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Stability and Bifurcation Analysis of Nonlinear Public Debt Dynamics with Regulatory Intervention and Political Influence.

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ABSTRACT
In this article we present a dynamical analysis of public debt that integrates regulatory interference and political pressure. We have modified the classical debt-growth model by including saturation effects, regulatory inertia, and bounded political pressures. Using Runge-Kutta 4th order integration, we have investigated the stability and bifurcation characteristics of the proposed financial dynamics system to understand how key parameters influence the system's long-term behaviour. The results highlight critical thresholds for regulatory strategies and political pressures, providing intuitions for debt management and economic policy design.

Keywords: *Stability; Bifurcation; Debt-Growth, Political intervention.*

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I. Introduction

Public debt dynamics and fiscal stability have become very relevant areas of research in nonlinear financial dynamical systems due to increasing debt accumulation and economic uncertainty in modern economies. Excess debt growth may significantly impact institutional regulations and push a stable economic system towards instability.

The Goodwin growth model and Kaldor business cycle are considered to be classical contributions in financial dynamics, which describe nonlinear interactions in economic systems [1],[2]. Furthermore, extended nonlinear methods for financial stability, bifurcation, and chaos have been discussed in [3–10].

Lorenz [8] and Strogatz [9] demonstrated how nonlinear feedback mechanisms may generate oscillations, bifurcation, and chaotic motion in a stable dynamical system. A mathematical framework for bifurcation theory was developed to analyse stability transitions [11]. Bifurcation and chaos theory have provided important insights into the stability and dynamical behaviour of complex models [12]. It has been observed that small changes in system parameters may lead to instability and bifurcation in a stable system [13],[14]. In addition to bifurcation and stability, a nonlinear system may exhibit chaos under parameter variation, as reported by Qin Gao and Junhai Ma [15].

Sovereign economic instability may be accelerated by excess debt growth, high borrowing costs, and fiscal deficits. It is emphasised that public debt sustainability depends on the dynamic interaction between debt accumulation and interest rates. Also, public debt has a nonlinear effect on economic growth. Studies on nonlinear public debt dynamics with other state variables have reported that excessive debt accumulation may destabilize fiscal systems and generate complex behaviour [16],[18]. The adoption of monetary policies is necessary, in addition to fiscal rules, to control the pattern of debt ratios. Bacchiocchi, Bellocchi, Bischi, and Travaglini, reported that active budget adjustment rules have a stabilizing effect on the dynamical relationship between the public debt ratio and inflation [19],[20]. The impact of budget deficit conditions on public debt and its solvency during pandemics and global financial crises has been discussed in [21],[22].

By introducing delay factors, several studies highlighted the importance of determining essential parameters using Hopf bifurcation, Hopf–pitchfork bifurcation, and chaos analysis [23–25]. These studies indicate that parameter variation plays a crucial role in determining the qualitative behaviour and stability transitions of nonlinear financial systems.

Although there have been significant advances in financial dynamics involving the combined effects of several state variables such as public debt, regulatory intervention, and other economic variables, the role of political pressure has received limited attention in the analysis of financial dynamics.

Keeping this gap in mind, the present study attempts to demonstrate the nonlinear behaviour arising from the interaction among public debt, regulatory intervention, and political pressure. Using stability theory and bifurcation analysis, the proposed framework investigates long-term fiscal stability and the occurrence of Hopf bifurcation. Furthermore, numerical simulations based on the RK4 method are carried out to validate the theoretical analysis.

II. Model Overview:

In this model we describe the dynamics of three key variables where $R(t)$, $D(t)$ and $P(t)$ represent the intensity of regulatory intervention by the authority, the public debt accumulated by the Government and political and economic pressure influenced by regulation and debt management respectively for an instant, $t > 0$.

Then the proposed model is expressed by a system of three coupled ordinary differential equations.

$$\begin{aligned} \dot{R} &= P + (D-\alpha)R + M - \eta R \\ \dot{D} &= rD\left(1 - \frac{D}{K}\right) - \beta P \\ \dot{P} &= \frac{\gamma D}{1+\gamma D} - R - \nu P \end{aligned} \tag{2.1}$$

The first equation of (2.1) characterize how political pressure act as a driving force increasing regulatory intervention, more political pressure pushes the regulators to impose more regulations strongly. Here $(D-\alpha)R$ is the interaction between debt and existing regulations. If $D > \alpha$ then it amplifies regulation that reflects the high debt and strict regulatory actions. If $D < \alpha$ then the regulatory effort may reduce. M here is an external support or baseline regulatory effort which is independent of all other variables, $-\eta R$ is a natural decay or limits on how fast regulation can increase.

The second equation is the interaction between the debt and economic capacity. Debt tends to grow but it is influenced by economic limits and political pressure. $rD(1-\frac{D}{K})$ is the logistic growth of debt with fundamental growth rate and carrying capacity K. $-\beta P$ is the political intervention which reduces the debt accumulation.

The third equation characterizes how political pressure build up with worsening debt and regulatory interaction but is controlled by effective regulations and natural political cycle. The term $\frac{\gamma D}{1+\gamma D}$ indicates how political pressure raises with the interaction of regulations and debt but saturates to avoid runway growth bounded by γ. $-R$ reduces political pressure as regulatory measure may claim political concern. $-\nu P$ is a natural decay of political pressure over time.

Parameters:

- r: debt growth rate
- K: maximum sustainable debt
- α: debt threshold for regulation
- M: external support
- β: influence of political pressure on debt
- γ: feedback from regulation-debt interaction on political pressure
- ν: natural decay of political pressure
- η: regulatory inertia

To better understand the model formulation (2.1) we would like to refer the works [7],[26].

III. Equilibrium:

To analyse the dynamical behaviour of the system (2.1), we first investigate the equilibria of it by equating the first derivative of (2.1) to zero and solving we get

$$P = \frac{r}{\beta} D\left(1 - \frac{D}{K}\right)$$
$$R = \frac{\gamma D}{1+\gamma D} - \frac{\nu r}{\beta} D\left(1 - \frac{D}{K}\right). \text{ and}$$
$$F(D) = a_4 D^4 + a_3 D^3 + a_2 D^2 + a_1 D + a_0 = 0$$

Where

$$a_4 = -\frac{r}{\beta}\frac{\gamma\nu}{K}$$
$$a_3 = \gamma - \frac{r}{\beta}\left[\frac{\nu}{K} + \gamma\nu - \frac{\gamma\nu(\alpha+\eta)}{K}\right]$$
$$a_2 = -\gamma(\alpha+\eta) - \frac{r}{\beta}\left[1 - \nu(\alpha+\eta) - \frac{\gamma}{K} + \frac{\gamma\nu(\alpha+\eta)}{K}\right]$$
$$a_1 = -\frac{r}{\beta}[\nu(\alpha+\eta) - 1] - \gamma M$$
$$a_0 = M$$

Hence we obtained the following proposition for existence of equilibrium points.

Proposition 3.1

The system has

(i) A trivial debt free equilibrium point $E_0\ (0,0,0)$ if and only if $M = 0$
(ii) An interior equilibrium $E^*(R^*, D^*, P^*)$ whenever the quadric polynomial

$F(D) = a_4 D^4 + a_3 D^3 + a_2 D^2 + a_1 D + a_0$ has at least one positive real root, $D^* > 0$ $\left(a_0, a_1, a_2, a_3, a_4 \text{ are defined above.}\right)$

The corresponding non trivial equilibrium is

$$E^* = \left(\frac{\gamma D^*}{1+\gamma D^*} - \frac{\nu r}{\beta} D^*\left(1 - \frac{D^*}{K}\right), D^*, \frac{r}{\beta} D^*\left(1 - \frac{D^*}{K}\right)\right)$$

IV. Stability and Bifurcation analysis

In this section we investigate the stability of the system (2.1) at the interior equilibrium point $E^*(R^*, D^*, P^*)$. We shift the equilibrium point $E^*(R^*, D^*, P^*)$ to $E_0\ (0,0,0)$. We obtain the Jacobian matrix,

$$J_{E_0(R,D,P)} = \begin{pmatrix} D - \alpha - \eta & R & 1 \\ 0 & r(1 - 2D/K) & -\beta \\ -1 & \frac{\gamma}{(1+\gamma D)^2} & -\nu \end{pmatrix}$$

Again
$$J_{E^*(R^*,D^*,P^*)} = \begin{pmatrix} D^* - \alpha - \eta & \frac{\gamma D^*}{1+\gamma D^*} - \frac{\nu r}{\beta} D^*\left(1 - \frac{D^*}{K}\right) & 1 \\ 0 & r(1 - 2D^*/K) & -\beta \\ -1 & \frac{\gamma}{(1+\gamma D^*)^2} & -\nu \end{pmatrix}$$

Now the characteristic equation governing the behaviour of the system (2.1) is

$$|J_{E^*} - \lambda I| = 0$$

Substituting the matrix J_{E^*} we get

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \qquad (4.1)$$

Where

$$a_1 = \nu - r\left(1 - \frac{2D^*}{K}\right) - (D^* - \alpha - \eta)$$
$$a_2 = (D^* - \alpha - \eta)\left[r\left(1 - \frac{2D^*}{K}\right) - \nu\right] - \frac{\beta\gamma}{(1+\gamma D^*)^2} + \nu r\left(1 - \frac{2D^*}{K}\right) + 1$$
$$a_3 = -(D^* - \alpha - \eta)\left[\frac{\beta\gamma}{(1+\gamma D^*)^2} - \nu r\left(1 - \frac{2D^*}{K}\right)\right] - \frac{\beta\gamma D^*}{1+\gamma D^*} + \nu r D^*\left(1 - \frac{D^*}{K}\right) - r\left(1 - \frac{2D^*}{K}\right)$$

The proposed system is stable at E^* if a_1, a_2, a_3 satisfies Routh–Hurwitz Criterion.[27]

Now if $\lambda = i\omega, \omega > 0$ be a root of the equation (4.1)
Then $(i\omega)^3 + a_1(i\omega)^2 + a_2(i\omega) + a_3 = 0$

Simplifying and separating real and imaginary parts, we get

$a_3 = a_1\omega^2$ and $\omega^2 = a_2$ which implies $a_1 a_2 = a_3$

Therefore, the condition for occurrence of Hopf bifurcation is $a_1 a_2 = a_3$

The critical value $r = r_c$ where the Hopf bifurcation occurs can be determined by

$$a_1(r_c)a_2(r_c) = a_3(r_c)$$

Where the characteristic equation possesses a pair of purely imaginary eigen values

$\lambda_{1,2} = \pm i\omega$ where $\omega = \sqrt{a_2}$, and $\lambda_3 < 0$.

Now to determine the presence of Hopf bifurcation around E^* we must check the transversality condition. Differentiating (4.1) with respect to r, we get

$$\frac{d\lambda}{dr} = -\frac{\frac{da_1}{dr}\lambda^2 + \frac{da_2}{dr}\lambda + \frac{da_3}{dr}}{3\lambda^2 + 2a_1\lambda + a_2}$$

Considering $\lambda = i\omega, \omega > 0$

$$\frac{d\lambda}{dr} = -\frac{-\omega^2\frac{da_1}{dr} + i\omega\frac{da_2}{dr} + \frac{da_3}{dr}}{-3\omega^2 + 2ia_1\omega + a_2}$$

For occurrence of Hopf bifurcation, $\omega^2 = a_2$

Thus

$$\frac{d\lambda}{dr} = -\frac{-\omega^2 a_1' + i\omega a_2' + a_3'}{-3\omega^2 + 2ia_1\omega + a_2} \text{ where } a_j' = \frac{da_j}{dr}\ ; j = 1,2,3$$
$$Re\left(\frac{d\lambda}{dr}\right)\Big|_{r=r_c} = -\frac{2a_2(a_2 a_1' + a_1 a_2' - a_3')}{4a_2^2 + 4a_1^2\omega^2} > 0 \text{ provided } \frac{d}{dr}(a_1 a_2 - a_3)\Big|_{r=r_c} < 0$$

Which shows that, as the debt growth rate increases through the critical value r_c the pair of complex conjugate eigen values crosses imaginary axis. As a result, the interior equilibrium loses stability and Hopf bifurcation occurs

Thus, we get the following result

Theorem 1. The public debt system at the interior equilibrium E^* is

(i) Stable if $a_1 > 0, a_3 > 0, and\ a_1 a_2 > a_3$

(ii) Undergoes Hopf bifurcation around E^* at $r = r_c$ when
   (a) $a_1 > 0, a_2 > 0, a_3 > 0\ and\ a_1 a_2 = a_3$
   (b) $\frac{d}{dr}(a_1 a_2 - a_3)\Big|_{r=r_c} < 0$.

Remark: The eigenvalues of the Jacobian matrix determine the local stability properties of the equilibrium point, while the associated eigenvectors determine the local orientation of trajectories near the equilibrium state.[9],[11].

V. Sensitivity analysis and model significance:

To validate the theoretical analysis, we illustrate the dynamical behaviour of the system, the proposed model is solved numerically using classical fourth order Runge-Kutta(RK4) Method.

In order to perform it we take a set of parameter values given in table 1 with the initial condition $R(0) = 0.5, D(0) = 1, P(0) = 0.5$ the numerical investigation is performed with step size $h = 0.0001$ over the finite interval time $t \in [0,80]$ aligning with the Theorem 1. With the parameter values and the above-mentioned initial condition, the eigen value of the characteristic equation associated with $E^*(0.0000002,0.0000095,0.0000001)$ are $(-4.4761 + 0.85741i, -4.4761 - 0.85741i, -0.0077 + 0.0000i)$ indicating the system stability around E^*. The numerical parametrization is chosen within the range that are commonly adopted to illustrate the dynamical behaviour of a nonlinear financial dynamical system. [4],[5],[7],[28],[29]. The values are assumed such that the stability condition holds and the simulation remain bounded.

The simulation shows that, increase in η and ν enhance stability. It supresses the fluctuation in institutional regulations and political intervention. Also, it is observed that increased in the external support$M$ improves the overall stability by strengthening institutional regulations.

For a smaller value of r the system remains stable and the debt growth variable $D(t)$ evolve

Smoothly. On the other hand, as increase in r the interaction between debt accumulation and regulation become stronger and as a result of this the system moves towards instability. The parameter γ when it increases amplifies political pressure generated by rising debt and increase fiscal vulnerability.

At critical debt growth rate, the bifurcation analysis shows that excessive debt growth can destabilize the system and weaken regulation control. The simulation against the given parameter set is portrayed in fig:1, fig:2, fig:3 and fig:4.

Table1

parameter	value
α	2
η	3
β	1
γ	0.2
ν	4
K	8
r	0.04
M	0.2

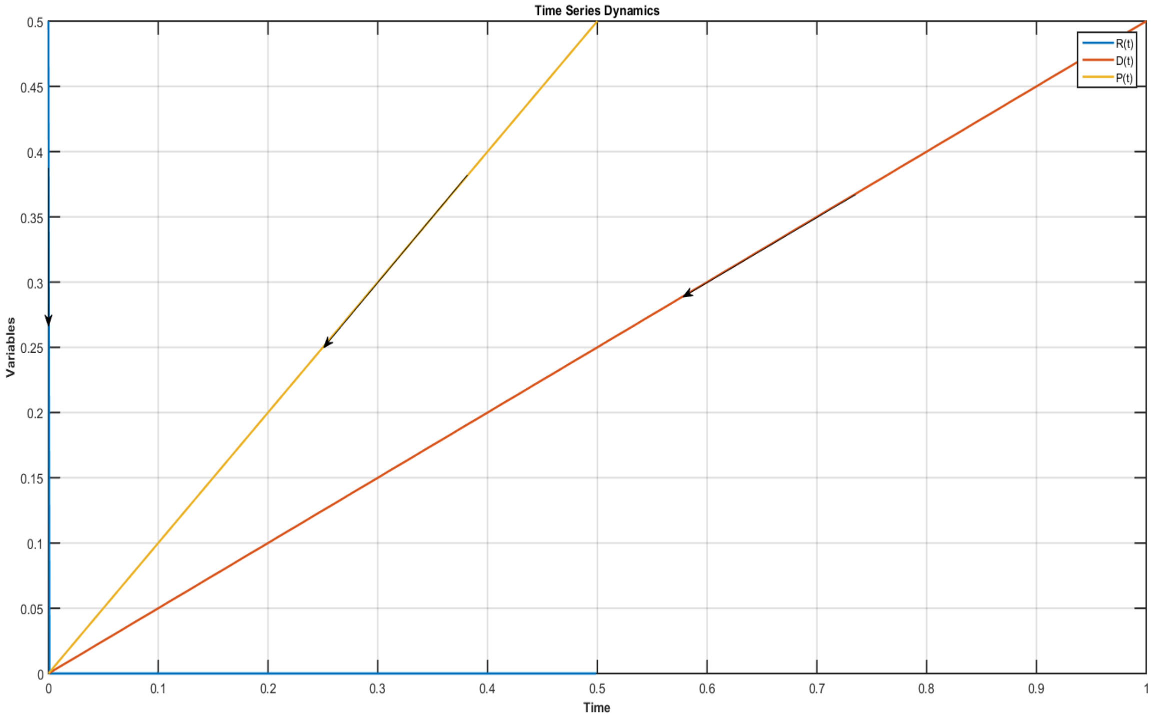

Figure 1. Time-series dynamics of the nonlinear fiscal system showing the evolution of regulation $R(t)$, debt $D(t)$, and political pressure $P(t)$under stable parameter conditions.

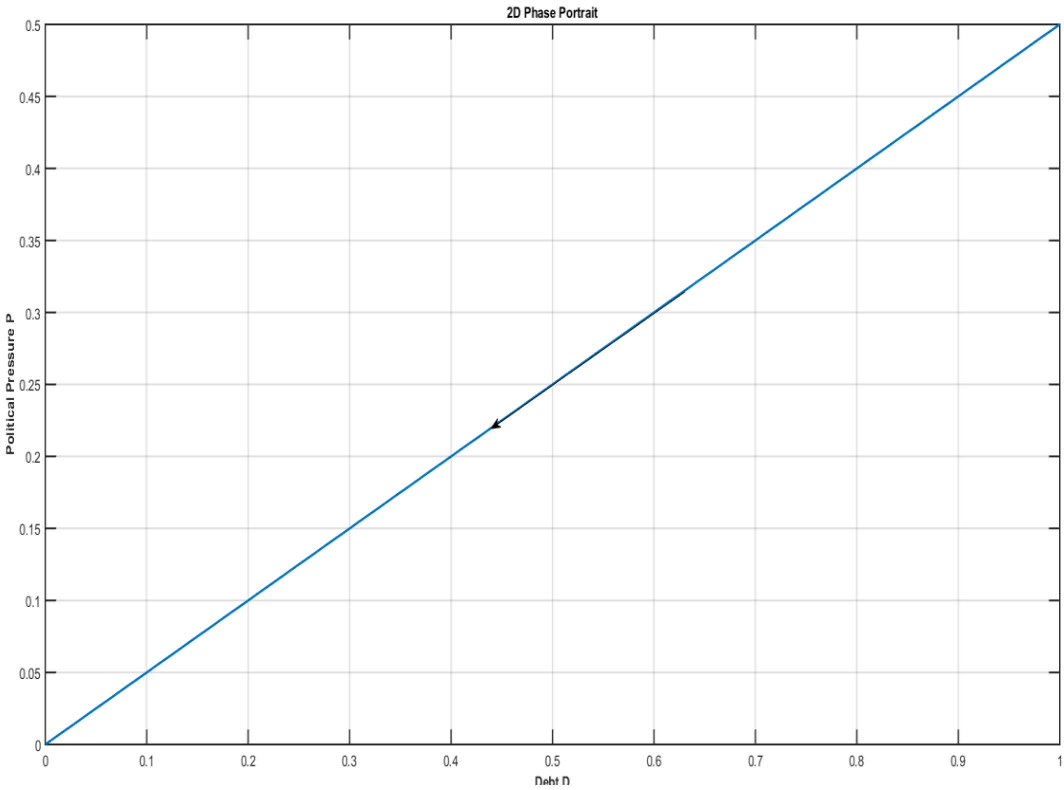

Figure 2. Two-dimensional phase portrait in the (D, P)-plane illustrating the bounded nonlinear interaction between debt and political pressure.

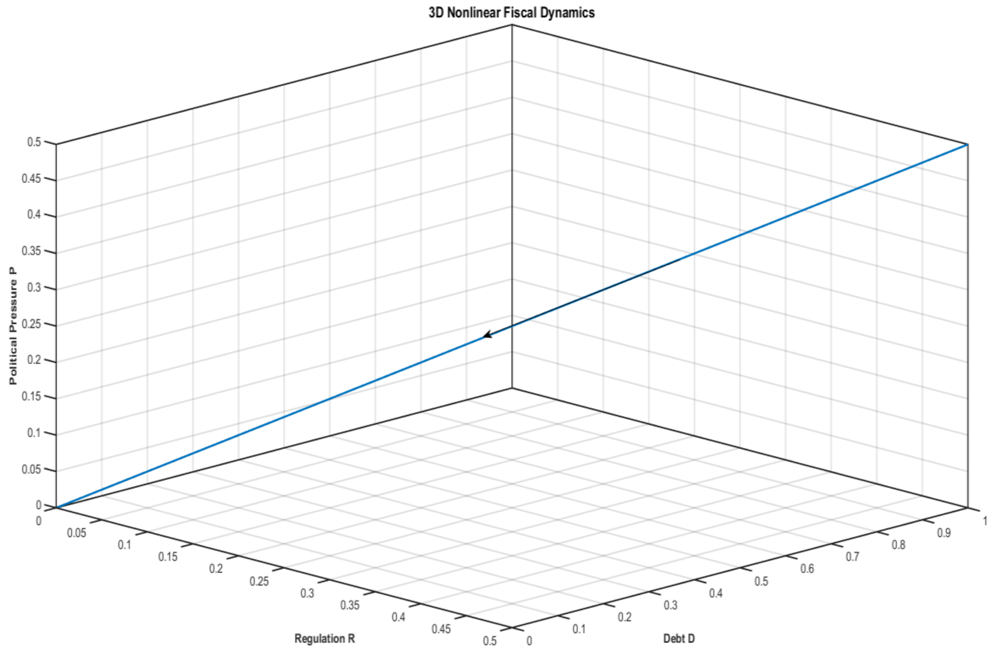

Figure 3. Three-dimensional nonlinear trajectory of the fiscal system demonstrating the transient evolution and convergence behaviour of regulation, debt, and political pressure.

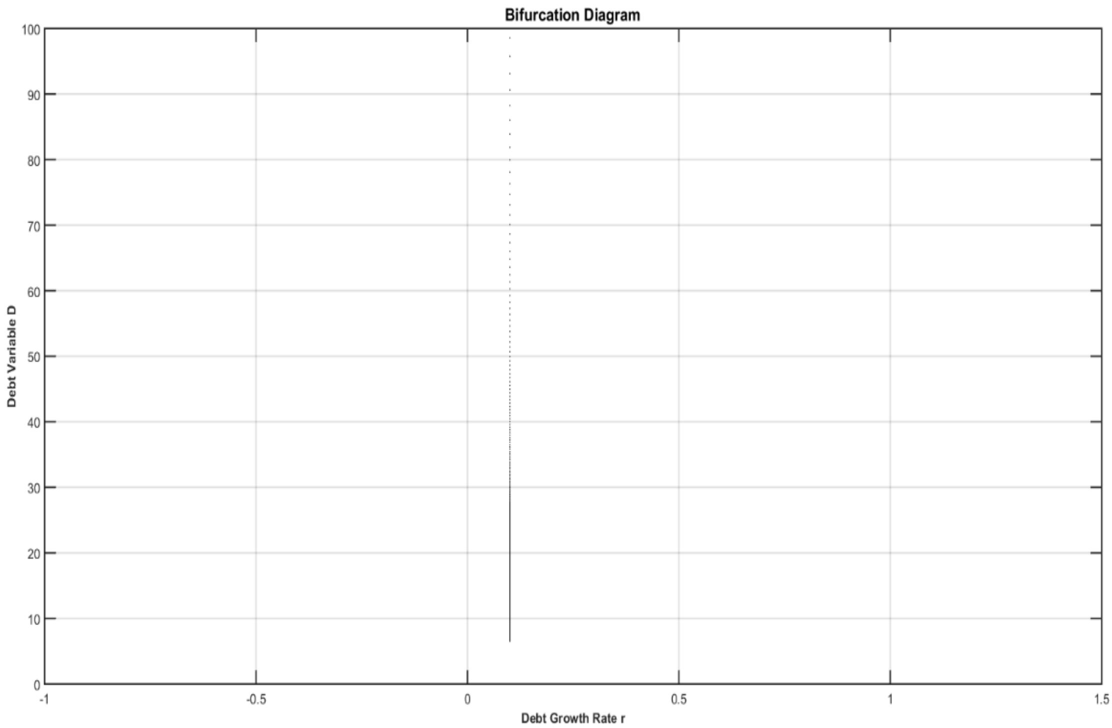

Figure 4. Bifurcation diagram with respect to the debt growth parameter r, illustrating the transition of the fiscal system from stable dynamics toward instability.

VI. Conclusion:

The study shows that strong but balanced regulation keeps public debt under control, even when there are short-term political or fiscal shocks. Effective institutions prevent debt from growing out of hand and reduce the risk created by political pressure. The system remains calm and stable, without repeated crises or wild fluctuations, indicating a healthy governance structure.

However, the results also warn that too much regulation can be harmful. If regulatory action becomes excessive, it may weaken political processes and create new instability, even when debt appears controlled. In simple terms, stability comes from balance, not from over-control. Sustainable governance requires regulation that is firm, but not extreme.

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