

Exact k -Connected m -Domination in Generating Graphs of Finite p -Groups

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Abstract:

We determine exact k -connected m -domination parameters for the non-isolated generating graphs of finite p -groups with rank-two Frattini quotient. Burnside's basis theorem identifies these graphs with equal complete multipartite graphs whose parts correspond to projective lines in the Frattini quotient. Motivated by this reduction, we solve the associated optimization problem for $K(a, \dots, a)$. The selected set is encoded by its part occupancies and by the number of saturated parts. This yields an exact finite phase formula. Within every phase, balanced occupancies maximize the domination level that can be sustained without increasing cardinality. These robustness values determine the exact right endpoint of each cardinality plateau and, more strongly, give a Plateau Filtration Theorem describing all optimal phases throughout a plateau. Boundary cases $a=1$ and $s=2$ are treated explicitly. The theory is specialized to generating graphs, and Q_8 is worked out completely. Keywords. generating graph; finite p -group; Frattini subgroup; k -connected m -domination; complete multipartite graph; saturation; robustness; plateau filtration.

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I. INTRODUCTION

The generating graph $\Gamma(G)$ of a finite group G has the elements of G as vertices, with two distinct elements adjacent when they generate G . Its non-isolated subgraph $\Delta(G)$ is often the more informative object, because it removes elements that cannot participate in a generating pair. For finite p -groups, Burnside's basis theorem makes this graph particularly transparent when the Frattini quotient has rank two. If $G/\Phi(G)$ is isomorphic to C_p^2 , two elements generate G precisely when their images form a basis of the two-dimensional vector space. The nonzero vectors are partitioned by the $p+1$ one-dimensional subspaces, and two nonzero vectors form a basis precisely when they belong to different projective lines. After lifting to G , $\Delta(G)$ is therefore an equal complete multipartite graph. This projective structure is the starting point of the present paper.

We study a simultaneous domination-connectivity requirement. A set D must m -dominate every vertex outside D and, at the same time, $X[D]$ must be k -vertex-connected. We call such a set a k -connected m -dominating set. The general k -connected m -dominating set problem has an algorithmic literature, including approximation results. Our purpose is different: we seek exact formulas and exact structural information on the highly symmetric multipartite graphs arising from generating graphs. There is also a direct connection with exact multiple domination in complete multipartite graphs. Lu and Xu determined the p -domination number of general complete multipartite graphs. Their framework already exploits part occupancies and completely selected classes. The contribution here is therefore not the bare idea of recording saturated classes. The new feature is the interaction between saturation and the additional internal condition $\kappa(X[D]) \geq k$. That interaction creates a finite family of saturation phases, and it permits a stability analysis of the minimizers as m varies.

The main contributions are as follows. First, an exact occupancy criterion compresses the problem by the saturated classes. Second, in the equal-part case, symmetry reduces the optimization to $s+1$ phases and gives an exact closed formula. Third, a balanced representative maximizes domination robustness inside each phase. Fourth, the robustness values determine exact cardinality plateaus. Finally, the Plateau Filtration Theorem identifies every optimal phase at every point of a plateau. These results are then transferred to generating graphs through the Frattini quotient.

1.1. Relation with earlier multipartite domination

Feature	Lu-Xu p -domination	Present problem
Underlying graph	general complete multipartite	equal multipartite; generating-graph specialization
External requirement	at least p neighbors in D	at least m neighbors in D
Condition on $X[D]$	none	k -vertex-connected
Saturated classes	used in occupancy analysis	used together with connectivity cost
Exact cardinality	yes	yes
Phase robustness	not the objective	exact
Plateau filtration	not the objective	exact

The table is intended only to locate the present contribution. In Lu and Xu, the parameter p is the required number of neighbors in the dominating set for each outside vertex; it therefore plays the same external-domination role that m plays here. Our parameter k has no counterpart in their p -domination condition: it imposes the additional internal requirement that $X[D]$ be k -vertex-connected. Thus the essential new constraint is the vertex connectivity of the selected induced subgraph.

II. GENERATING GRAPHS AND PROJECTIVE MULTIPARTITE STRUCTURE

Let $\Gamma(G)$ be the generating graph of G and $\Delta(G)$ its induced subgraph on non-isolated vertices. Throughout the group-theoretic part, G is finite.

Proposition 2.1. Let G be a finite noncyclic p -group. Then $\Gamma(G)$ has an edge if and only if G is two-generated. Equivalently, $G/\Phi(G)$ is isomorphic to C_{p^2} .

Proof. By Burnside's basis theorem, a subset generates G if and only if its image generates $G/\Phi(G)$. A noncyclic p -group is generated by two elements exactly when its Frattini quotient has dimension two over F_p . Thus an edge exists exactly in this case.

Proposition 2.2. Suppose $G/\Phi(G)$ is isomorphic to C_{p^2} and set $A=(p-1)|\Phi(G)|$. Then the isolated vertices of $\Gamma(G)$ are precisely $\Phi(G)$, and

$$\Delta(G) \text{ is isomorphic to } K_{(A,\dots,A)} \text{ with } p+1 \text{ equal parts. (2.1)}$$

Proof. An element of $\Phi(G)$ maps to zero and cannot occur in a generating pair, so it is isolated.

Conversely, every element outside $\Phi(G)$ has nonzero image v . Choose a quotient vector w not lying in the one-dimensional subspace generated by v . Then v, w form a basis and any lifts generate G . Thus the non-isolated vertices are exactly $G/\Phi(G)$. The $p+1$ projective lines partition the nonzero quotient vectors. Each line has $p-1$ nonzero vectors, each with $|\Phi(G)|$ lifts, hence each part has A vertices. Two lifts are adjacent exactly when their images belong to different projective lines. Consequently, exact k -connected m -domination on $\Delta(G)$ is reduced to the equal complete multipartite problem. We now solve that problem independently, because the resulting formulas are useful beyond the group-theoretic application.

III. DEFINITIONS AND OCCUPANCY FORMULATION

Throughout, k and m are positive integers. For $D \subseteq V(X)$, we write $X[D]$ for the subgraph of X induced by D . A graph H is k -vertex-connected if it has at least $k+1$ vertices and remains connected after deletion of fewer than k vertices; in particular, K_2 is 1-connected.

Definition 3.1. A set $D \subseteq V(X)$ is a k -connected m -dominating set if $X[D]$ is k -vertex-connected and every vertex in $V(X) \setminus D$ has at least m neighbors in D . Its minimum cardinality is denoted $\gamma_{(k,m)}(X)$; if no such set exists, the value is ∞ .

Let $X=K_{(a_1,\dots,a_s)}$ with partite classes $V_1,\dots,V_s$. Write $z_i=|D \cap V_i|$ and $t=|D|=\sum_i z_i$. If D meets at least two parts, $X[D]$ is itself complete multipartite with nonzero part sizes among the z_i . Its vertex connectivity is

$$\kappa(X[D])=t-\max_i z_i. \quad (3.1)$$

Also, a vertex outside D in V_i is adjacent to every selected vertex outside V_i and to none inside V_i . Hence it has exactly $t-z_i$ neighbors in D .

Theorem 3.2 (exact occupancy criterion). An occupancy vector $(z_1,\dots,z_s)$ is feasible if and only if D meets at least two parts and

$$t-\max_i z_i \geq k, \quad t-z_i \geq m \text{ whenever } z_i < a_i. \quad (3.2)$$

Proof. The first inequality is exactly $\kappa(X[D]) \geq k$ by (3.1). The second is exactly the m -domination condition for every part containing an outside vertex. If $z_i=a_i$, there is no outside vertex in that class and no external domination inequality is required there.

Observation 3.3 (the effective occupancy threshold). Put $q=\max\{k,m\}$. Connectivity gives

$z_i \leq \max_j z_j \leq t-k$ for every class. If class i is incomplete, m -domination also gives $z_i \leq t-m$. Hence every incomplete class satisfies $z_i \leq t-q$. This explains why $q = \max\{k, m\}$ is the common capacity threshold in all subsequent phase formulas.

Let $S = \{i: z_i = a_i\}$ be the set of saturated classes and $A_S = \sum_{i \in S} a_i$. For i not in S , Observation 3.3 and incompleteness give $z_i \leq \min\{a_i - 1, t - q\}$.

Theorem 3.4 (saturation-set compression). Let S be a proper subset of $\{1, \dots, s\}$. Cardinality t is realizable with exactly the saturated classes S if and only if

$$A_S \leq t, a_i \leq t - k \text{ for } i \in S, t - A_S \leq \sum_{i \notin S} \min\{a_i - 1, t - q\}, \quad (3.3)$$

and the selected vertices meet at least two partite classes.

Proof. Necessity follows from the preceding bounds. Conversely, the last inequality says that the remainder $t - A_S$ can be distributed among the nonsaturated classes without exceeding any individual capacity $\min\{a_i - 1, t - q\}$. Together with the saturated occupancies a_i , this gives a vector satisfying (3.2). The additional two-part condition rules out the degenerate one-class induced graph.

IV. EXACT SATURATION PHASES FOR EQUAL PARTS

Now let $X = K_{(a, \dots, a)}$ have s equal parts. We first assume $a \geq 2$. Since all classes are symmetric, the identity of the saturated classes is irrelevant; only their number f matters.

Theorem 4.1 (exact phase formula). If $k > a(s-1)$, then $\gamma_{(k,m)}(X) = \infty$. Assume $k \leq a(s-1)$ and $q = \max\{k, m\}$. For $f=0$ define

$$L_0 = \text{ceil}(sq/(s-1)), \text{ feasible iff } L_0 \leq s(a-1). \quad (4.1)$$

For $1 \leq f \leq s-2$ define

$$L_f = \max\{fa, a+k, \text{ceil}(((s-f)q-fa)/(s-f-1))\}, \quad (4.2)$$

and phase f is feasible if and only if $L_f \leq as - (s-f)$. For $f=s-1$ define

$$L_{(s-1)} = \max\{(s-1)a, a+k\}, \quad (4.3)$$

which is feasible if and only if $m \leq (s-1)a$ and $L_{(s-1)} \leq as-1$. Finally $L_s = as$. Then

$$\gamma_{(k,m)}(X) = \min\{L_f : \text{phase } f \text{ is feasible}\}. \quad (4.4)$$

Proof of Theorem 4.1

Fix an intermediate phase f , $1 \leq f \leq s-2$. Let $h = s-f \geq 2$ be the number of incomplete classes. The saturated classes contribute fa selected vertices. If t is the total size, the incomplete classes must receive $R = t - fa$ vertices. By Theorem 3.4 every incomplete class has common capacity $c(t) = \min\{a-1, t-q\}$. Thus the distribution exists exactly when $R \leq h c(t)$, in addition to the connectivity requirement for a saturated class, namely $a \leq t-k$ or $t \geq a+k$.

The inequality $R \leq h(a-1)$ is equivalent to $t \leq as - h = as - (s-f)$. This is the phase upper bound: every incomplete class must leave at least one vertex outside D . The other inequality $R \leq h(t-q)$ is

$$t - fa \leq (s-f)(t-q), \text{ or equivalently}$$

$(s-f-1)t \geq (s-f)q - fa$. (4.5) Therefore the smallest integer t satisfying the lower bounds is exactly (4.2), and it belongs to the phase precisely when it does not exceed $as - (s-f)$. Conversely, if these inequalities hold, R can be distributed among h identical integer capacities, so Theorem 3.4 gives feasibility.

For $f=0$, all s classes are incomplete. The same calculation gives $(s-1)t \geq sq$ and $t \leq s(a-1)$, yielding (4.1). For $f=s-1$ only one class is incomplete. Its outside vertices, if any, see all $(s-1)a$ selected vertices in the saturated classes, so $m \leq (s-1)a$; connectivity requires $t \geq a+k$, while exact incompleteness gives $t \leq as-1$. The full phase $f=s$ is $D=V(X)$.

4.1. Boundary cases

If $a=1$, then $X = K_s$. Every t -subset induces K_t , so it is k -connected exactly when $t \geq k+1$. Every outside vertex is adjacent to all t selected vertices. Therefore, for $k \leq s-1$,

$$\gamma_{(k,m)}(K_s) = \min\{s, \max\{k+1, m\}\}, \quad (4.6)$$

and the value is ∞ when $k > s-1$. If $s=2$ and $a \geq 2$, the intermediate family is empty. The only phases are $f=0, 1, 2$, with $L_0=2q$, $L_1=\max\{a, a+k\}$, and $L_2=2a$, subject to the corresponding feasibility conditions.

4.2. Exact phase-survival thresholds

For an intermediate phase put $U_f = as - (s-f) = s(a-1) + f$. The q -dependent lower bound in (4.2) is compatible with the phase exactly when

$$\text{ceil}(((s-f)q-fa)/(s-f-1)) \leq U_f. \quad (4.7)$$

Because U_f is integral, this is equivalent to

$$(s-f)q - fa \leq (s-f-1)U_f. \text{ Hence phase } f \text{ survives exactly up to}$$

$$Q_f = \text{floor}((fa + (s-f-1)[s(a-1) + f]) / (s-f)). \quad (4.8)$$

For $f=0$ the corresponding threshold is $q \leq (s-1)(a-1)$. Thus the phase formula not only gives the optimum at a fixed pair (k, m) ; it also gives the exact disappearance threshold of every intermediate saturation pattern.

V. BALANCED MINIMIZERS AND DOMINATION ROBUSTNESS

The exact phase formula determines cardinality. We now ask how robust a size- t solution is when m is increased while t and the saturation phase are held fixed.

Definition 5.1. For a proper feasible set D with occupancies z_i , define its domination capacity by $\mu(D) = \min\{t - z_i : z_i < a\}$. (5.1) Thus D is an m' -dominating set for every $m' \leq \mu(D)$, and it fails external domination at $\mu(D) + 1$. Set $\mu(V(X)) = \infty$.

Theorem 5.2 (balanced representative). Suppose phase $f < s$ is feasible at size t . Put $h = s - f$ and $R = t - fa$. If $R = hb + r$ with $0 \leq r < h$, then there is a feasible representative whose incomplete occupancies are r copies of $b + 1$ and $h - r$ copies of b .

Proof. Feasibility of the phase means R can be distributed among h identical capacities. The balanced distribution has maximum $\text{ceil}(R/h)$, no larger than the common capacity because $R \leq hc(t)$. Hence it is feasible.

Theorem 5.3 (maximum phase robustness). Among all feasible size- t sets in phase $f < s$,

$$\max_D \mu(D) = \mu_f(t) := t - \text{ceil}((t - fa)/(s - f)). \quad (5.2)$$

Proof. The incomplete occupancies sum to R . Every integer distribution has maximum at least $\text{ceil}(R/h)$.

Since $\mu(D) = t - \max z_i$ over the incomplete classes, this gives $\mu(D) \leq t - \text{ceil}(R/h)$. The balanced representative from Theorem 5.2 attains equality.

The result gives a canonical representative for stability questions: among all minimum solutions with a prescribed number of saturated classes, the balanced one survives the largest increase in the domination requirement.

VI. CARDINALITY PLATEAUS AND THE PLATEAU FILTRATION THEOREM

Proposition 6.1. For fixed a, s, k , the function $m \rightarrow \gamma_{(k,m)}(X)$ is nondecreasing.

Proof. Every k -connected $(m+1)$ -dominating set is also k -connected m -dominating.

Let $t = \gamma_{(k,m)}(X)$, and let $F^*(m)$ be the set of feasible phases attaining t .

Theorem 6.2 (exact plateau endpoint). Define

$M^*(k, m) = \max_{(f \in F^*(m))} \mu_f(t)$, with $\mu_s(as) = \infty$. (6.1) If M^* is finite, then $\gamma_{(k,r)}(X) = t$ for every integer $m \leq r \leq M^*$, whereas $\gamma_{(k, M^*+1)}(X) > t$. If $M^* = \infty$, the full-vertex solution produces the terminal plateau.

Proof. Choose an optimal phase attaining M^* and a balanced minimum representative. By Theorem 5.3 it remains feasible through domination level M^* , so monotonicity forces $\gamma_{(k,r)} = t$ throughout that interval. If a size- t solution existed at $M^* + 1$, it would already be minimum at level m and would belong to an optimal phase with capacity at least $M^* + 1$, contradicting the definition of M^* .

Theorem 6.3 (Plateau Filtration Theorem). Let $[m_0, m_1]$ be a maximal plateau of $m \rightarrow \gamma_{(k,m)}(X)$, with common value t . Then for every r in $[m_0, m_1]$,

$$F^*(r) = \{f \in F^*(m_0) : \mu_f(t) \geq r\}. \quad (6.2)$$

Proof. Suppose f belongs to $F^*(r)$. There is a size- t feasible set of phase f at level r . Since $r \geq m_0$, the same set is feasible at level m_0 . Because $\gamma_{(k, m_0)} = t$, phase f is optimal at m_0 . This proves the inclusion from left to right. Conversely, suppose f belongs to $F^*(m_0)$ and $\mu_f(t) \geq r$. Theorem 5.3 supplies a balanced size- t representative in phase f with domination capacity at least r . It is therefore feasible at level r . Since r lies in the same cardinality plateau, $\gamma_{(k,r)} = t$, so this representative is minimum and f belongs to $F^*(r)$.

Corollary 6.4. Along a fixed plateau,

$$F^*(m_0) \supseteq F^*(m_0 + 1) \supseteq \dots \supseteq F^*(m_1). \quad (6.3)$$

Hence no new optimal phase can appear before the next cardinality jump. In particular, $\min F^*(r)$ is nondecreasing inside a plateau. This is only a local statement: after $\gamma_{(k,m)}$ increases, lower-saturation minimizers can reappear at the new cardinality.

Corollary 6.5. The phases surviving at the right endpoint m_1 are exactly the phases in $F^*(m_0)$ maximizing $\mu_f(t)$. Thus a cardinality plateau is an exact robustness filtration of its initial optimal phases.

VII. SPECIALIZATION TO GENERATING GRAPHS

Return to a finite p -group G with $G/\Phi(G)$ isomorphic to C_{p^2} , and put $A = (p-1)|\Phi(G)|$. Assume first $A \geq 2$. Substituting $s = p+1$ and $a = A$ into Theorem 4.1 gives the following.

Theorem 7.1. If $k > pA$, then $\gamma_{(k,m)}(\Delta(G)) = \infty$. Otherwise, with $q = \max\{k, m\}$, define

$$M_0 = \text{ceil}((p+1)q/p), \quad (7.1)$$

and, for $1 \leq f \leq p-1$,

$$M_f = \max\{fA, A+k, \text{ceil}(((p+1-f)q-fA)/(p-f))\}. \quad (7.2)$$

The phase $f=0$ is feasible exactly when $M_0 \leq (p+1)(A-1)$. For $1 \leq f \leq p-1$, phase f is feasible exactly when $M_f \leq (p+1)A - (p+1-f)$. For $f=p$,

$$M_p = \max\{pA, A+k\},$$

with feasibility $m \leq pA$ and $M_p \leq (p+1)A-1$. Finally $M_{(p+1)} = (p+1)A$. The minimum feasible M_f is $\gamma_{(k,m)}(\Delta(G))$.

For $1 \leq f \leq p-1$ the exact projective phase-survival threshold is

$$Q_f^*G = \text{floor}((fA + (p-f)[(p+1)(A-1) + f]) / (p+1-f)). \quad (7.3)$$

For $f \leq p$ the maximum robustness of a size- t solution in phase f is

$$\mu_f^*G(t) = t - \text{ceil}((t-fA)/(p+1-f)). \quad (7.4)$$

Corollary 7.2 (projective plateau filtration). If $[m_0, m_1]$ is a plateau of $\gamma_{(k,m)}(\Delta(G))$ with value t , then

$$F_G^*(r) = \{f \text{ in } F_G^*(m_0) : \mu_f^*G(t) \geq r\}, \quad m_0 \leq r \leq m_1. \quad (7.5)$$

Here f has a direct group-theoretic meaning: it is the number of projective classes in $G/\Phi(G)$ whose entire lift-class is selected.

7.1. Exceptional value $A=1$

The equality $A=1$ forces $p=2$ and $|\Phi(G)|=1$. Under the rank-two hypothesis, G is $C_2 \times C_2$ and $\Delta(G) = K_3$. Hence the complete-graph formula (4.6) applies directly. Thus the group-theoretic specialization contains no uncovered degenerate case.

VIII. WORKED EXAMPLE: Q_8

Let $G = Q_8$. Its Frattini subgroup is $\Phi(Q_8) = \{1, -1\}$, of order 2, and $Q_8/\Phi(Q_8)$ is isomorphic to C_2^2 . Hence $p=2$ and $A=(2-1)2=2$. Proposition 2.2 gives

$$\Delta(Q_8) \text{ is isomorphic to } K_{(2,2,2)}. \quad (8.1)$$

The three projective parts are $\{i, -i\}$, $\{j, -j\}$, and $\{k, -k\}$. Fix connectivity parameter $k=1$. Evaluating the three nonterminal phases and the full phase gives the following table. At $m=3$ the optimum is $t=4$. In phase $f=1$,

m	$\gamma_{(1,m)}$	$F^*(m)$	Structural description
1	2	$\{0\}$	two selected vertices in different parts
2	3	$\{0, 1\}$	unsaturated and one-saturated minima tie
3	4	$\{1, 2\}$	two optimal saturation patterns
4	4	$\{2\}$	the less robust phase has disappeared
5	6	$\{3\}$	all non-isolated vertices selected

$$\mu_1(4) = 4 - \text{ceil}((4-2)/2) = 3,$$

whereas in phase $f=2$,

$$\mu_2(4) = 4 - \text{ceil}((4-4)/1) = 4.$$

Therefore Theorem 6.2 predicts that the cardinality 4 persists exactly through $m=4$, and Theorem 6.3 predicts

$$F^*(3) = \{1, 2\}, F^*(4) = \{2\}. \quad (8.2)$$

Thus the cardinality plateau $[3, 4]$ contains a genuine structural transition: the less robust saturation pattern disappears while the more robust pattern remains. At $m=5$ the optimum jumps to all six non-isolated vertices. This illustrates why the filtration theorem contains information not visible in $\gamma_{(k,m)}$ alone.

IX. DISCUSSION

The exact formula in Theorem 4.1 should be viewed as the first layer of the result rather than the endpoint. The connectivity requirement couples all occupancies through $t - \max z_i$, while external domination acts only on incomplete classes. Saturation therefore changes the set of active constraints. Equal-part symmetry collapses these possibilities to finitely many phases, and balanced occupancy then solves the secondary robustness problem exactly.

The Plateau Filtration Theorem provides a stability description of the minimizers. Within a fixed cardinality plateau, optimal phases cannot appear; they can only disappear when the requested domination level exceeds their phase capacity. This filtration resets at a cardinality jump, so no global monotonicity of the minimum saturation phase is asserted. For generating graphs, the reduction to an equal multipartite graph is elementary once Burnside's basis theorem is available, but it is structurally decisive. The projective lines of the Frattini quotient become the saturation classes. Consequently, the abstract phase parameter has a direct algebraic interpretation.

The rank-two hypothesis is intrinsic for ordinary generating graphs of noncyclic p -groups. If $\dim(G/\Phi(G)) \geq 3$, no pair spans the quotient and the ordinary generating graph has no edges. Higher-rank analogues would naturally require graphs or hypergraphs based on generating tuples.

X. CONCLUSION

We obtained an exact theory of k -connected m -domination for the equal complete multipartite graphs arising as non-isolated generating graphs of rank-two finite p -groups. Saturation compression gives exact phase formulas and survival thresholds. Balanced occupancies maximize phase robustness, and these capacities determine both exact cardinality plateaus and the complete filtration of optimal phases within

each plateau. The Q_8 example exhibits the projective meaning of the theory explicitly. The results suggest two natural directions: extending the occupancy method to unequal multipartite families under additional structure, and replacing generating pairs by generating tuples in higher Frattini rank.

REFERENCES

- [1] Y. Lu and J.-M. Xu, The p -domination number of complete multipartite graphs, *Discrete Mathematics, Algorithms and Applications* 6 (2014), no. 4, 1450063. DOI: 10.1142/S1793830914500633.
- [2] Z. Nutov, Improved approximation algorithms for k -connected m -dominating set problems, *Information Processing Letters* 140 (2018), 30-33. DOI: 10.1016/j.ipl.2018.08.003.
- [3] Z. Nutov, Approximating k -connected m -dominating sets, 28th Annual European Symposium on Algorithms (ESA 2020), *LIPIcs* 173, Article 73. DOI: 10.4230/LIPIcs.ESA.2020.73.
- [4] C. Acciarri and A. Lucchini, Graphs encoding the generating properties of a finite group, *Mathematische Nachrichten* 293 (2020), 1644-1674. DOI: 10.1002/mana.201900144.
- [5] A. Lucchini and A. Maroti, Some results and questions related to the generating graph of a finite group, in *Ischia Group Theory 2008*, World Scientific, 2009. DOI: 10.1142/9789814277808_0014.
- [6] A. Lucchini and D. Nemmi, Forbidden subgraphs in generating graphs of finite groups, *Algebraic Combinatorics* 5 (2022), no. 5, 925-946. DOI: 10.5802/alco.229.
- [7] A. Lucchini and D. Nemmi, On the connectivity of the generating and rank graphs of finite groups, *Journal of Algebra* 665 (2025), 131-144. DOI: 10.1016/j.jalgebra.2024.10.030.
- [8] D. J. S. Robinson, *A Course in the Theory of Groups*, 2nd ed., Graduate Texts in Mathematics 80, Springer, New York, 1996. DOI: 10.1007/978-1-4419-8594-1.