IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-0661, p-ISSN: 2278-8727, Volume 22, Issue 1, Ser. 1 (Jan. – Feb. 2026), PP 41-45
www.iosrjournals.org

Sensitivity Aanalysis Of A Mathematical Model For The Transmission Of Measles With Passive Immunity

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Abstract
A mathematical model of the transmission dynamics of measles incorporating passively immune class is developed. In the formulation of this model, a set of ordinary differential equations have been used to express the dynamics of the disease. A sensitivity analysis of a deterministic compartmental model for measles transmission with passive immunity is presented. The analysis identifies parameters that significantly influence the basic reproduction number and disease dynamics. Both analytical and numerical sensitivity analyses are performed to guide effective measles control strategies.
Keywords: *Equilibrium points, basic reproduction number, sensitivity analysis.*

Date of Submission: 02-02-2026 Date of Acceptance: 12-02-2026

I. Introduction

Measles is an infectious and highly contagious viral respiratory disease. It is spread through coughing and/or sneezing, a close personal contact of the susceptible individual with an infected person or direct contact of the susceptible individuals with nasal or throat secretions from an infected individual. The infectious secretions remain in the air or on infected surfaces for up to two hours after an infected person sneezes or coughs. Worldwide, measles is fifth leading cause of death among under-five children with reported 109,638 deaths in 2017 [1]. There is no known specific treatment for measles. However, infection and subsequent recovery confers permanent and lifelong immunity on an individual [2, 3]. Because of the health burden and the high death rates the disease causes, it is important to develop effective control strategies. A mathematical model is a powerful tool in the analysis of measles transmission dynamics. A sensitivity analysis of a deterministic compartmental model for measles transmission with passive immunity is presented. The analysis identifies parameters that significantly influence the basic reproduction number and disease dynamics. Both analytical and numerical sensitivity analyses are performed to guide effective measles control strategies.

II. Formulation Of The Mathematical Model Of Measles

In this section, we describe and develop the model of measles dynamics based on the several assumptions.

Model assumptions
In developing the model, the following assumptions were made;
1. Individuals get into the system by birth only.
2. Al new born infants acquire passive immunity from their mothers hence they are disease free.
3. All the variables and parameters used in the model are non-negative.
4. All recovered individuals acquire permanent immunity.
5. There is free interaction within the population.

Model equations

The population is divided into five compartments: passively immune (M), susceptible (S), exposed (E), infectious (I), and recovered (R). The model assumes homogeneous mixing, permanent immunity after recovery, and vaccination at birth. The measles transmission model with passive immunity incorporates several epidemiological and demographic parameters. Each parameter represents a specific biological or public health process and is defined as follows:

Parameter	Description
β	Effective contact rate between susceptible and infectious individuals
σ	Rate of loss of passive immunity

Parameter	Description
θ	Proportion of individuals successfully vaccinated at birth
α	Progression rate from exposed to infectious class
γ	Recovery rate of infectious individuals
μ	Natural death rate
δ	Disease-induced death rate

Table 1: Parameter Description

The total population is given by:

$$N(t) = M(t) + S(t) + E(t) + I(t) + R(t)$$

The schematic diagram below illustrates how the disease spreads in different compartments.

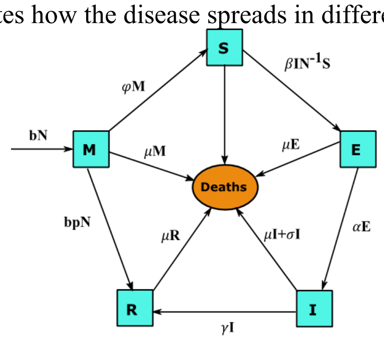


Figure 1: Schematics depicting transitions between different compartments, transmission and mortalities rates (Authors 2019)

The mathematical model is described by the following system of equations

$$\begin{aligned} dM/dt &= bN - bpN - (\varphi + \mu)M \\ dS/dt &= \varphi M - \beta IS/N - \mu S \\ dE/dt &= \beta IS/N - (\mu + \alpha)E \\ dI/dt &= \alpha E - (\mu + \sigma + \gamma)I \\ dR/dt &= bpN + \gamma I - \mu R \end{aligned} \qquad (1)$$

Basic Reproduction Number (R_0)

Basic reproduction number, R_0, is a significant threshold in determining whether the disease dies out or persists in the population. It is a measure of the speed with which a disease spreads through a population. In this study, we compute R_0, using the next generation matrix method as formulated by [8] in which we determine the dominant eigenvalue of the steady state Jacobian matrix of the model after linearization.

$$R_0 = \frac{\alpha\beta\varphi(1-p)}{(\alpha+b)(\sigma+\gamma+b)(\varphi+b)} \qquad (2)$$

III. Sensitivity Analysis

The normalized forward sensitivity index of R0 with respect to a parameter p is defined as:

$$\Upsilon_p = (p/R_0)(\partial R_0/\partial p). \qquad (3)$$

Analytical Sensitivity Analysis

The sensitivity of the basic reproduction number R_0 to changes in model parameters is examined using the normalized forward sensitivity index. This approach quantifies the relative impact of each parameter on disease transmission dynamics.

Sensitivity Index Definition

The normalized forward sensitivity index of the basic reproduction number R_0 with respect to a parameter p is defined as:

$$\Upsilon_p^{R_0} = (p / R_0) \times (\partial R_0 / \partial p) \qquad (4)$$

This index measures the proportional change in R_0 resulting from a proportional change in the parameter p. For the measles transmission model with passive immunity, the basic reproduction number is given by:

$$R_0 = \beta(1 - \theta) / [(\alpha + \mu)(\gamma + \mu + \delta)]$$

Analytical Sensitivity Indices
Using R_0, the sensitivity indices with respect to the key model parameters are computed as follows.

Sensitivity with respect to the contact rate β
The partial derivative of R_0 with respect to β is:

$$\partial R_0 / \partial \beta = (1 - \theta) / [(\alpha + \mu)(\gamma + \mu + \delta)] \quad (5)$$

Hence, the sensitivity index is:

$$\Upsilon_{\beta}^{R_0} = 1 \quad (6)$$

This indicates that the basic reproduction number is directly proportional to the contact rate.

Sensitivity with respect to vaccination coverage θ
The partial derivative of R_0 with respect to θ is:

$$\partial R_0 / \partial \theta = -\beta / [(\alpha + \mu)(\gamma + \mu + \delta)] \quad (7)$$

Thus, the sensitivity index is:

$$\Upsilon_{\theta}^{R_0} = -\theta / (1 - \theta) \quad (8)$$

This negative value implies that an increase in vaccination coverage significantly reduces measles transmission.

Sensitivity with respect to the progression rate α
The partial derivative of R_0 with respect to α is:

$$\partial R_0 / \partial \alpha = -\beta(1 - \theta) / [(\alpha + \mu)^2(\gamma + \mu + \delta)] \quad (9)$$

Therefore, the sensitivity index is:

$$\Upsilon_{\alpha}^{R_0} = -\alpha / (\alpha + \mu) \quad (10)$$

This result shows that increasing the progression rate from exposed to infectious reduces R_0 by shortening the latent period.

Sensitivity with respect to the recovery rate γ
The partial derivative of R_0 with respect to γ is:

$$\partial R_0 / \partial \gamma = -\beta(1 - \theta) / [(\alpha + \mu)(\gamma + \mu + \delta)^2] \quad (11)$$

Hence, the sensitivity index is:

$$\Upsilon_{\gamma}^{R_0} = -\gamma / (\gamma + \mu + \delta) \quad (12)$$

This indicates that faster recovery decreases the average infectious period and lowers disease transmission.

Sensitivity with respect to the disease-induced death rate δ
The partial derivative of R_0 with respect to δ is:

$$\partial R_0 / \partial \delta = -\beta(1 - \theta) / [(\alpha + \mu)(\gamma + \mu + \delta)^2] \quad (13)$$

Thus, the sensitivity index is:

$$\Upsilon_{\delta}^{R_0} = -\delta / (\gamma + \mu + \delta) \quad (14)$$

Parameter	Description	Sensitivity Index
β	Contact rate	+1
θ	Vaccination at birth	−θ/(1−θ)
α	Progression rate	−α/(α+μ)
γ	Recovery rate	−γ/(γ+μ+δ)
δ	Disease-induced death rate	−δ/(γ+μ+δ)

Table 2: Sensitivity Indices of R_0

Contact rate β has a sensitivity index of +1, indicating direct proportionality with R_0. Vaccination coverage θ has a negative sensitivity index, highlighting its role in reducing transmission. Recovery and progression rates also reduce R_0 when increased.

Numerical Simulations

Numerical simulations were conducted and the effect on R_0 is illustrated in Figures 1–3.

Equilibrium points and basic reproduction number

The equilibrium points of our model were analyzed [8]. In this section, we carried out numerical analysis of the equilibrium points for the system at high contact rate using Matlab software. From the results, it was found that the DFE is,

$$E_0^* = (823.08, 605.20,0,0,99.521)$$

While the EE point for our model $\beta = 0.09091$ is,

$$E_1^* = (833.02, 488.10, 0.0011, 0.0022, 800.8)$$

the EE point for our model at $\beta = 1.6667$ is,

$$E_1^* = (814.7.02, 10.1134, 0.0011, 0.0045, 1277.7)$$

The basic reproductive number $R_0 = 0.44097$ for $\beta = 1.6667$ and $R_0 = 0.024026$ for $\beta = 0.09091$.

Sensitivity index

Using parameter values in Appendix A, the sensitivity of R_0 to all the parameters are given out in the table 3 below. This indices will be used to identify parameters with high impact on R_0 and to which intervention strategies are to target.

Parameter	Sensitivity index
β	+1.00000
b	-0.962819
α	+0.000686
μ	0
σ	-0.016918
φ	+0.961543
p	- 0.00051
γ	-0.982492

Table 3: the constant sensitivity indices of model parameters

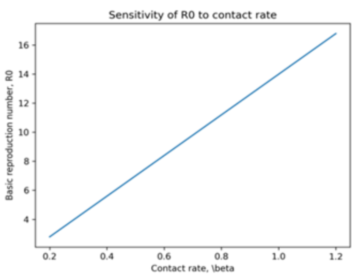
The signs and magnitude of the sensitivity indices, helps to show the direction of the response to small variation in the input parameters and importance of each parameter in the model output respectively. From the computed sensitivity indices in table 3, the parameters β, α, and φ have positive indices. This indicates that R_0 increases when these parameters are increased, when all the other parameters are held constant. The parameters b, σ, p and γ have negative indices. This indicates increase in these parameters while holding constant all the other parameter, decreases R_0 value. The parameter that is most sensitive is the contact rate, β, followed by φ then α, while the least sensitive one is the recovery rate, γ.

Sensitivity of the model variables to all the parameters at endemic equilibrium point was also carried out numerically and the results are shown in the table below.

	m	s	e	i	r
b	-4.7545	-1.5413	$+0.0018$	$+1.5452$	$+3683.8$
p	$+0.0001348$	-0.00002168	-0.000001963	-0.00000209	-0.0060
α	-0.4611	-0.2241	-1.0002	$+0.2268$	$+540.654$
β	-0.6234	-1.3170	$+0.0002871$	$+0.3067$	$+730.9030$
φ	$+0.0727$	$+0.0168$	$+0.0000152$	-0.0162	-38.7167
γ	-3419.5	-1662.0	$+3.7581$	$+1682.1$	$+4011600$
σ	$+58.5589$	-27.4477	$+0.0653$	$+27.8060$	$+66272$
μ	0	0	0	0	0

Table 4: The sensitivity indices for the model variables to the parameters at EE.

The effect on R_0



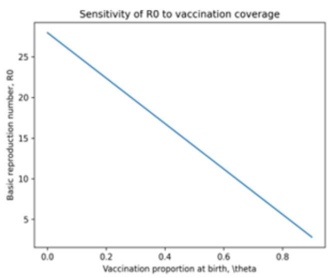


Figure 1: Sensitivity of R_0 to contact rate **Figure 2: Sensitivity of R_0 to Vaccination**

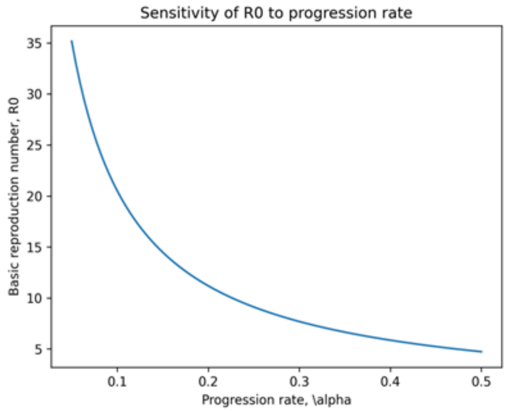


Figure 3: Sensitivity of R_0 to progression rate

IV. Discussion And Conclusion

The results show that vaccination coverage and contact rate are the most influential parameters. Strengthening vaccination programs and reducing contact rates are key to measles elimination.

Acknowledgment
The authors wish to sincerely thank the (anonymous reviewers).

Competing interest
The authors have declared that no competing interests exist.

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