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Properties Of Quasi-Similarity On Isometries, Co-Isometries And Partial Isometries In Hilbert Spaces

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Abstract
Classes of operators in Hilbert spaces have recently been studied in relation to quasi-similarity. Quasi-similarity has been shown to be an equivalence relation. Further, it has been established that normality, hyponormality, M-hyponormality, w-hyponormality, log-hyponormality and (p,k)-quasihyponormality are all preserved by quasi-similarity. It has also been demonstrated how quasi-similarity relates to other equivalence relations such as similarity and almost similarity. Specifically, similarity implies quasi-similarity which in turn implies almost similarity under certain conditions. That is, if $A, B \in B(H)$ are normal operators where H is an infinite dimensional Hilbert space such that A and B are quasi-similar, then A and B are almost similar. However, there exist other operators whose quasi-similarity properties have not been established. In this study, we focus on isometries, co-isometries and partial isometries operators and determine their quasi-similarity properties.

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I.Introduction

Let H denote a complex Hilbert space and $B(H)$ denote a banach algebra of bounded linear operators on a Hilbert space H. An operator $T \in B(H)$ is a structure preserving map or a symbol that represents a transformation that can be performed on a function. If $P \in B(H)$, then P^* denotes the adjoint of an operator P and P^{-1} denotes the inverse of P. If $A, B \in B(H)$ then $(AB)^{-1} = B^{-1}A^{-1}$. Sitati *et al*. (2012) demonstrated that quasi-similarity is an equivalence relation on the classes of all operators. Sitati *et al*. (2015) gave the condition under which similarity implies quasi-similarity which in turn implies almost similarity. Kamau *et al*. (2024) noted that quasi-similarity preserves normality, hyponormality and M-hyponormality of an operator. Properties of partial isometries have also been determined on other classes of equivalence relations such as almost similarity, unitary equivalence and unitary quasi-equivalence. Luketero & Khalagai (2020) proved that unitary equivalence preserves partial isometries of operators. Jibril (1996) established that almost similar operators preserve isometries and partial isometries. Lilian *et al*. (2023) established that unitary quasi-equivalence operators preserves isometry, co-isometry and partial isometry. Lilian *et al*. (2023) also established that if $K, L \in B(H)$ are self-adjoint and unitary quasi-equivalent operators and K^2 is partial isometric, then L^2 is also partial isometric.

We also recall the following key definitions that are useful in the subsequent sections.

Definition 1.1: A Hilbert space *H* (Halmos, 2017). A Hilbert space H is a complete inner product space.

Definition 1.2: Quasi-affinity operator (Nzimbi *et al*., 2016). An operator $X \in B(H, K)$ is said to be quasi-affinity or quasi-invertible if it is injective with dense range.

Definition 1.3: Quasi-similar operators (Nagy *et al*., 2010). Two operators S and T of bounded linear operators $B(H)$ and $B(K)$ on Hilbert spaces H and K respectively are said to be quasi-similar if there exist quasi-affinity operators $X \in B(H, K)$ and $Y \in B(K, H)$ satisfying the equations$XS = TX$ and $SY = YT$.

Definition 1.4: (Nzimbi & Wanyonyi, 2020). An operator $A \in B(H)$ is said to be

(i) Binormal if $(AA^*)(A^*A) = (A^*A)(AA^*)$.
(ii) Isometry if $A^*A = I$.
(iii) Co-Isometry if $AA^* = I$.
(iv) Unitary if $A^*A = AA^* = I$.

Definition 1.5: A partial isometry (Halmos, 2017). An operator $S \in B(H)$ is said to be partial isometry if $S = SS^*S$. that is, S^*S and SS^* are projections.

Definition 1.6: An orthogonal projection (Halmos, 2017). An operator $S \in B(H)$ is said to be an orthogonal projection if $S^2 = S$ and $S^* = S$. That is, S is idempotent and self adjoint.

II.Methodology

In order to achieve the desired results, the following outcomes were useful:

Theorem 2.1: (Helmberg, 2008). Let A and B be bounded linear operators on a Hilbert space H. Then

(i) $A^{**} = A$.
(ii) $(\propto A)^* = \overline{\propto} A^*$.
(iii) $(A + B)^* = A^* + B^*$
(iv) $(AB)^* = B^*A^*$
(v) If A is invertible, then so is A^* and $(A^*)^{-1} = (A^{-1})^*$.

Theorem 2.2: (Salhi & Zerouali, 2020). For $S \in B(H)$, the following statements are equivalent;

(i) S is a partial isometry,
(ii) S^* is a partial isometry,
(iii) SS^* is a projection,
(iv) S^*S is a projection,
(v) $S^*SS^* = S^*$,
(vi) $SS^*S = S$.

Theorem 2.3: (Luketero &Khalagai, 2020). Let $A \in B(H)$ be a partial isometry and $B \in B(H)$ be any other operator such that either,

i) $A = UBU^*$ where U is an isometry or
ii) $A = U^*BU$ where U is a co-isometry

Then B is a partial isometry.

Lemma 2.4: (Luketero & Khalagai, 2020). Let $S \in B(H)$ be a partial isometry operator and $T \in B(H)$ be any other operator such that, either $S = UTU^*$ or $S = U^*TU$ where U is unitary. Then $T \in B(H)$ is also a partial isometry operator.

Lemma 2.5: (Lilian et al., 2023). If $S, T \in B(H)$ are self-adjoint and unitary quasi-equivalent operators and S^2 is partial isometric, then T^2 is also partial isometric.

III.Main Results

In Theorem 3.1, we present the equivalence property of quasi-similarity on isometric operators.

Theorem 3.1

Let $A, B \in B(H)$ be quasi-similar operators, then A is an isometry if and only if B is an isometry.

Proof. Suppose that an operator A is an isometry. Since A and B are quasi-similar, then there exist two operators X and Y that are quasi-affine/ quasi-invertible such that

$$XA = BX \tag{1}$$

and

$$AY = YB \tag{2}$$

Consider equation (1), since X is quasi-invertible, then multiplying both sides from the right by X^{-1} we have

$$XAX^{-1} = BXX^{-1}$$
$$\Rightarrow B = XAX^{-1}$$

Consequently, we have

$$B^* = (XAX^{-1})^* = (X^{-1})^*A^*X^*$$

and

$$B^*B = (X^{-1})^*A^*X^*XAX^{-1}$$

Since X is quasi-invertible, then $X^*X = I$ and hence

$$B^*B = (X^{-1})^*A^*IAX^{-1} = (X^{-1})^*A^*AX^{-1}$$

But A is an isometry, thus by definition $A^*A = I$. Hence

$$B^*B = (X^{-1})^*X^{-1} \tag{3}$$

Since X is invertible then $(X^{-1})^* = (X^*)^{-1}$ and equation (3) becomes

$$B^*B = (X^*)^{-1}X^{-1} = (XX^*)^{-1} = I$$

Thus $B^*B = I$ which implies that B is an isometry.

Also considering equation (2), since Y is quasi-invertible, then multiplying both sides of equation (2) from the left by Y^{-1} we have

$$Y^{-1}AY = Y^{-1}YB$$
$$\Rightarrow B = Y^{-1}AY$$

Taking the adjoint of B, we have

$$B^* = (Y^{-1}AY)^* = Y^*A^*(Y^{-1})^*$$

and consequently,

$$B^*B = Y^*A^*(Y^{-1})^*Y^{-1}AY = Y^*A^*(Y^*)^{-1}Y^{-1}AY = Y^*A^*(YY^*)^{-1}AY = Y^*A^*AY$$

Since A is an isometry, then by definition

$$A^*A = I$$

Hence

$$B^*B = Y^*IY = Y^*Y$$

But Y is quasi-invertible, hence

$$B^*B = Y^*Y = I$$

Thus

$$B^*B = I$$

which implies that B is an isometry.
Conversely, suppose that B is an isometry. Since A and B are quasi-similar, then there exist two operators X and Y that are quasi-affine/ quasi-invertible such that

$$XA = BX \quad (4)$$

and

$$AY = YB \quad (5)$$

Considering equation (4), since X is quasi-invertible, then multiplying both sides of equation (4) from the left by X^{-1} we have

$$X^{-1}XA = X^{-1}BX$$
$$\Rightarrow A = X^{-1}BX$$

Also,

$$A^* = (X^{-1}BX)^* = X^*B^*(X^{-1})^*$$

Thus,

$$A^*A = X^*B^*(X^{-1})^*X^{-1}BX = X^*B^*(X^*)^{-1}X^{-1}BX = X^*B^*(XX^*)^{-1}BX$$

But $XX^* = I$, hence

$$A^*A = X^*B^*IBX = X^*B^*BX$$

But B is an isometry, thus by definition $B^*B = I$ and hence

$$A^*A = X^*IX = X^*X$$

But X is quasi-invertible, hence

$$A^*A = X^*X = I$$

Therefore

$$A^*A = I$$

which implies that A is an isometry.
Also considering equation (5), since Y is quasi-invertible, then multiplying both sides of equation (5) from the right by Y^{-1} we have

$$AYY^{-1} = YBY^{-1}$$
$$\Rightarrow A = YBY^{-1}$$

Consequently,

$$A^* = (YBY^{-1})^* = (Y^{-1})^*B^*Y^*$$

and

$$A^*A = (Y^{-1})^*B^*Y^*YBY^{-1} = (Y^{-1})^*B^*IBY^{-1} = (Y^{-1})^*B^*BY^{-1}$$

But

$$B^*B = I$$

Hence

$$A^*A = (Y^{-1})^*IY^{-1} = (Y^{-1})^*Y^{-1} = (Y^*)^{-1}Y^{-1} = (YY^*)^{-1} = I$$

That is,

$$A^*A = I$$

implying that A is an isometry.

Having considered the equivalence property of quasi-similarity on isometric operators in Theorem 3.1, and utilizing the relationship between isometry and co-isometry we explore results of theorem 3.1 in Theorem 3.2.

Theorem 3.2
Let $A, B \in B(H)$ be quasi-similar operators, then A is co-isometry if and only if B is co-isometry.

Proof. Suppose that an operator A is co-isometry. Since A and B are quasi-similar, then there exist two operators X and Y that are quasi-affine or quasi-invertible such that

$$XA = BX \quad (6)$$

and

$AY = YB \qquad (7)$

Considering equation (6), since X is quasi-invertible, then multiplying both sides of equation (6) from the right by X^{-1} we have

$$XAX^{-1} = BXX^{-1}$$
$$\Rightarrow B = XAX^{-1}$$

and the adjoint of B becomes

$$B^* = (XAX^{-1})^* = (X^{-1})^* A^* X^*$$

Consequently,

$$\begin{aligned} BB^* &= XAX^{-1}(X^{-1})^* A^* X^* \\ &= XAX^{-1}(X^*)^{-1} A^* X^* \\ &= XA(X^*X)^{-1} A^* X^* \\ &= XAA^* X^* \end{aligned}$$

But A is a co-isometry, that is, $AA^* = I$ and hence

$$BB^* = XIX^* = XX^* = I$$

Therefore

$$BB^* = I$$

implying that B is a co-isometry.

Also considering equation (7), since Y is quasi-invertible, then multiplying both sides of equation (7) from the left by Y^{-1} we get

$$Y^{-1}AY = Y^{-1}YB$$
$$\Rightarrow B = Y^{-1}AY$$

Hence

$$B^* = (Y^{-1}AY)^* = Y^*A^*(Y^{-1})^*$$

and

$$\begin{aligned} BB^* &= Y^{-1}AYY^*A^*(Y^{-1})^* \\ &= Y^{-1}AIA^*(Y^{-1})^* \\ &= Y^{-1}AA^*(Y^{-1})^* \end{aligned}$$

But A is co-isometry, thus by definition $AA^* = I$. Hence

$$BB^* = Y^{-1}(Y^{-1})^* = Y^{-1}(Y^*)^{-1} = (Y^*Y)^{-1} = I$$

That is,

$$BB^* = I$$

and therefore B is also co-isometry.

Conversely, suppose that B is a co-isometry. Since A and B are quasi-similar, then by definition there exist two operators X and Y that are quasi-affine/ quasi-invertible such that

$$XA = BX \qquad (8)$$

and

$AY = YB \qquad (9)$

Considering equation (8), since X is quasi-invertible, then by multiplying both sides of equation (8) from the left by X^{-1} we have

$$X^{-1}XA = X^{-1}BX$$
$$\Rightarrow A = X^{-1}BX$$

Taking the adjoint of $A = X^{-1}BX$, we have

$$A^* = (X^{-1}BX)^* = X^*B^*(X^{-1})^*$$

As a result,

$$\begin{aligned} AA^* &= X^{-1}BXX^*B^*(X^{-1})^* \\ &= X^{-1}BIB^*(X^{-1})^* \\ &= X^{-1}BB^*(X^{-1})^* \\ &= X^{-1}BB^*(X^*)^{-1} \end{aligned}$$

But B is a co-isometry, hence $BB^* = I$. Therefore

$$AA^* = X^{-1}I(X^*)^{-1} = X^{-1}(X^*)^{-1} = (X^*X)^{-1} = I$$

That is,

$$AA^* = I$$

which implies that A is a co-isometry.

Also considering equation (9), since Y is quasi-invertible, then multiplying both sides of equation (9) from the right by Y^{-1} we have

$$AYY^{-1} = YBY^{-1}$$
$$\Rightarrow A = YBY^{-1}$$

Consequently,

$$A^* = (YBY^{-1})^* = (Y^{-1})^*B^*Y^*$$

and therefore

$$\begin{aligned} AA^* &= YBY^{-1}(Y^{-1})^*B^*Y^* \\ &= YBY^{-1}(Y^*)^{-1}B^*Y^* \\ &= YB(Y^*Y)^{-1}B^*Y^* \\ &= YBIB^*Y^* \\ &= YBB^*Y^* \end{aligned}$$

But B is a co-isometry, thus by definition $BB^* = I$. Hence

$$AA^* = YIY^* = YY^* = I$$

Thus

$$AA^* = I$$

implying that A is also a co-isometry.

Similary, the results of Theorem 3.1 and Theorem 3.2 are considered in the case of partial isometric operators in Theorem 3.3.

Theorem 3.3
Let $A, B \in B(H)$ be quasi-similar operators and let A be a partial isometry, then the operator B is also a partial isometry operator.

Proof. Since A and B are quasi-similar operators, then there exist two operators X and Y that are quasi-affine/ quasi-invertible such that

$$XA = BX \qquad (10)$$

and

$$AY = YB \qquad (11)$$

Considering equation (10), since X is quasi-invertible then multiplying both sides of equation (10) from the left by X^{-1} we have

$$X^{-1}XA = X^{-1}BX$$
$$\Rightarrow A = X^{-1}BX$$

which follows that,

$$A^* = (X^{-1}BX)^* = X^*B^*(X^{-1})^*$$

Since A is a partial isometry, then by definition A^*A is a projection, thus by definition of a projection it implies that

$$(A^*A)^2 = A^*A$$

This implies that

$$\begin{aligned} A^*A &= X^*B^*(X^{-1})^*X^{-1}BX \\ &= X^*B^*(X^*)^{-1}X^{-1}BX \\ &= X^*B^*(XX^*)^{-1}BX \\ &= X^*B^*IBX \\ &= X^*B^*BX \end{aligned}$$

Also,

$(A^*A)^2 = X^*B^*BXX^*B^*BX = X^*B^*BB^*BX = X^*(B^*B)^2X$

Thus

$$X^*B^*BX = X^*(B^*B)^2X \qquad (12)$$

Multiplying both sides of equation (12) from the left and from the right by X and X^{-1} respectively, we get

$$XX^*B^*BXX^{-1} = XX^*(B^*B)^2XX^{-1}$$

But $XX^* = I$ and $XX^{-1} = I$.

Thus

$$IB^*BI = I(B^*B)^2I$$

That is,

$$B^*B = (B^*B)^2$$

This implies that B^*B is a projection and thus by definition of partial isometry, B is also a partial isometry operator.

Also considering equation (11), since Y is quasi-invertible, then multiplying both sides of equation (11) from the right by Y^{-1} we get

$$AYY^{-1} = YBY^{-1}$$

$$\Rightarrow A = YBY^{-1}$$

which follows that

$$A^* = (YBY^{-1})^* = (Y^{-1})^* B^* Y^*$$

Since A is a partial isometry, then by definition A^*A is a projection, thus by definition of a projection it implies that

$$(A^*A)^2 = A^*A$$

Consequently, we have

$$\begin{aligned} A^*A &= (Y^{-1})^* B^* Y^* Y B Y^{-1} \\ &= (Y^{-1})^* B^* I B Y^{-1} \\ &= (Y^{-1})^* B^* B Y^{-1} \end{aligned}$$

and

$$\begin{aligned} (A^*A)^2 &= (Y^{-1})^* B^* B Y^{-1} (Y^{-1})^* B^* B Y^{-1} \\ &= (Y^{-1})^* B^* B Y^{-1} (Y^*)^{-1} B^* B Y^{-1} \\ &= (Y^{-1})^* B^* B (Y^* Y)^{-1} B^* B Y^{-1} \\ &= (Y^{-1})^* B^* B B^* B Y^{-1} \\ &= (Y^{-1})^* (B^* B)^2 Y^{-1} \end{aligned}$$

Thus

$$(Y^{-1})^* B^* B Y^{-1} = (Y^{-1})^* (B^* B)^2 Y^{-1} \quad (13)$$

Multiplying both sides of equation (13) from the left and from the right by Y^{-1} and Y respectively, we get

$$\begin{aligned} Y^{-1}(Y^{-1})^* B^* B Y^{-1} Y &= Y^{-1}(Y^{-1})^* (B^* B)^2 Y^{-1} Y \\ \Rightarrow Y^{-1}(Y^*)^{-1} B^* B Y^{-1} Y &= Y^{-1}(Y^*)^{-1} (B^* B)^2 Y^{-1} Y \\ \Rightarrow (Y^* Y)^{-1} B^* B Y^{-1} Y &= (Y^* Y)^{-1} (B^* B)^2 Y^{-1} Y \\ \Rightarrow I B^* B I &= I (B^* B)^2 I \end{aligned}$$

That is,

$$\Rightarrow B^* B = (B^* B)^2$$

This implies that B^*B is a projection, thus by definition of partial isometry it means that B is a partial isometry operator.

As an immediate consequence of Theorem 3.3, as well as applying the concept of symmetric property, this study established that the equivalence relation of quasi-similarity to square operators of partial isometry mappings as illustrated in Theorem 3.4.

Theorem 3.4

Let $A, B \in B(H)$ be symmetry and quasi-similar operators. If A^2 is a partial isometry, then B^2 is also a partial isometry operator.

Proof. Since A and B are quasi-similar, then there exist two operators X and Y that are quasi-affine/quasi-invertible such that

$$XA = BX \quad (14)$$

and

$$AY = YB \quad (15)$$

Since A^2 is a partial isometry, we have

$$A^2 = A^2 (A^*)^2 A^2$$

and by projection property we also have

$$AA^* = AA = A^2$$

Considering equation (14), since X is quasi-invertible, then multiplying both sides of equation (14) by X^{-1} from the left we have

$$\begin{aligned} X^{-1} X A &= X^{-1} B X \\ \Rightarrow A &= X^{-1} B X \end{aligned}$$

which implies that

$$A^* = (X^{-1} B X)^* = X^* B^* (X^{-1})^*$$

and

$$\begin{aligned} A^2 &= (X^{-1} B X)^2 \\ &= X^{-1} B X X^{-1} B X \\ &= X^{-1} B I B X \\ &= X^{-1} B B X \\ &= X^{-1} B^2 X \end{aligned}$$

But

$$A^2 = A^2(A^2)^*A^2$$

and

$$(A^*)^2 = (A^2)^*$$
$$\Rightarrow (A^2)^* = (A^*)^2$$
$$= X^*B^*(X^{-1})^*X^*B^*(X^{-1})^*$$
$$= X^*B^*(XX^{-1})^*B^*(X^{-1})^*$$
$$= X^*B^*IB^*(X^{-1})^*$$
$$= X^*B^*B^*(X^{-1})^*$$
$$= X^*(B^*)^2(X^{-1})^*$$

Thus

$$X^{-1}B^2X = X^{-1}B^2XX^*(B^*)^2(X^{-1})^*X^{-1}B^2X$$
$$= X^{-1}B^2I(B^*)^2(X^*)^{-1}X^{-1}B^2X$$
$$= X^{-1}B^2(B^*)^2(XX^*)^{-1}B^2X$$
$$= X^{-1}B^2(B^*)^2IB^2X$$
$$= X^{-1}B^2(B^*)^2B^2X$$
$$XX^{-1}B^2XX^{-1} = XX^{-1}B^2(B^*)^2B^2XX^{-1}$$

But $XX^{-1} = X^{-1}X = I$

$$\Rightarrow\ IB^2I = IB^2(B^*)^2B^2I$$
$$\Rightarrow\ B^2 = B^2(B^*)^2B^2$$
$$= B^2(B^2)^*B^2$$

Hence B^2 is also a partial isometry operator.

Also considering equation (15), since Y is quasi-invertible, then multiplying both sides of equation (15) by Y^{-1} from the right we have

$$AYY^{-1} = YBY^{-1}$$
$$\Rightarrow A = YBY^{-1}$$

which follows that

$$A^* = (YBY^{-1})^* = (Y^{-1})^*B^*Y^*$$

and

$$A^2 = (YBY^{-1})^2$$
$$= YBY^{-1}YBY^{-1}$$
$$= YBIBY^{-1}$$
$$= YBBY^{-1}$$
$$= Y(B)^2Y^{-1}$$

But $A^2 = A^2(A^2)^*A^2$

Note that

$$(A^2)^* = (A^*)^2$$
$$\Rightarrow (A^2)^* = (A^*)^2$$
$$= (Y^{-1})^*B^*Y^*(Y^{-1})^*B^*Y^*$$
$$= (Y^{-1})^*B^*(Y^{-1}Y)^*B^*Y^*$$
$$= (Y^{-1})^*B^*IB^*Y^*$$
$$= (Y^{-1})^*B^*B^*Y^*$$
$$= (Y^{-1})^*(B^*)^2Y^*$$

Thus

$$Y(B)^2Y^{-1} = Y(B)^2Y^{-1}(Y^{-1})^*(B^*)^2Y^*Y(B)^2Y^{-1}$$
$$= Y(B)^2Y^{-1}(Y^*)^{-1}(B^*)^2I(B)^2Y^{-1}$$
$$= Y(B)^2(Y^*Y)^{-1}(B^*)^2(B)^2Y^{-1}$$
$$= Y(B)^2I(B^*)^2(B)^2Y^{-1}$$
$$= Y(B)^2(B^*)^2(B)^2Y^{-1}$$

and consequently

$$Y^{-1}Y(B)^2Y^{-1}Y = Y^{-1}Y(B)^2(B^*)^2(B)^2Y^{-1}Y$$
$$\Rightarrow I(B)^2I = I(B)^2(B^*)^2(B)^2I$$

Thus

$$(B)^2 = (B)^2(B^*)^2(B)^2$$

Therefore B^2 is a partial isometry operator.

IV.Conclusion

From the above results, it is established that quasi-similar operators preserve isometries, co-isometries and partial isometries.

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