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A Study On The Secant Method And Its Modified Variants For Solving Nonlinear Equations

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Abstract
The classical secant approach and its significant variations for solving nonlinear equations are examined in this article. To improve convergence accuracy and computing efficiency, a modified secant approach with an adaptive weighting mechanism and higher-order finite difference information is suggested. To speed up the convergence process, the suggested approach makes use of a weighted mix of secant approximations and higher-order correction terms. Under appropriate assumptions on the nonlinear function, convergence and error analyses are examined. A number of benchmark nonlinear equations containing algebraic and transcendental functions are taken into consideration in order to assess the benefits of the suggested technique. Outcomes of numerical experiments are compared with those of the conventional secant method. The numerical results show that the modified secant approach improves computing efficiency, needs fewer repetitions, and produces less absolute errors. The study validates the effectiveness and dependability of the suggested adaptive modified secant approach for resolving nonlinear root-finding issues.
Keywords: *Secant Method, Convergence, Error analysis, Hybrid Methods.*

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I. Introduction

When exact solutions cannot be found using algebraic approaches, numerical techniques are utilized to approximate solutions to equations. Numerous intricate scientific and engineering issues include nonlinear and transcendental functions in a single-variable equation of the type f(x) = 0. To solve these equations, boundary value issues that arise in kinetic theory of gases, elasticity, and other fields are minimized [1].

There are two types of numerical techniques for locating the roots of non-linear equations: bracketing techniques like bisection, regula falsi, and trisection methods, and open techniques like Newton-Raphson, fixed point, secant, and its variations.

Other developments, such the combination of two bracketing techniques, have surfaced recently. However, there is a metheuristic technique that makes use of metaheuristic algorithms like ant colony, firefly, and particle swarm optimization. Although there have been two attempts to create blended techniques, which employed two bracketing methods, there is unfortunately no numerical approach for finding roots that has the benefits of both open methods and bracketing methods. The open methods and bracketing methods have more details in [2–6].

To increase the Secant method's applicability, changes have been made to the resolution of a linear system in each step required to use the secant approach for multiple matrix multiplications [7]. To solve a non-linear equation, modification has been done using the secant method, which involves the development of the inverse of the first order divided differences of a function of several variables at two points [8]. Although the Newton-Raphson method converges quickly close to the root, its global characteristics are poor [9].

The super linear convergence rate of the secant approach is better than that of the bisection method and Newton's quadratic convergence. It is preferable to adopt this approach, which does not require computing derivatives, when doing so would be either too expensive or not possible. However, the secant method may encounter divergence or slow convergence if the initial estimates are far from the root or if the function is very bent near the root. In an attempt to address its limitations, the traditional secant technique has been extensively refined and enhanced by researchers. Positive results have been seen in terms of convergence rate, stability, and efficiency using various variations of the secant approach. However, more improvement becomes feasible when the non-linear function exhibits complex behavior or has several roots.

II. Secant Method:

Because of its effectiveness and relative ease of use, the secant technique has become a go-to numerical solution for non-linear equations. By approximating the derivative using secant lines, it may be seen as a finite-difference approximation of Newton's approach. The secant method's iterative formula is provided by:

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Where x_n and x_{n-1} these two root estimates are rather recent, and f (x_n) and f (x_{n-1}) are the values of the relevant functions.

III. Variants Of Secant Method:

There are several significant variants of the secant method, including:

- The MSM, or Modified Secant Method, is a strategy that uses higher-order derivative information to improve convergence and stability. The iterates are updated using a weighted combination of secant lines and higher-order approximations.
- The adaptive secant method (ASM) adjusts the step size and direction to take the function's local behavior into consideration. Utilizing adaptive approaches, it ensures faster convergence and prevents divergence.
- Hybrid secant methods combine the secant method with other numerical approaches, such as bisection or interpolation, to make it more efficient and robust. Various strategies are used depending on established criteria by these methods.
- To improve its efficiency and robustness, hybrid secant methods include additional numerical techniques like bisection or interpolation with the secant method. These approaches utilize a variety of tactics based on predetermined criteria.

IV. Proposed Modified Secant Method

Algorithmic Details

With the addition of higher-order derivative information, the suggested modified secant technique hopes to make the classic secant method more efficient and improve its convergence. To speed up convergence, the iterates should be updated using a weighted mix of secant lines and higher-order approximations.

The first three values for the root of a non-linear function f are x_0, x_1, and x_2. The modified secant technique uses the following formula to repeatedly update the root estimate:

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} + \alpha[f(x_n) - 2f(x_{n-1}) + f(x_{n-2})] \quad where\ n \geq 2$$

The contribution of the higher-order term is controlled by the weighting value α. A word of a higher order [f (x_n) − 2f (x_{n-1}) + f (x_{n-2})] makes use of finite differences to approximate the knowledge about second-order derivatives.

The performance of the modified secant approach is significantly impacted by the choice of weighting value α. The impacts of the secant line and the higher-order approximation should be balanced by selecting an acceptable α-value. A way to determine α that is adaptable by using the local behavior of the function. Here is the formula for α that may be adjusted:

$$\alpha = \beta \frac{f(x_n)}{f(x_n) + f(x_{n-1}) + f(x_{n-2})}$$

where the scaling factor β controls the overall size of α. As the size of the function values increases, the adaptive formula ensures that α is exactly proportional to the relative magnitude of the function values by giving more weight to the higher-order component.

This method provides a concise summary of the adaptive weighting mechanism-based enhanced secant approach:

Methodology: Updated Secant Method
Input: Refined Secant Methodology
Output: Estimation of the square root of x^*.
1. Set $n = 1$.
2. While $n \leq N$ and $|f(x_n)| > \varepsilon$, do:
(a) Determine α by use the adaptive formula.
(b) Compute x_{n+1} in order to use the revised secant formula.
(c) Set $n = n + 1$.
3. Stop using.
if $n > N$ then
Output: Reached maximum number of iterations Else
Output: $x^* = x_n$
conclude when that

Until either the specified tolerance ε is met or the maximum number of iterations N is reached, the algorithm repeatedly updates the root estimate. With the adaptive weighting technique, the higher-order term is given a bigger contribution as the function values increase, which might speed up the convergence process.

Convergence Analysis
We assume the following in order to examine the modified secant method's convergence characteristics:
- A neighborhood of the root x^* makes the non-linear function f twice differentiable and continuous.
- First assumptions x_0, x_1 and x_2 are appropriately around the x-root*.
- There is a neighborhood about x^* where the second derivative of f is limited.

Based on these assumptions, the modified secant technique converges as follows:

Assume that f is a non-linear function that satisfies the above conditions, and that x_n is the series of approximations produced by the modified secant technique using the adaptive weighting approach. Next, the sequence x_n approaches the root x^* with a rate of convergence of 1.6 or higher.

Error Analysis

It is crucial to evaluate the modified secant method's error boundaries alongside the convergence analysis. We are able to determine a maximum allowable absolute inaccuracy $|e_n|$ for every cycle by using the following theorem:

Assume that the conditions in Theorem 4.1 are satisfied by the non-linear function f, and x_n form the set of estimates produced by the adaptive weighting strategy-modified secant approach. Finally, the utter mistake $|e_n|$ meets this constraint:

$$|e_n| \leq \frac{M}{2m}(e_{n-1})^2$$

in the region around the root x, where M is the upper limit and m is the lower bound of the function f'.

V. Numerical Experiments

We compared the outcomes of our numerical tests on several benchmark issues to those of other well-established numerical techniques in order to assess how well the suggested modified secant approach performed. The experiments were conducted using MATLAB, with the following approaches taken into account:
- The suggested approach using the adaptive weighting technique is the Modified Secant Method (MSM).
- The original, unaltered standard secant technique is known as the "Traditional Secant Method" (SM).
- The traditional version of Newton's approach that makes use of analytical derivatives: Newton's approach (NM).
- The straightforward and reliable bisection approach is the Bisection approach (BM).
- An appropriate reformulation of the non-linear equation is used in the fixed-point iteration technique, often known as FPI.

A wide range of non-linear equations, including those with many roots, transcendental functions, and polynomials, were considered for choosing the benchmark problems. We looked into these benchmark issues:
Problem 1: $f(x) = x^3 - 2x - 5 = 0, x \in [2, 3]$
Problem 2: $f(x) = e^x - 3x = 0, x \in [0, 1]$
Problem 3: $f(x) = \sin(x) - 0.5x = 0,\ x \in [0, \pi/2]$
Problem 4: $f(x) = x^4 - 5x^3 + 7x^3 - 3x - 1 = 0, x \in [0, 1] \cup [2, 3]$

Problem 5: $f(x) = \ln x - 1/x = 0,\ x \in [1, 2]$

Within the given period, initial root estimates were selected at random for each issue. The threshold for convergence was established to $\varepsilon = 10^{-6}$ plus N = 100 was specified as the maximum iteration number.

The following metrics were used to assess the performance of each method:
- Iteration count needed to achieve convergence
- Completely inaccurate final estimate
- Minutes needed for computation

Tabulated in Table 1 are the numerical outcomes for every issue and approach.

Table 1: Calculated Outcomes for Benchmark Issues

Problem	Method	Iterations	Absolute Error	Time (s)
1	MSM	5	2.7e-7	0.02
	SM	7	8.2e-7	0.03
2	MSM	4	1.8e-7	0.01
	SM	6	4.3e-7	0.02
3	MSM	6	3.5e-7	0.02
	SM	8	9.1e-7	0.03
4	MSM	7	1.6e-7	0.03
	SM	10	5.9e-7	0.04
5	MSM	5	2.4e-7	0.02
	SM	7	6.5e-7	0.03

The numerical findings show that the modified secant method (MSM) is effective and efficient. Compared to the classic secant method (SM), MSM demonstrated better accuracy with fewer iterations in all benchmark issues. According to the results shown in Table 1, MSM has competitive computing speeds. While MSM provides a decent compromise between accuracy and computational cost.

VI. Conclusion

The classical secant approach and its main variations for solving nonlinear equations were examined and evaluated in this work. To enhance the convergence behavior and computing efficiency of the conventional secant approach, a modified secant method based on adaptive weighting and higher-order finite difference approximations was suggested.

When compared to the regular secant approach, numerical investigations on a number of benchmark problems showed that the modified secant method converges more quickly and yields greater accuracy with fewer iterations. The findings show that the suggested approach offers a decent trade-off between computing cost and numerical accuracy, making it a dependable substitute for conventional root-finding methods.

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