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Modified Class Of Exponential Dual To Ratio Cum Intercept Estimator For Estimating Population Mean

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Abstract:
In this manuscript, we put forward a modified difference estimator with Exponential Dual to Ratio (EDR) as intercept for estimation of population mean with an aim to study mean square error (MSE)and efficiency of the suggested class of estimator over classical estimators. The proposed model has been discussed along with the numerical illustration. The estimators have higher percent relative efficiency in comparison to the existing estimators.
Keywords: *Difference estimator, Auxiliary variable, Exponential dual to ratio, Bias, Mean square error (MSE), Efficiency.*

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I. Introduction:

In Sample Survey, ratio method has achieved a lot of importance over years,the information available on ancillary variable is used for estimation procedure. On the other hand, by the stronger perceptive demand, statisticians are more tending towards the dual to ratio cum product estimators. Possibly that is why a widespread work has been carried in the way of refining the performance of estimators. From last few year, a large no of estimators have been proposed by several authors on dual to ratio cum estimators. This paper is based on the new modifications in the dual to Ratio Cum estimators by introducing an intercept to the classical estimators and compared its influence over existing estimators.

Most of the work which is based on transformation of auxiliary variable,is done by Bandyopadhyay and Srivenkataramana [5][6]on dual.The latest references on dual to ratio is given by singh [3],Tailor,Sharma and Kin[4],Handique[1],choudhary and singh[2],lone and Tailor[7]and singh[8].Singh [11] and Ahmed,Alafara and Olamilekan[12].

We draw a sample of *n* units from population of N to estimate the population mean of the study variable $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ .The sampling units are measured. Let (y_i, x_i) where $i = 1,2,3.......N$ denotes the set of the observation. Let $\bar{x}$ and $\bar{y}$ the sample means be unbiased estimators respectively

The estimator of $\bar{Y}$ is given as $\bar{Y}_R = \frac{\bar{y}}{\bar{x}}\bar{X}$ and regression estimator is given as $\bar{y}_{reg} = \bar{y} + \hat{b}(\bar{X} - \bar{x})$, where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\hat{b} = \frac{S_{xy}}{S_x^2}$. [5]obtained estimator $\bar{Y}_{dR} = \bar{y}\frac{\bar{x}^*}{\bar{X}}$, $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N-n} = (1+g)\bar{X} - g\bar{x}, g = \frac{n}{N-n}$.

The mean square error of [5] is given as:

$$MSE(\bar{Y}_{dR}) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y{}^2 + C_x{}^2 g(g-2K)\right] \tag{1.1}$$

The exponential type of the above estimator has been proposed by Tailor and Tailor [11] is given as,

$$\bar{y}_{ER}{}^* = \bar{y}\exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right) \tag{1.2}$$

The mean square error is given as,

$$MSE\left(\bar{y}_{ER}{}^{*}\right)=\gamma.\bar{Y}^{2}\left(C_y^2+gC_x^2\left(\frac{g}{4}-K_{yx}\right)\right) \quad (1.3)$$

Where $k=\frac{\rho C_y}{C_x}$

II. Suggested Estimator

In order to minimize MSE, we suggested a modified exponential estimator with dual to Ratio by introducing intercept given as:

$$\bar{Y}_p{}^{*}=\bar{y}\exp\left(\frac{\bar{x}^{*}-\bar{X}}{\bar{x}^{*}+\bar{X}}\right)+\alpha(\bar{X}-\bar{x}^{*}), \quad (2.1)$$

Where α is a suitable chosen scalar used as the design parameter, and

$$\bar{x}^{*}=\frac{N\bar{X}-n\bar{x}}{N-n}=(1+g)\bar{X}-g\bar{x},$$

$$g=\frac{n}{N-n}$$

To evaluate expected value of $\bar{Y}_p{}^{*}$,let $\bar{y}=\bar{Y}(1+e_0)$ and such that $E(e_0)$ = $E(e_1)$ = 0 and under *SRSWOR* scheme $E({e_0}^2)=\frac{1-f}{n}C_y^2$

$$E({e_1}^2)=\frac{1-f}{n}C_x^2$$

$$E(e_0e_1)=\frac{1-f}{n}\rho C_xC_y$$

Where $f=\frac{n}{N}$ is the sampling fraction, $C_y^2=\frac{S_y^2}{\bar{Y}^2}, C_x^2=\frac{S_x^2}{\bar{X}^2}$

Expressing equation (2.1) in "e's",

$$\bar{Y}_P{}^{*}=\bar{Y}(1+e_0)\exp\left(\frac{(1+g)\bar{X}-g\bar{x}-\bar{X}}{(1+g)\bar{X}-g\bar{x}+\bar{X}}\right)+\alpha(\bar{X}-(1+g)\bar{X}-g\bar{x})$$

$$=\bar{Y}(1+e_0)\exp\left(\frac{(1+g)\bar{X}-g(X(1+e_1))-\bar{X}}{(1+g)\bar{X}-g(X(1+e_1))+\bar{X}}\right)+\alpha\left(\bar{X}-(1+g)\bar{X}-g(X(1+e_1))\right)$$

$$=\bar{Y}(1+e_0)\exp\left(\frac{-g\bar{X}e_1}{2\bar{X}-g\bar{X}e_1}\right)+\alpha g\bar{X}e_1.$$

After solving the above expression, we get

$$\left(\bar{Y}_P^{*}-\bar{Y}\right)=\bar{Y}\left[e_0-\frac{ge_1}{2}\right]+\alpha g\bar{X}e_1\ . \quad (2.2)$$

Squaring and expectation of [2.2],we get

$$E\left(\bar{Y}_P^{*}-\bar{Y}\right)^2=E\left[\bar{Y}^2{e_0}^2+\bar{Y}^2\frac{g^2{e_1}^2}{4}+\hat{b}g\bar{X}e_1-\bar{Y}^2ge_1e_o-\alpha g^2\bar{X}\bar{Y}C_x^2+2\alpha g\bar{X}\bar{Y}\rho C_yC_x\right]$$

$$E(\left(\bar{Y}_P^{*}-\bar{Y}\right)^2=\frac{1-f}{n}\left[\bar{Y}^2C_y^2+C_x^2g^2\left(\frac{\bar{Y}}{2}-\alpha\bar{X}\right)^2-2g\bar{Y}\rho C_yC_x\left(\frac{\bar{Y}}{2}-\alpha\bar{X}\right)\right] \quad (2.3)$$

$$MSE(\left(\bar{Y}_P^{*}\right)=\frac{1-f}{n}\left(\bar{Y}^2C_y^2+C_x^2g^2\left(\frac{\bar{Y}}{2}-\alpha\bar{X}\right)^2-2g\bar{Y}\rho C_yC_x\left(\frac{\bar{Y}}{2}-\alpha\bar{X}\right)\right) \quad (2.3)$$

Differentiating (2.3) with respect to α we get the optimal value of α_{opt}

$$\alpha_{opt}=\frac{R}{2}-\frac{b}{g},$$

Put the value of α_{opt} in (2.3),we get

$$MSE(\bar{y}_P^*) = \frac{1-f}{n}\bar{Y}^2 C_y^2 (1-\rho^2)$$

which is the usual regression estimator

III. Special case:

For α =0, the suggested estimator condensed to exponential dual to ratio estimator $\bar{y}_{ER}{}^* = \bar{y}\exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right)$

The mean square error is as,

$$MSE\left(\bar{y}_{ER}{}^*\right) = \frac{1-f}{n}.\bar{Y}^2\left(C_y^2 + gC_x^2\left(\frac{g}{4} - K_{yx}\right)\right)$$

IV. Efficiency Comparison:

In order to minimize the MSE, a comparison is made between the proposed estimator and the traditional estimators.

$$MSE(\bar{y}) = \frac{1-f}{n}\bar{Y}^2 C_y^2 \qquad (4.1)$$

comparing (2.3) and (4.1), the suggested estimator $\bar{y}_P{}^*$ is improved than that the usual $\bar{y}$ if,

$$MSE_{\min}\left(\bar{y}_P{}^*\right) < MSE(\bar{y}) \; \frac{1-f}{n}\left(\bar{Y}^2C_y^2 + C_x^2 g^2\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right)^2 - 2g\bar{Y}\rho C_y C_x\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right)\right) < \frac{1-f}{n}\bar{Y}^2 C_y^2$$

$$\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right)\left(-2g\bar{Y}\rho C_y C_x + C_x^2 g^2\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right)\right) < 0$$

This holds if and only if ,

Case 1 $\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right) < 0$ and $-2g\bar{Y}\rho C_y C_x + C_x^2 g^2\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right) > 0$

Or

Case 2 $\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right) > 0$ and $-2g\bar{Y}\rho C_y C_x + C_x^2 g^2\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right) < 0$

The Range of α is given as,

$$\min\left\{R, R\left(1 - \frac{2\rho C_y}{gC_x}\right)\right\}, \max\left\{R, R\left(1 - \frac{2\rho C_y}{gC_x}\right)\right\}$$

We compare the $\left(\bar{y}_P{}^*\right)$estimator with $\left(\bar{y}_R{}^*\right)$

$MSE_{\min}\left(\bar{y}_P{}^*\right) < MSE\left(\bar{y}_R^*\right)$that is

$$\frac{1-f}{n}\left(\bar{Y}^2C_y^2 + C_x^2 g^2\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right)^2 - 2g\bar{Y}\rho C_y C_x\left(\frac{\bar{Y}}{2} - \alpha\bar{X}\right)\right) < \frac{1-f}{n}\left(\bar{Y}^2C_y^2 + \bar{Y}^2 g^2 C_x^2 - 2\bar{Y}^2 g\rho C_y C_x\right)$$

$$-\alpha\bar{X}\left(-2g\bar{Y}\rho C_y C_x + C_x^2 g^2\left(\bar{Y} - \alpha\bar{X}\right)\right) < 0$$

This holds if and only if ,

Case 1 $-\alpha\bar{X} < 0$ and $-2g\bar{Y}\rho C_y C_x + C_x^2 g^2\left(\bar{Y} - \alpha\bar{X}\right) > 0$

Or

Case 2 $-\alpha\bar{X} > 0$ and $-2g\bar{Y}\rho C_y C_x + C_x^2 g^2\left(\bar{Y} - \alpha\bar{X}\right) < 0$

The range of α is

$$\min\left\{0, 2R\left(1 - \frac{\rho C_y}{gC_x}\right)\right\}, \max\left\{0, 2R\left(1 - \frac{\rho C_y}{gC_x}\right)\right\}$$

We compare $\left(\bar{y}_P^{\ *}\right)$with $\bar{y}^*_{ER}$

$MSE_{\min}\left(\bar{y}_P^{\ *}\right) < MSE\left(\bar{y}^*_{ER}\right)$ that is

$$\frac{1-f}{n}\left(\bar{Y}^2C_y^2 + C_x^2g^2\left(\frac{\bar{Y}}{2}-\alpha\bar{X}\right)^2 - 2g\bar{Y}\rho C_yC_x\left(\frac{\bar{Y}}{2}-\alpha\bar{X}\right)\right) < \frac{1-f}{n}\left(\bar{Y}^2C_y^2 + \bar{Y}^2g^2C_x^2 - 2\bar{Y}^2g\rho C_yC_x\right)$$

$$-\alpha\bar{X}\left(-2g\bar{Y}\rho C_yC_x + C_x^2g^2\left(\alpha\bar{X}\right)\right) < 0$$

This holds if and only if ,

Case 1 $-\alpha\bar{X} < 0$ and $-2g\bar{Y}\rho C_yC_x + C_x^2g^2\left(\alpha\bar{X}\right) > 0$

Or

Case 2 $-\alpha\bar{X} > 0$ and $-2g\bar{Y}\rho C_yC_x + C_x^2g^2\left(\alpha\bar{X}\right) < 0$

The range of α is

$$\min\left\{0, \frac{-2R\rho C_y}{gC_x}\right\}, \max\left\{0, \frac{-2R\rho C_y}{gC_x}\right\}$$

Table 2: Data Statistics

Parameters	Population I	Population II
N	104	278
n	20	30
$\bar{Y}$	625.37	*39.0680*
ρ	0.865	0.7213
C_y	1.866	*1.4451*
C_x	1.653	*1.6198*

Table 3: Pre's of the proposed estimators

Estimator	**Population I**	**Population II**
$\left(\bar{y}\right)$ **Usual SRS**	100	100
$\left(\bar{y}_R^{\ *}\right)$ **Bandhopadhy**	144	122
$\left(\bar{y}_{ER}^{\ *}\right)$ **Tailor**	151	136
$\left(\bar{y}_p^{\ *}\right)$ **Proposed**	**397**	**209**

V. Conclusion:

In the context of the above illustration and discussions, we infer that the proposed estimator performs better than the classical estimators by comparing the percent relative efficiency.The suggested estimator has higher PRE's given in table 3. Further it would be recommended for future references in sampling.

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