $A \subseteq K\}$ and

int $(A) = \cup \{G: G$ is an IFOS in X and $G \subseteq A\}$

Definition 2.7[7, 9] Let (X, τ) and $(Y, \emptyset)$ be two IFTSs and $f: X \to Y$ a function. Then

(i) f is fuzzy continuous iff the preimage of each IFOS in Y is an IFOS in X.

(ii) f is fuzzy completely continuous iff the preimage of each IFOS in Y is an IFROS in X.

Definition 2.8[9] Let $f: X \to Y$ be a function. The graph $g: X \to X \times Y$ of f is defined by

$$g(x) = (x, f(x)), \forall x \in X.$$

Lemma 2.9[9] Let $g: X \to X \times Y$ be the graph of a function $f: X \to Y$. If A is an IFS of X and B is an IFS of Y, then $g^{-1}(A \times B)(x) = (A \cap f^{-1}(B)(x)$.

III. Intuitionistic fuzzy semiopen sets:

Definition 3.1 Let A be an IFS of IFTS (X, τ), then A is said to be (i) an intuitionistic fuzzy semiopen (IFSO) set of X if there exists a IFO set $B \in \tau$ such that $B \leq A \leq cl(B)$,
and (ii) an intuitionistic fuzzy semiclosed (IFSC) set of X if there exists a IFC set B such that $int(B) \leq A \leq B$.

It can be easily shown that, closure of an IFO (IFC) set of (X, τ) is IFSO (IFSC) set.

Remark 3.2 It is obvious that every IFO (IFC) set in an IFTS (X, τ) is IFSO (IFSC) but the converse is not true.

Example 3.3 Let $X = [a, b]$, consider the IFSs on X are

$A = \{(a, 0.5, 0.2), (b, 0.5, 0.4)\}$; $B = \{(a, 0.4, 0.5); (b, 0.6, 0.3)\}$;

$C = \{(a, 0.5, 0.2), (b, 0.6, 0.3)\}$; $D = \{((a, 0.4, 0.5), (b, 0.5, 0.4)\}$;

E= $\{(a, 0.6, 0.1), (b, 0.7, 0.2)\}$; $F = \{((a, 0.1, 0.5), (b, 0.2, 0.6)\}$;

Then $\tau = \{0_\sim, 1_\sim, A, B, C, D\}$ is a IFT on X.

Also we see that $A \leq E \leq cl(A) = 1_\sim$ and $0_\sim = int(\bar{D}) \leq F \leq \bar{D}$.

Thus E is IFSO set but not IFO of $(X, \tau)$and F is IFSC set but not IFC of (X, τ).

Theorem 3.4 Let (X, τ) be an IFTS. Then the following are equivalent:

(i) A is IFSC set

(ii) $\bar{A}$ is IFSO set

(iii) $int\ cl(A) \leq A$

(iv) $cl\ int(\bar{A}) \geq \bar{A}$

Proof:

(i)⇒(ii) By definition there exists a closed set B in (X, τ) such that $int(B) \leq A \leq B$, this implies, $\overline{int(B)} \geq \bar{A} \geq \bar{B}$, using proposition 3.15 of D.Coker[7], we get, $\bar{B} \leq \bar{A} \leq$ cl$(\bar{B})$ where $\bar{B}$ is an IFO set in (X, τ). Therefore, $\bar{A}$ is IFSO in (X, τ).

(ii)⇒(i) Similar

(i)⇒(iii) By definition there exists a IFC set B in (X,τ) such that $int(B) \leq A \leq B$ and hence using proposition 3.16 of D. Coker[7], we get, $int(B) \leq A \leq cl(A) \leq B$. Since $int(B)$ is the largest open set contained in B, we have $int\, cl(A) \leq int(B) \leq A$ i.e, $int\, cl(A) \leq A$.

(iii)⇒(i) This follows by taking $B = cl(A)$ i.e, $int(B) \leq A$. Also we know that $A \leq cl(A) = B$ and hence $int(B) \leq A \leq B$, as $B = cl(A)$ is a closed set, therefore A is IFSC.

(ii) ⇒(iv) can similarly be proved.

Theorem 3.5 Union of a finite number of IFSO sets is a IFSO set and intersection of a finite number of IFSC sets is a IFSC set.
Proof:
1st part Let $A_1, A_2, \ldots \ldots \ldots, A_n$ be IFSO sets of (X,τ) then their exist IFO sets $B_1, B_2, \ldots \ldots \ldots . B_n$ of (X,τ) such that $B_i \leq A_i \leq cl(B_i)$, $i = 1(2)n$. Generalizing the idea of proposition 3.16, D.Coker[7] we get,

$$\cup B_i \leq \cup A_i \leq \cup\, cl(B_i) = cl(\cup B_i)$$

Also $\cup B_i \in \tau$, Hence $\cup A_i$ is IFSO.
2nd part Similar.

Proposition 3.6 Let (X,τ) be an IFTS and let $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$ be IFSO sets in X, then $\mu_B = \{\langle x, \mu_B(x)\rangle : x \in X\}$ is semiopen in (X, τ_1) and $\gamma_B = \{\langle x, \gamma_B(x)\rangle : x \in X\}$ is semiclosed in (X, τ_2), where $\tau_1 = \{\mu_G : G \in \tau\}$ and $\tau_2 = \{1 - \gamma_G : G \in \tau\}$ are FTS on X in Chang's sense.
Proof: Given B is IFSO, then there exists IFO set A such that $A \subseteq B \subseteq cl(A)$. Let $A = \langle x, \mu_A, \gamma_A\rangle$ and suppose that the family of IFC sets containing A are indexed by $\{\langle x, \gamma_{G_i}, \mu_{G_i}\rangle : i \in J\}$. Then we have, $Cl(A) = \langle x, \wedge\gamma_{G_i}, \vee\mu_{G_i}\rangle$ and $\mu_A \leq \gamma_{G_i}, \gamma_A \geq \mu_{G_i}$ for each $i \in J$. By definition of IFSO sets, we get

$\langle x, \mu_A, \gamma_A\rangle \subseteq \langle x, \mu_B, \gamma_B\rangle \subseteq \langle x, \gamma_{G_i}, \mu_{G_i}\rangle$, each $i \in J$

i.e, $\mu_A \leq \mu_B \leq \gamma_{G_i}$ and $\gamma_A \geq \gamma_B \geq \mu_{G_i}$, each $i \in J$

i.e, $\mu_A \leq \mu_B \leq (1 - \mu_{G_i})$ and $(1 - \gamma_{G_i}) \leq \gamma_B \leq \gamma_A$, each $i \in J$

i.e, $\mu_A \leq \mu_B \leq cl(\mu_A)$ and $int(\gamma_A) \leq \gamma_B \leq \gamma_A$

i.e, $\mu_B = \{\langle x, \mu_B(x)\rangle : x \in X\}$ is semiopen in (X, τ_1) and $\gamma_B = \{\langle x, \gamma_B(x)\rangle : x \in X\}$ is semiclosed in (X, τ_2).

Example 3.7 Any FTS (X,τ_0) in the sense of Chang is obviously an IFTS in the form $\tau = \{A : \mu_A \in \tau_0\}$ whenever we identify a fuzzy set in X whose membership function is μ_A with its counter part $A = \langle x, \mu_A, 1 - \mu_A\rangle$. If μ_B is fuzzy semiopen in (X,τ_0) then $B = \langle x, \mu_B, 1 - \mu_B\rangle$ is IFSO in (X,τ).

Since μ_B is semi open in (X,τ_0), there exists a open set in (X,τ_0) such that

$$\mu_A \leq \mu_B \leq cl(\mu_A),$$

i.e $1 - \mu_A \geq 1 - \mu_B \geq 1 - cl(\mu_A) = int(1 - \mu_A)$,

i.e $int(1 - \mu_A) \leq 1 - \mu_B \leq 1 - \mu_A$

Therefore $(1 - \mu_B)$ is semi closed in (X,τ_0).

Thus we conclude that $A = \langle x, \mu_A, 1 - \mu_A\rangle \leq B = \langle x, \mu_B, 1 - \mu_B\rangle \leq Cl(A)$.

Hence B is IFSO in (X,τ).

IV. Intuitionistic fuzzy semicontinuous functions:

Definition 4.1 Let (X,τ) and $(Y,\emptyset)$ be two IFTSs and $f: X \to Y$ a function. Then f is said to be intuitionistic fuzzy semicontinuous (IFSCn) iff the preimage of each IFO set in $\emptyset$ is an IFSO in τ.
Remark 4.2 Every intuitionistic fuzzy continuous function is intuitionistic fuzzy semicontinuous but the converse is not true.
Example 4.3 Let $X = [a,b]$, consider the IF sets on X are $A = \{(a, 0.5, 0.2), (b, 0.5, 0.4)\}$; $B = \{(a, 0.5, 0.2), (b, 0.5, 0.2)\}$. Then $\tau = \{0_\sim, 1_\sim, A\}$ is a IFT on X. Let us consider a function $f: (X,\tau) \to (X,\tau)$ defined by $f(a) = f(b) = a$, then f is IFS continuous but it is not IF continuous, because $f^{-1}(A) = B \notin (X,\tau)$.
Definition 4.4 Let (X,τ) and $(Y,\emptyset)$ be two IFTSs and $f: X \to Y$ a function. Then f is said to be intuitionistic fuzzy semiopen mapping if the image of each IFO set in τ is an IFSO set in $\emptyset$.

It is obvious that every IF open mapping [7] is IF semiopen mapping.

Example 4.5 Let (X,τ_0) and $(Y,\emptyset_0)$ be two FTSs in the sense of Chang.

(a) If $f: X \to Y$ is fuzzy semicontinuous in the usual sense, then in this case, f is IFS continuous. Here we consider the IFTs on X and Y are as follows:
$\tau = \{\langle x, \mu_G, 1 - \mu_G\rangle : \mu_G \in \tau_0\}$ and $\emptyset = \{\langle y, \lambda_H, 1 - \lambda_H\rangle : \lambda_H \in \emptyset_0\}$.
In this case we have, for each $\langle y, \lambda_H, 1 - \lambda_H\rangle \in \emptyset$, $\lambda_H \in \emptyset_0$

$f^{-1}(\langle y,\lambda_H, 1-\lambda_H\rangle) = \langle x, f^{-1}(\lambda_H), f^{-1}(1-\lambda_H)\rangle = \langle x, f^{-1}(\lambda_H), 1-f^{-1}(\lambda_H)\rangle$ is IF semiopen set in τ. As $\lambda_H \in \emptyset_0$ and f is fuzzy semiopen, therefore $f^{-1}(\lambda_H)$ is fuzzy semiopen in (X,τ_0) and $1-f^{-1}(\lambda_H)$ is fuzzy semiclosed in (X,τ_0).

(b) Let $f: X \to Y$ be a fuzzy semiopen function in the usual sense. Then f is also IF semiopen function in the sense of above definition.

Theorem 4.6 A function $f:(X,\tau) \to (Y,\emptyset)$ is IFSCn iff $f^{-1}(B)$ is an IFSC set in X, for each IFC set B in Y.

Proof: **1st part**

Let f is IFSCn and also let B be any IF closed set in Y , then $\bar{B}$ is an IFO set in Y. By definition of IFSCn function, $f^{-1}(\bar{B})$ is IFSO in (X,τ) i.e $\overline{f^{-1}(B)}$ is IFSO in (X,τ) i.e $f^{-1}(B)$ is IFSC in (X,τ).

2nd part similar

Theorem 4.7 If f is IFSCn function and g is IF continuous then gof is IFS continuous.

Proof: obvious.

Theorem 4.8 Let $f:(X,\tau) \to (Y,\emptyset)$ be a function and $g: X \to X \times Y$, be the graph of the function f, then f is IFS continuous if g is so.

Proof: Let B be any IFO set in Y, then $f^{-1}(B) = f^{-1}(1_\sim \times B)$=$1_\sim \cap f^{-1}(B) = g^{-1}(1_\sim \times B)$. Since B is IFO in Y, $1_\sim \times B$ is IFO in $X \times Y$. Also, the fact that g is IFSC implies that $g^{-1}(1_\sim \times B)$ is IFSO in X. Hence $f^{-1}(B)$ is IFSO in X and so f is IFS continuous.

Definition 4.9 Let (X,τ) be an IFTS and A an IFS in X.

(a) If a family $\{\langle x,\mu_{G_i},\gamma_{G_i}\rangle : i \in J\}$ of IFSO sets in X satisfies the condition $A \subseteq \cup\{\langle x,\mu_{G_i},\gamma_{G_i}\rangle : i \in J\}$, then it is called a fuzzy semi open cover of A. A finite subfamily of the fuzzy semi open cover $\{\langle x,\mu_{G_i},\gamma_{G_i}\rangle : i \in J\}$ of A, which is also a fuzzy semi open cover of A, is called a finite subcover of $\{\langle x,\mu_{G_i},\gamma_{G_i}\rangle : i \in J\}$.

(b) An IFS $A = \langle x,\mu_A,\gamma_A\rangle$ in an IFTS (X,τ) is called IF semi compact iff every IF semi open cover of A has a finite subcover.

Theorem 4.10 Let $f:(X,\tau) \to (Y,\emptyset)$ be an IF semicontinuous function. If A is IF semicompact in (X,τ) then $f(A)$ is IF compact in $(Y,\emptyset)$.

Proof: Let $\mathcal{B} = \{G_i : i \in J\}$, where $G_i = \{\langle y,\mu_{G_i},\gamma_{G_i}\rangle : i \in J\}$ is a IF open cover of $f(A)$. Then $\mathcal{A} = \{f^{-1}(G_i) : i \in J\}$ is a IF semi open cover of A. Since A is IF semi compact, there exists a finite subcover of $\mathcal{A}$ i.e there exists $G_i, (i = 1,2,\ldots,n)$ such that

$A \subseteq \cup_{i=1}^{n} f^{-1}(G_i)$. Hence $f(A) \subseteq f(\cup_{i=1}^{n} f^{-1}(G_i)) = \cup_{i=1}^{n} f(f^{-1}(G_i)) \subseteq \cup_{i=1}^{n}(G_i)$.

Therefore, $f(A)$ is also IF compact.

Definition 4.11 An IFTS (X,τ) is said to be IF semi C_5-connected if the only IF sets which are both IFSO and IFSC are $0_\sim$ and $1_\sim$.

(X,τ) is said to be IF semi C_5-disconnected if it is not IF semi C_5-connected.

Theorem 4.12 Let $f:(X,\tau) \to (Y,\emptyset)$ be an IF semicontinuous surjection. If (X,τ) is fuzzy semi C_5-connected, then $(Y,\emptyset)$ is fuzzy C_5-connected.

Proof: On the contrary, suppose that $(Y,\emptyset)$ is fuzzy C_5-disconnected. Then there exists an IF open and closed set G such that $G \neq 1_\sim$ and $G \neq 0_\sim$. Since f is IF semicontinuous , $f^{-1}(G)$ is both an IFSO and IFSC set. The equalities $f^{-1}(G) = 1_\sim$ or $f^{-1}(G) = 0_\sim$ cannot hold. [Because, otherwise we have $G = f(f^{-1}(G)) = f(1_\sim) = 1_\sim$ and $G = f(f^{-1}(G)) = f(0_\sim) = 0_\sim$] Hence $(Y,\emptyset)$ is IF C_5-connected.

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