

Modelling of Depth-Averaged Polluted Flow, Sediment Load And Bed Topography in An Open Channel

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Abstract

Environmental pollution has become a major global concern, posing significant challenges to environmental scientists and policymakers seeking effective strategies for its mitigation and management. Water pollution has increased substantially, adversely affecting aquatic ecosystems and disrupting human activities. The introduction of pollutants into flowing water alters key hydraulic and morphological characteristics, including flow density, flow depth, sediment transport, bed elevation, and channel bed topography. Particulate pollutants contribute to sediment accumulation, thereby altering bed elevation, flow depth, and channel morphology. Although considerable research has been conducted on the effects of either soluble or particulate pollutants individually, studies that simultaneously incorporate both forms of pollution remain limited. This gap in the literature necessitates further investigation into the combined effects of pollutants on flow and sediment dynamics. In this study we investigate the influence of water pollution on sediment transport, and bed topography in open-channel systems. The governing equations describing sediment transport, and bed evolution equation were formulated by incorporating the relevant physical conditions of the flow problem. The resulting equations were non-dimensionalized and subsequently discretized using the finite difference method, subject to appropriate initial and boundary conditions. Numerical simulations were performed using Python, and the resulting data were analyzed

and presented graphically. The simulation results show that pollutant accumulation causes an increase in bed elevation, resulting in a corresponding decrease in flow depth. In addition, increased fluid density is associated with higher bed-load transport rates. These findings demonstrate the significant influence of pollution on hydraulic behavior and channel morphology. The outcomes of this study provide valuable insights for water resources engineers and environmental managers involved in water quality assessment, river management, and the design of hydraulic structures. Furthermore, the developed model contributes to a better understanding of the downstream evolution of open-channel flows and may support disaster management efforts through improved prediction of hydraulic responses associated with extreme events such as floods and dam-break scenarios.

Keywords: Pollution, Depth-averaged equations, Finite Difference Method, Numerical Simulation. Bed-load transport rate.

Date of Submission: 02-10-2026

Date of Acceptance: 12-10-2026

I. Introduction

The depth-averaged equations are widely used to model hydraulic and environmental flows [1]. Their applications include river dynamics, flood propagation, sediment transport, and coastal circulation. Finite difference numerical methods has been used for solving depth-averaged flow equation. The study of open-channel flows requires an understanding of the flow in order to accurately capture its characteristics. Designing and studying engineering projects on rivers, understanding the hydraulic phenomena of the watercourse for water quality control, and predicting exceptional natural events associated with water flow constitute the main objectives of studying open channel flow simulation. The effects of channel curvature and riparian vegetation on flow dynamics and sediment transport have been investigated using a depth-averaged two-dimensional model [2]. According to [3], water flow and sediment transport interact in rivers, floodplains, and coastal areas. Human activities and extreme natural phenomena both have an impact on the interactions between these processes. This causes channel and harbor aggravation and degradation, as well as deterioration of water quality and fisheries, among other environmental effects, as well as a variety of other ecological disturbances such as dam removal, dam breakage, and extreme storm events that cause rapidly varying flow and sediment flushing. The disturbance is complex because of the uneven and changing bottom topography, irregular boundaries, rapid and strong erosion with abrupt bed and flow variations, and complicated and uncertain flow-sediment transport mechanisms. Under these conditions, a one-dimensional strategy is generally not sufficient, and two-dimensional space approaches capable of handling complicated geometry and rapidly varying flow are necessary. This research builds on recent advances in numerical solutions to problems in engineering and computational fluid dynamics to flow, sediment, and bed morphology interactions in rivers. According to [4], bank erosion caused by non-cohesive sediments typically involves four cyclic processes: bed scouring at the toe of the side bank, bank collapse due to instability of the scoured bank, deposition of collapsed bank material at the front of the banks, and transportation of the deposited material. [5] describe fluvial erosion and mass failure as the two primary types of bank erosion interaction processes. These descriptions agree with the experimental observation of bank-line recession due to dam-break surges as a sequence of discrete slumps of bank material into the stream.

The flow parameters: flow height $h(x,y)$, bed height $z(x,y)$, and flow velocities u and v in both x and y directions respectively can be illustrated using figure 1 below

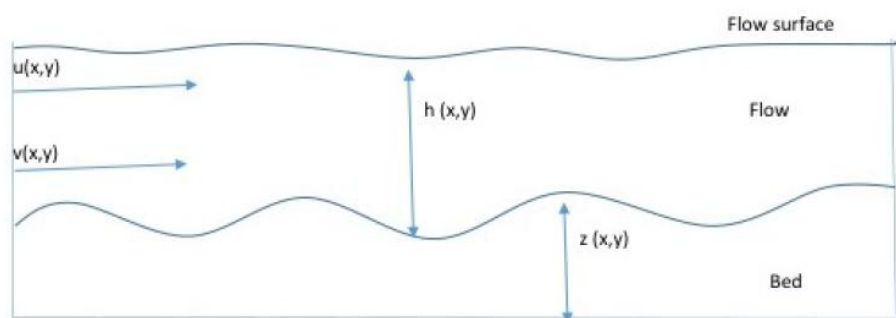


Figure 1: Two-Dimensional Flow Profile

II. Mathematical Model

2.1 Sediment Transport Equations

According to [6], the sediment transport equation is shown in equation (1).

$$\frac{\partial hc}{\partial t} + \frac{\partial uhc}{\partial x} + \frac{\partial vhc}{\partial y} = E - D \quad (1)$$

In which c is the volumetric sediment concentration, h is the flow depth, E is the sediment entrainment due to turbulence and is as shown in equation (2), and D is sediment deposition due to gravitational action as shown in equation (3).

$$E = \begin{cases} \Phi (\theta - \theta_c) |u| d^{-0.2} h^{-1}, & \text{if } \theta \geq \theta_c, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

Φ is a coefficient to control the erosion force, which may be obtained using empirical formulas for intense and strong turbulence bursting entrainment processes. The parameter Φ is calibrated for a relatively weak sediment entrainment process. u is flow velocity, d is sediment grain diameter, h is water depth, and θ and θ_c are Shields parameter and critical Shields parameter, respectively. In this study, we ensure that $\theta \geq \theta_c$ to enhance sediment entrainment at a rate proportional to excess shear stress, flow velocity, sediment grain size, and water depth as shown in equation (3)

$$D = W_0 (1 - C_\alpha)^m C_\alpha \quad (3)$$

m is an exponent representing the effect of hindered settling due to sediment concentration. C_α represent sediment concentration at a reference level α . W_0 is the settling velocity of a single particle in tranquil water, as shown in equation (4).

$$W_0 = \sqrt{\left(\frac{13.95\Omega}{d}\right)^2 + 1.09sgd - \frac{13.95\Omega}{d}} \quad (4)$$

Ω is the kinematic viscosity of water; d is sediment particle diameter; s is the specific density ratio; and g is the gravitational acceleration.

In the sediment transport equation, the influx of sediments in terms of pollutants increases the volumetric sediment concentration. Volumetric sediment concentration is gradually increased by pollutants to $1.5c$ and to $2c$ to obtain equations (5) and (6), respectively.

$$\frac{\partial hc}{\partial t} + \frac{\partial uhc}{\partial x} + \frac{\partial vhc}{\partial y} = \frac{2}{3}(E - D) \quad (5)$$

$$\frac{\partial hc}{\partial t} + \frac{\partial uhc}{\partial x} + \frac{\partial vhc}{\partial y} = \frac{1}{2}(E - D) \quad (6)$$

h is the flow depth; E is the sediment entrainment due to turbulence as illustrated in equation (2), and D is sediment deposition due to gravitational action as shown in equation (3).

2.2 Non-Dimensionalizing Sediment Transport Equation

In sediment transport equation we utilize the following Non-dimensional variables,

$$h^* = \frac{h}{H^*}, \quad x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}, \quad u^* = \frac{u}{U^*}, \quad v^* = \frac{v}{V^*}, \quad t^* = \frac{t}{T^*}, \quad E^* = \frac{E}{S^*},$$

$$c^* = \frac{c}{C^*}, \quad D^* = \frac{D}{S}$$

Computing for derivatives

$$\frac{\partial}{\partial t} = \frac{U}{L} \frac{\partial}{\partial t^*}, \quad \frac{\partial}{\partial x} = \frac{1}{L} \frac{\partial}{\partial x^*}, \quad \frac{\partial}{\partial y} = \frac{1}{L} \frac{\partial}{\partial y^*},$$

$$hc = (HC)h^*c^*, \quad uhc = (UHC)u^*h^*c^*, \quad vhc = (VHC)v^*h^*c^*$$

Substituting the non-dimensional variables into the equation (1) and factorizing we obtain equation (7)

$$\frac{\partial(h^*c^*)}{\partial t^*} + \frac{\partial(u^*h^*c^*)}{\partial x^*} + \frac{\partial(v^*h^*c^*)}{\partial y^*} = S_r(E^* - D^*) \quad (7)$$

where S_r is the dimensionless source number $S_r = \frac{LS}{UHC}$.

In this study we choose S_r be equal to advective scale implying $S_r = 1$ and hence equation (7) becomes equation (8)

$$\frac{\partial(h^*c^*)}{\partial t^*} + \frac{\partial(u^*h^*c^*)}{\partial x^*} + \frac{\partial(v^*h^*c^*)}{\partial y^*} = (E^* - D^*) \quad (8)$$

Similarly substituting the non-dimensional variables into equations (5) and (6) and factorizing we obtain equations (9) and (10) respectively.

$$\frac{\partial(h^*c^*)}{\partial t^*} + \frac{\partial(u^*h^*c^*)}{\partial x^*} + \frac{\partial(v^*h^*c^*)}{\partial y^*} = \frac{2}{3}(E^* - D^*) \quad (9)$$

$$\frac{\partial(h^*c^*)}{\partial t^*} + \frac{\partial(u^*h^*c^*)}{\partial x^*} + \frac{\partial(v^*h^*c^*)}{\partial y^*} = \frac{1}{2}(E^* - D^*) \quad (10)$$

2.3 Sediment Load Equation in Finite Difference Method

The finite difference discretization is done on a Cartesian grid with indices (i,j) and time levels n for current grid and n+1 for next grid. $\Delta x, \Delta t$ are the grid and time spacing.

We first define the computational grid as,

$$x_i = i\Delta x^*, \quad y_i = i\Delta y^*, \quad \text{and} \quad t^n = n\Delta t^*$$

The time derivative

$$\frac{\partial(h^*c^*)}{\partial(x^*)} \Big|_{i,j}^n \approx \frac{(h^*c^*)_{i,j}^{n+1} - (h^*c^*)_{i,j}^n}{\Delta t^*}$$

and spatial derivative

$$\frac{\partial(u^*h^*c^*)}{\partial(x^*)} \Big|_{i,j}^n \approx \frac{(u^*h^*c^*)_{i+1,j}^n - (u^*h^*c^*)_{i-1,j}^n}{2\Delta x^*}$$

$$\frac{\partial(v^*h^*c^*)}{\partial(y^*)} \Big|_{i,j}^n \approx \frac{(v^*h^*c^*)_{i,j+1}^n - (v^*h^*c^*)_{i,j-1}^n}{2\Delta y^*}$$

Using the above derivatives, we discretize equation (7) to obtain equation (11).

$$\begin{aligned} & \frac{(h^*c^*)_{i,j}^{n+1} - (h^*c^*)_{i,j}^n}{\Delta t^*} + \frac{1}{2\Delta x^*}[(u^*h^*c^*)_{i+1,j}^n - (u^*h^*c^*)_{i-1,j}^n] \\ & + \frac{1}{2\Delta y^*}[(v^*h^*c^*)_{i+1,j}^n - (v^*h^*c^*)_{i,j-1}^n] = (E^* - D^*)_{i,j}^n \end{aligned} \quad (11)$$

Discretizing equation (9) for increased pollutants to 1.5c, we obtain equation (12).

$$\begin{aligned} & \frac{(h^*c^*)_{i,j}^{n+1} - (h^*c^*)_{i,j}^n}{\Delta t^*} + \frac{1}{2\Delta x^*}[(u^*h^*c^*)_{i+1,j}^n - (u^*h^*c^*)_{i-1,j}^n] \\ & + \frac{1}{2\Delta y^*}[(v^*h^*c^*)_{i+1,j}^n - (v^*h^*c^*)_{i,j-1}^n] = \frac{2}{3}(E^* - D^*)_{i,j}^n \end{aligned} \quad (12)$$

Discretizing equation (10) for increased pollutants to 2c, we obtain equation (13).

$$\begin{aligned} & \frac{(h^*c^*)_{i,j}^{n+1} - (h^*c^*)_{i,j}^n}{\Delta t^*} + \frac{1}{2\Delta x^*}[(u^*h^*c^*)_{i+1,j}^n - (u^*h^*c^*)_{i-1,j}^n] \\ & + \frac{1}{2\Delta y^*}[(v^*h^*c^*)_{i+1,j}^n - (v^*h^*c^*)_{i,j-1}^n] = \frac{1}{2}(E^* - D^*)_{i,j}^n \end{aligned} \quad (13)$$

2.4 Bed-load Equation

According to Liu et al.(2015), the general governing equation for bed-load is as shown in equation (14)

$$\frac{\partial z}{\partial t} + \xi \frac{\partial q_{bx}}{\partial x} + \xi \frac{\partial q_{by}}{\partial y} = \frac{D - E}{1 - \sigma} \quad (14)$$

where q_b is shown by equation (15)

$$q_b = \sqrt{(s-1)gd^3(\theta - \theta_c)^{\frac{3}{2}}} \quad (15)$$

$\xi = \frac{1}{1-\sigma}$ Where σ the bed porosity.

s is the specific gravity of sediment given by $s = \frac{\rho_s}{\rho_f}$, where ρ_s is the density of sediment particle, ρ_f is the density of water, d is the mean particle diameter, and g is the gravitational acceleration. θ_c is the critical bed shear stress, and $\theta_c = 0.047$ [7]. and θ is the dimensionless shear, which can be calculated using equation (16).

$$\theta = \frac{hs'}{(s-1)d} \quad (16)$$

where $s' = s_f = \sqrt{s_{fx}^2 + s_{fy}^2}$ equation (16) becomes equation (17)

$$\theta = \frac{n_b^2}{(s-1)d h^{\frac{1}{3}}} |u|^2 \quad (17)$$

$|u| = \sqrt{u^2 + v^2}$ and h is the flow depth

The water's turbidity changes as a result of pollution, which also affects the water's density. Different values of bed height and flow height are gradually determined, and the density of sediment is gradually changed. The discharge of flow without pollution is represented by equation (15); the introduction of a pollutant gradually alters sediment density.

When the sediment density rises to 1.5 , equation (15) becomes equation (18)

$$q_b = \sqrt{(1.5s - 1)gd^3(\theta - \theta_c)^{\frac{3}{2}}} \quad (18)$$

When the sediment density increases to $2s$, equation (15) becomes equation (19)

$$q_b = \sqrt{(2s - 1)gd^3(\theta - \theta_c)^{\frac{3}{2}}} \quad (19)$$

s is the specific gravity of sediment given by $s = \frac{1.5\rho_s}{\rho_f}$ and $s = \frac{2\rho_s}{\rho_f}$, respectively, where ρ_s is the density of sediment particle, ρ_f is the density of water, d is the mean particle diameter, and g is the gravitational acceleration. θ_c is the critical bed shear stress, and $\theta_c = 0.047$ (Smart, 1984), and θ is the dimensionless shear, which can be calculated using equation (20)

$$\theta = \frac{hs'}{(1.5s - 1)d} \quad (20)$$

Using $s' = s_f = \sqrt{s_{fx}^2 + s_{fy}^2}$ equation (20), is transformed into equation (21)

$$\theta = \frac{n_b^2}{(1.5s - 1)dh^{\frac{1}{3}}} |u|^2 \quad (21)$$

and simplifying we obtain equation (22)

$$\theta = \frac{hs'}{(2s - 1)d} \quad (22)$$

Using $s' = s_f = \sqrt{s_{fx}^2 + s_{fy}^2}$ equation (22) is transformed into equation (23).

$$\theta = \frac{n_b^2}{(2s - 1)dh^{\frac{1}{3}}} |u|^2 \quad (23)$$

in which $|u|^2 = \sqrt{u^2 + v^2}$ and h is the flow depth.

2.5 Bed-Load Equation in Finite Difference Method

We discretize bed-load equation using the upwind scheme to avoid oscillations; we shall use the following grid and notations.

$$z_{i,j}^n(x, y, t^n), \Delta x^*, \Delta y^*, \Delta t^*$$

$$\text{time derivative } \frac{z_{i,j}^{n+1} - z_{i,j}^n}{\Delta t^*}$$

upward discretization in the x -direction

$$\left[\frac{\partial q_{bx}^*}{\partial x^*}\right]_{i,j}^n \approx \begin{cases} \frac{q_{bx,t,j}^n - q_{bx,t-1,j}^n}{\Delta x^*}, q_{bx,i,j}^n > 0, \\ \frac{q_{bx,t+1,j}^n - q_{bx,t,j}^n}{\Delta x^*}, q_{bx,i,j}^n < 0, \end{cases} \quad (24)$$

upward discretization in the y-direction

$$\left[\frac{\partial q_{by}^*}{\partial y^*}\right]_{i,j}^n \approx \begin{cases} \frac{q_{by,t,j}^n - q_{by,t-1,j}^n}{\Delta y^*}, q_{by,i,j}^n > 0, \\ \frac{q_{by,t+1,j}^n - q_{by,t,j}^n}{\Delta y^*}, q_{by,i,j}^n < 0, \end{cases} \quad (25)$$

Using the above grid, notations, and derivatives, the fully discretized upwind scheme bed-load equation becomes equation (26).

$$\frac{z_{i,j}^{n+1} - z_{i,j}^n}{\Delta t^*} + \left(\frac{\partial q_{bx}^*}{\partial x^*}\right)_{i,j}^n + \left(\frac{\partial q_{by}^*}{\partial y^*}\right)_{i,j}^n = D_{i,j}^{*n} - E_{i,j}^{*n} \quad (26)$$

The introduction of insoluble pollutants allows sediment densities to gradually rise from 1.0 to 1.5 and ultimately to 2.0 at grid point (i,j) and time level n. Equations (27) and (28) are obtained by substituting sediment densities 1.5 and 2.0, respectively.

$$q_{b,i,j}^* = \sqrt{(1.5s - 1)}(\theta_{i,j}^n - \theta_c)^{\frac{3}{4}} \quad (27)$$

$$\theta_{i,j}^n \geq \theta_c \text{ and } q_{b,i,j}^* = 0 \text{ if } \theta_{i,j}^n < \theta_c$$

$$q_{b,i,j}^* = \sqrt{(2s - 1)}(\theta_{i,j}^n - \theta_c)^{\frac{3}{4}} \quad (28)$$

III. Initial and Boundary Conditions for Bed Evolution Equation

The following initial and boundary conditions are used for the numerical simulation of the bed evolution equation.

3.1 Initial Conditions

The initial bed elevation and bed-load transport rates are specified as

$$z(x, y, 0) = 0.5,$$

$$q_{bx}(x, y, 0) = 100,$$

$$q_{by}(x, y, 0) = 0,$$

with the initial deposition and erosion terms given by

$$D(x, y, 0) = 0, \quad E(x, y, 0) = 0.$$

3.2 Boundary Conditions

At the inflow boundary $x = 0$,

$$q_{bx}(0, y, t) = 100,$$

$$z(0, y, t) = 0.5.$$

At the outflow boundary $x = L_x$, zero-gradient boundary conditions are imposed:

$$\frac{\partial z}{\partial x} = 0,$$

$$\frac{\partial q_{bx}}{\partial x} = 0.$$

Along the lateral boundaries $y = 0$ and $y = L_y$,

$$q_{by}(x, 0, t) = 0, \quad q_{by}(x, L_y, t) = 0,$$

and

$$\frac{\partial z}{\partial y} = 0.$$

The initial and boundary conditions are chosen to be non-zero in order to represent a realistic hydraulic and sediment transport system. If all conditions were taken as zero, there would be no water motion, no sediment transport, and consequently no bed evolution within the computational domain. The initial bed elevation is specified as

$$z(x, y, 0) = 0.5,$$

to provide an initial bed profile from which erosion and deposition processes can evolve. A non-zero bed elevation also prevents numerical instability associated with a completely flat dry bed. The streamwise bed-load transport rate is prescribed as

$$q_{bx}(x, y, 0) = 100,$$

to represent the presence of active sediment transport driven by the flow in the longitudinal direction of the channel. This allows the simulation to capture realistic bed deformation processes. Therefore, the selected non-zero initial and boundary conditions ensure that the governing bed evolution equation describes physically meaningful sediment dynamics and produces stable numerical simulations.

3.3 Initial conditions

$$c^*(x, y, 0) = 1.5 \quad \text{and} \quad c^*(x, y, 0) = 2.0$$

$$u^*(x, y, 0) = 0.2$$

$$v^*(x, y, 0) = 0.2$$

3.4 Boundary conditions

inlet boundary condition

$$c^*(0, y, t^*) = 1.5 \quad \text{and} \quad c^*(0, y, t^*) = 2.0$$

outlet boundary condition

at the downstream boundary $x=L$

$$\frac{\partial c^*}{\partial x^*}(L, y, t^*) = 0$$

similarly

$$\frac{\partial c^*}{\partial x^*}(L, y, t^*) = 0$$

IV. Results and Discussion

4.1 Results and Discussion for Sediment Transport Equation

The discretized Sediment Transport Equation (11) has been simulated using python and the following results are obtained.

4.1.1 Effect of Sediment Concentration on Velocity

Simulating discretized equation (11) for a graph of mean sediment concentration against velocity we obtain the following results.

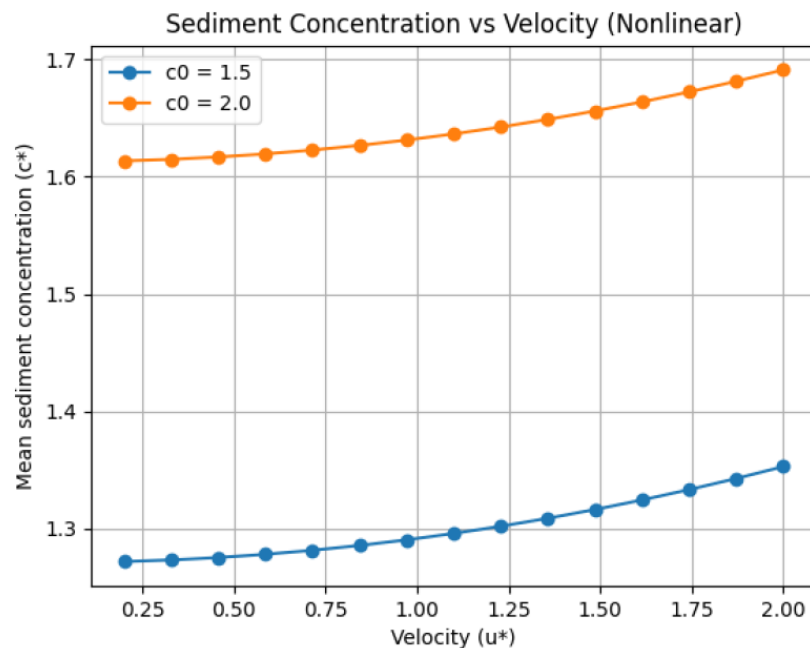


Figure 2: Mean Sediment Concentration vs Velocity

Figure 2 shows non linear gradual increase of sediment concentration with increase in velocity. From the graph, an increase of pollutants on flow causes significant increase in sediment concentration for flow with less velocity that enhances deposition. Comparing two densities 1.5 and 2.0 we realise that higher density 2.0 has higher sediment concentration than lower density 1.5 for equal velocity. Chemical pollutants weakens soil structure on the river bed and river bank leading to higher erosion increasing suspended sediment concentration in streams and rivers. Some chemical pollutants cause fine sediments to clump together and hence settle faster potentially reducing suspended sediment but increasing bottom deposits. Solid waste can trap sediments which might cause sediment build-up. Pollutants that harm aquatic plants reduce root systems that hold soil thus increasing bank erosion and sediment wash-in, raising sediment concentration in the flow

4.1.2 Sediment Flux vs Time

Simulating discretized equation (11) for a graph of sediment flux against time, we obtain the following results.

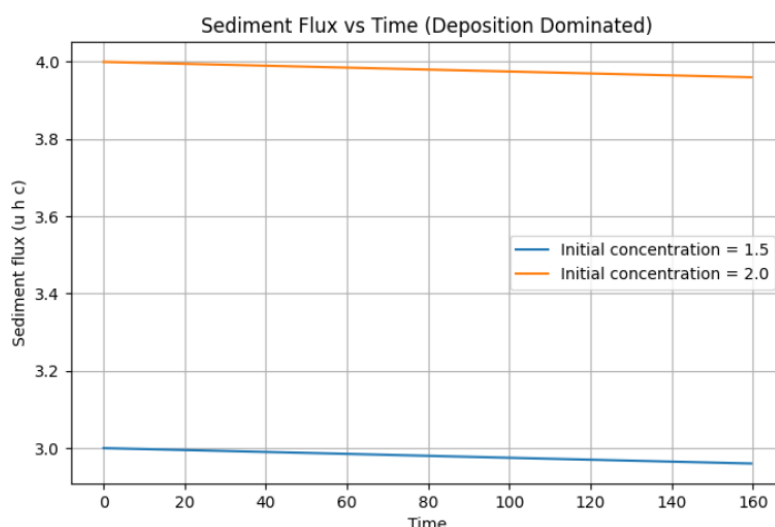


Figure 3: Sediment Flux vs Time

Figure 3 indicates the slight reduction of sediment flux with time. With an increase of time, there is a subsequent reduction of sediment flux. The graph also shows that the fluid with an initial concentration density of 2.0 has higher sediment flux than the one with an initial concentration density of 1.5. The sediment flux is reducing with time due to pollutant entrance into the flow, which increases shear stress on the bed, hence slowing down the amount of sediment passing through in a given time.

4.2 Results and Discussion of Bed-Load Equation

The discretized Bed-Load Equation (26) has been simulated using Python, and the following results are obtained.

4.2.1 Effect of pollutants on bed height

Simulating equation (26) for a graph of bed height against time, we obtain the following results.

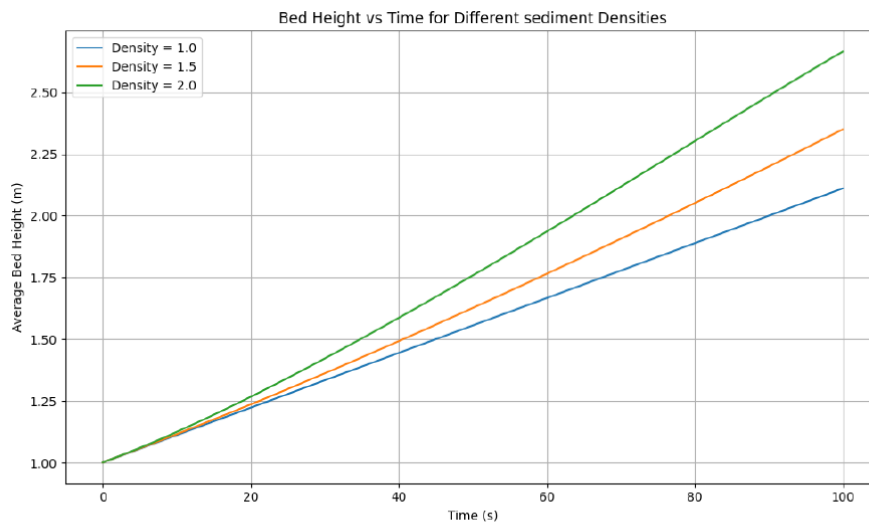


Figure 4: Bed height vs Time

Figure 4 illustrates the gradual increase in flow height over time. The graph also compares three different sediment densities, showing an increase in sediment density results in an increased bed height. The green curve, having a sediment density of 2.0, indicates a higher bed height than the red one, while the red one, having a sediment density of 1.5, has a higher bed height than the blue one with a sediment density of 1.0, which is considered the reference sediment density. The curves show that bed height is increasing over time due to sediment deposition favored by high sediment supply from upstream and shallow channels. When the flow velocity decreases, the energy of the flow reduces, causing heavier particles to settle on the bed, and over time, this deposition leads to an increase in bed height. The introduction of insoluble pollutants into the flow increases the sediment density; as a result, over time, the silt settles on the bed, which further increases the bed height. When the pollutants enter the flow and $D > E$, the velocity is slow enough to cause deposition, and hence sediments accumulate, increasing bed height and reducing flow height.

4.2.2 Effect of distance on bed-load transport rate for various densities.

Simulating discretized equation (13) for a graph of bed-load transport rate against time, we obtain the following results.

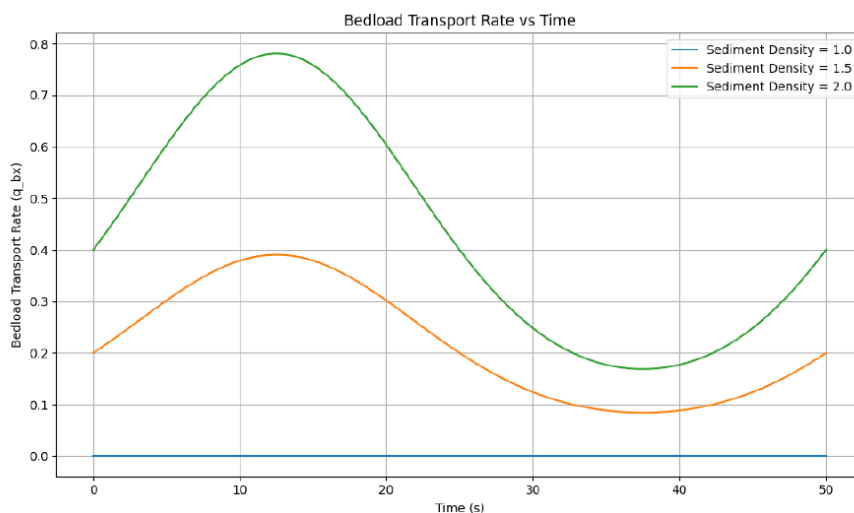


Figure 5: Bed-Load Transport rate vs time

Figure 5 generally indicates a non-linear reduction of bed-load transport rate with time. Bed-load transport varies significantly with distance along a river or open channels, as shown in Figure 2 above. It is highest in the upstream areas due to high velocity and abundant sediment supply, as the flow moves downstream, energy losses, and sediment deposition led to a gradual decrease in bed-load transport rate. Comparing different densities, an increase in sediment density enhances shear stress, reduces the threshold for particle entrainment, and increases the transport capacity of the flow. The green curve of sediment density 2.0 has the highest load transport rate; this is due to lower critical shear stress, large particle movement, and higher transport rate. Curve red of moderate density 1.5 has moderate shear stress and therefore moderate sediment-carrying capacity. Curve blue has the lowest density, 1.0, which signifies low shear stress, therefore limited particle movement, hence the least compared to others.

4.2.3 Effect of shear stress on bed-load transport rate for various densities

Simulating discretized equation (26) for a graph of bed-load transport rate against shear stress we obtain the following results.

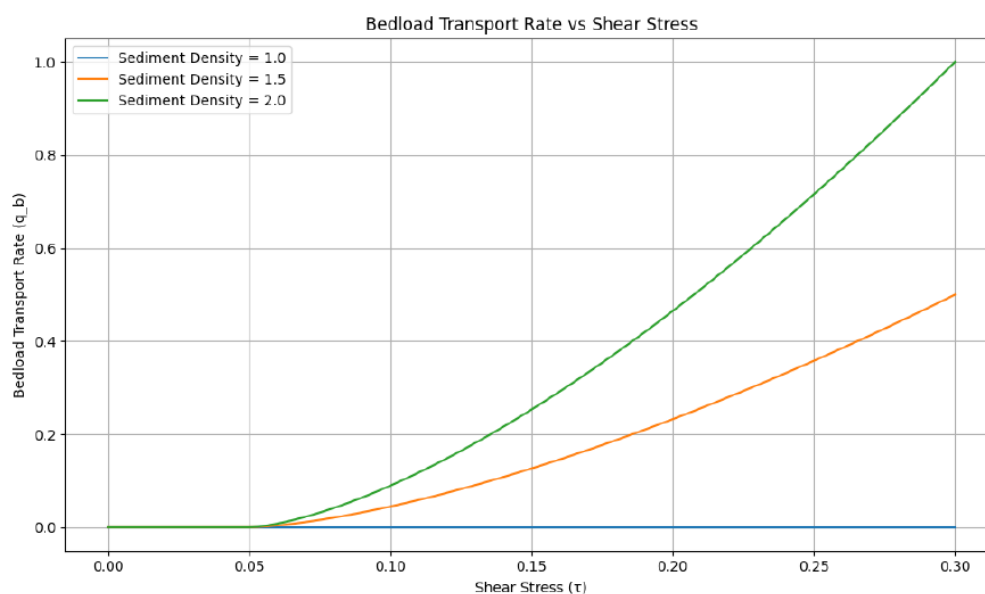


Figure 6: Bed-load transport rate vs stress

Figure 6 shows a non-linear growth of bed-load transport rate with growth of shear stress in both curves of sediment densities 1.5 and 2.0. Comparing sediment densities 2.0 and 1.5 it is shown that the higher the density 2.0 the higher the bed-load transport rate at specified shear stress.

4.3 Conclusion

In this study we explore the effect of pollution, which is associated with high sediment supply. When the flow velocity decreases, the energy of the flow reduces, causing heavier particles to settle on the bed, and due to time, deposition leads to an increase in bed height over time. Organic pollutants often increase biological activity, which can lead to sediment trapping and settling on the bed, hence increasing the bed height. After flooding there is an increase of bed height due to deposition of sediments as velocity drops. During the dry season there is a stable height due to minimal erosion or possible deposition. When comparing the bed-load transport rate over time, we consider the following factors: a consistent supply of sediments (pollutants), the assumption that pollutant and sediment particles are of the same size, and that the channel morphology remains unchanged within a given distance. The bed-load transport rate is significantly influenced by density variations of fluids.

The introduction of solid pollutants was found to influence sediment transport processes through mechanisms of erosion and deposition. The incorporation of these pollutant effects facilitated the development of a modified sediment transport model. The numerical results obtained were consistent with established principles of fluid dynamics and sediment transport theory, thereby demonstrating the validity and existence of the sediment load model for both concentration 1.5 and 2.0. The decrease in flow velocity associated with higher roughness values also influences sediment transport processes. Lower velocities reduce the erosive capacity of the flow, thereby decreasing the likelihood of bed and bank erosion. Conversely, excessively low velocities may promote sediment deposition, potentially affecting channel capacity and hydraulic performance. The introduction of soluble pollutants significantly affected the density of the flowing water. When the flow density changes from 1.0 to 1.5 and finally to 2.0 they produce the corresponding changes in key flow parameters, illustrating the sensitivity of flow behavior to pollutant concentration. The simulation results successfully captured the influence of density variations on the hydraulic characteristics of the flow. However, although the model accounted for erosion and deposition processes, the temporal evolution of bed topography was not clearly represented in the graphical results. Consequently, the precise development of bed forms and morphological changes over time requires further investigation and more detailed visualization techniques.

Acknowledgments

Special acknowledgment goes to Department of Physical science, Chuka University fraternity, for the facilities and resources availed for the work. To all my PhD lecturers thanks for insightful comments, and to my fellow PhD students, your help and support is appreciated.

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