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Discret Mathematics In General Solution And Displacement Of The Concrete Technology

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Abstract
In this paper the general solution provided a new method for solving the axisymmetric elastic space problem of cement concrete. Combining the Love method and the South well method, the South well operator was used in the variable selection process, the displacement function was introduced to express the displacement component by the Love method, the expression of stress indicated by the displacement function was obtained by combining the displacement components spatial problem where all variables are independent of θ and only related to r and z. Therefore, the shear stress, shear strain, and displacement along the θ direction are all zero [4, 5]. The general solution of the axial symmetry elastic space problem in cement concrete pavement is an elementary question., geometric equations, and physical equations, then the stress was substituted into the equilibrium equation, the biharmonic equation of displacement function was obtained by mathematical operation, and finally, a new general solution of the axisymmetric elastic space problem in cement concrete pavement was obtained. According to the proposed general solution, mechanical calculation.
Keywords*: axisymmetric, displacement, dimensions, amplitude. geometric equations, stress and strain.*

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I. Introduction

Substituting the obtained stress into the equilibrium equation, a harmonic equation about the displacement function is obtained, and solving this equation gives the expression of the displacement function; substituting the obtained expression into the displacement function to represent the displacement and stress, the general solution of the axisymmetric elastic space the introduction of the displacement function to represent the displacement component, and by combining the displacement component represented by the displacement function, and the geometric and physical equation, then the expression for stress represented by the displacement function is obtained. The problem is a special form of spatial problem where all variables are independent of θ and only related to r and z. Therefore, the shear stress, shear strain, and displacement along the θ direction are all zero [4, 5]. The solution of displacement and stress of the cement concrete pavement belongs to the axisymmetric elastic space problem [13]. Scholars have achieved many results in this area [14, 15]; however, it is difficult to find the stress function for pavement [16, 17]. The author adopted the South well operator without variable substitution but directly adopted the idea of the Love method to obtain a general solution of the axisymmetric elastic space problem, 2.The Derivation of General Solution**.**

The cylindrical coordinate system is adopted as solving the axisymmetric elastic space problem in pavement engineering. The basic equations are shown as follows.
Equilibrium equations are as follows:
Geometric equations are as follows:
Constitutive equations are as follows: where μ is Poisson's ratio, E is elasticity modulus, u and are displacement of pavement, and is shear modulus.
The compatibility equation is as follows:
The stress can be obtained as follows:
Substituting equation (5) into equations (1) and (4) yields
According to equations (2), (3), and (5), the displacement component can be expressed as follows:where , is the Southwell operator.
From equations (2), (3), (7), and (8), the expression of the displacement component and stress component is as follows:
Substituting equations (9)–(12) into equation (1), displacement must be satisfied by the following equation:
According to Hankel transform, we can obtain the following equation:
According to Hankel inverse transform, equation (14) can be rewritten as the following equation:

The differential operation is performed on both sides of equation (14) r and z, and we can get the following equations:
According to equations (16)–(18), the displacement of the cement concrete pavement can be expressed as follows:
Combining equations (13) and (20), we can draw the conclusion as the following equation:
According to Hankel transform, equation (21) can be rewritten as the following equation:
General solution is constructed to solve differential equation (22).
Substituting equation (23) into equation (14) yields:
Substituting equation (23) into equations (18) and (19) yields
Substituting equations (25) and (26) into equations (7)–(11), denoted by $A = \xi^2 A\xi$, $B = \xi\ B\xi$, $C = \xi^2 C\xi$, $D = \xi\ D\xi$, where .

II. The Relationship Between General Solutions

Parameters are identified as A_L, B_L, C_L, and D_L. Denoted by $A = A_L + (4\mu - 1)B_L$, $B = -B_L$, , and , we will just get the Love solution as follows by the equation of (27): where .Parameters are identified as A_S, B_S, C, and D_S. Denoted: by $A = A_s + 2\mu B_s$, $B = -B_s$, $C = C_s + 2\mu D_s$, and $D = D_s$, we will just get the Southwell solution as follows by the equation of (27):where .Denoted by $A_s = -A_L - (2\mu - 1)B_L$, $B_s = B_L$, $C_s = C_L + (2\mu - 1)D_L$, and $D_s = D_L$, we can convert the Southwell solution into Love solution.Combining these results leads to the general transformation relationship between solutions as shown in Table 1.

III. Application Of General Solution On Winkler Foundation As Shown In Figure 1, Δ Is Radius Of The Circle, And Is Vertical Circular Uniform Distributed Load.

Assumptions It is assumed the out-of-plane displacement w, is only differentiable in x and y axes. Consequently the vertical strain, $\varepsilon z = 0$, and the effect of the out-of-plane normal stress on the response of the plate is small with respect to the other stresses. Thus, it can be neglected (i.e $\sigma z = 0$. However, in contrast to the CPT, it is also assumed that the vertical line that is initially normal to the middle surface of the plate before bending becomes parabolic after bending. 2.2. Kinematic Relations In-plane displacements, u and v are defined mathematically as; $u = uc + us$; $v = vc + vs$ (2) us and vs are the shear deformation components of the in-plane displacements and are defined mathematically as, $us = f(z) \cdot \emptyset x$; $vs = f(z) \cdot \emptyset y$ (3) Where; $\emptyset x$ = shear rotation in x-direction and $\emptyset y$ = shear rotation in y-direction According to the classical plate theory (CPT) the in–plane displacements, uc and vc can be expressed as; $uc = -z\theta cx = -z\ dw\ dx$; $vc = -z\theta cy = -z\ dw\ dy$ (4) Therefore, $u = -z\ \partial w\ \partial x + f(z) \cdot \emptyset x$; $v = -z\ \partial w\ \partial y + f(z) \cdot \emptyset y$ (5) 2.3. Strain – Displacement Relations Strain–displacement relations suitable for an isotropic and homogenous material in three dimensions were developed by Ventsel and Krauthammer (2001) and applying the assumption that the vertical strain, $\varepsilon z = 0$ to the strain – displacement we have; $\boldsymbol{\varepsilon x}$ = d$u\ dx = -z\ \partial\ 2w\ \partial x\ 2 + f(z) \cdot \partial\emptyset x\ \partial x$; $\boldsymbol{\varepsilon y} = dv\ dy = -z\ \partial\ 2w\ \partial y2 + f(z) \cdot \partial\emptyset y\ \partial y$ (6) $\boldsymbol{\gamma xy} = du\ dy + dv\ dx = -z\ \partial\ 2w\ \partial x\ \partial y + f(z) \cdot \partial\emptyset x\ \partial y - z\ \partial\ 2w\ \partial x\ \partial y + f(z) \cdot \partial\emptyset y\ \partial x = -2z\ \partial\ 2w\ \partial x\ \partial y + f(z) \cdot \partial\emptyset x\ \partial y + f(z)$ (7) $\boldsymbol{\gamma xz} = du\ dz + dw\ dx = -\ \partial w\ \partial x + f(z) \cdot \partial\emptyset x\ \partial z + dw\ dx = f(z) \cdot \partial\emptyset x\ \partial z$ (8) $\boldsymbol{\gamma yz} = dv\ dz + dw\ dy = -\ \partial w\ \partial y + f(z) \cdot \partial\emptyset y\ \partial z + dw\ dy = f(z) \cdot \partial\emptyset y\ \partial z$ (9) Where u, v, w are displacements in x, y, and z directions respectively. 2.4. Constitutive (Stress–Strain) Boundary conditions are as follows: Combining equations (26) and (31), we can get Denoted by $z = 0$, from equations (30) and (31), we can getCombining equations (30) and (33), we can get the following equation: Substituting equations (33) and (34) into equation (26) yiel dwhere $x = \xi\delta$.5. The Proposed General Solution Apply to Winkler Foundation5.1. ModelThe arrangement of the circular uniform load on the Winkler foundation is shown in Figure 1.2.

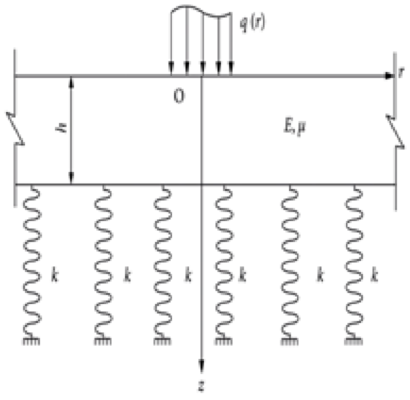
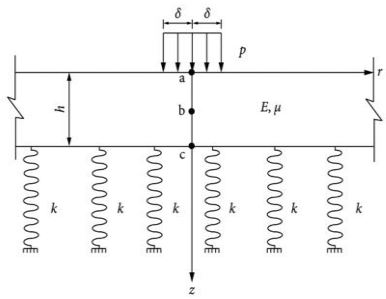


Figure 1.2: Calculation sketch

Boundary conditions are as follows: Substituting equation (26) into equations (36) and (37) yields According to Hankel transform, we can obtain the following equation:where h is the thickness of the cement concrete slab; *k* is Winkler foundation model. The solution of equation (39) can obtain the expression for *A*, *B*, *C*, and *D* about ξ, μ, *E*, *h*, and *k*. Substituting *A*, *B*, *C*, and *D* into equation (27), the displacement and stress of cement concrete pavement can be obtained.

Examples

The calculation diagram of cement concrete pavement is shown in Figure 3, the specific calculation parameters are E = *11500 MPa,* h = *0.18 m,* μ = *0.15,* k = *1.45 ×* 10^7 *N/*m^3*,* p = *700 kN/*m^2*,* r = *0.151 m, I choose integral interval 0–10, 0–20, 0–30, 0–40, 0–50, 0–500 and calculate displacement at point* a*, stress at point* a*,* b*,* c*, and the results are summarized in Tables 1–4.*



Based on Table 2, displacement at point *a* is about 0.86 mm. According to Tables 3−5, stress at point a is −1.1 M Pa, stress at point *b* is −0.028 M Pa, and stress at point *c* is 1.09 M Pa.

Table: 5 Stress at pint c

Variation of Displacement and Stress with r

The relative stiffness radius *l* of cement concrete pavement is as follows:

l ∈ (0.8, 1.5), stresses and displacement vary with cement concrete pavement board size, and the range of finite size slab instead of infinite size slab is referred to Z heng [6]:

Calculation range of *r* on the Winkler foundation is 0 to 4.4 m. According to the former calculation models, the displacement at point a on the top surface of the slab is calculated to vary with *r* as shown in Figure 1.4, and the stresses at points *a*, *b*, and *c* vary with *r* as shown in Figures respectively.

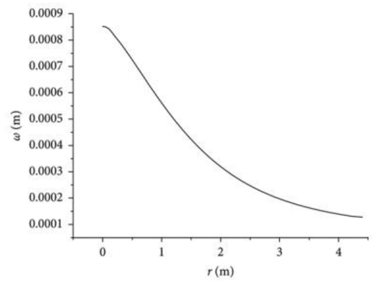


Figure 1.4: Displacement varied with r at point a

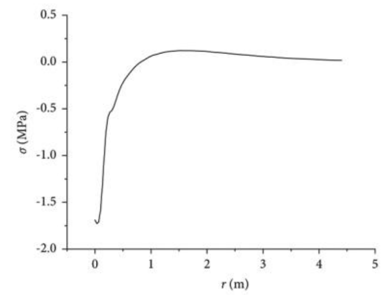


Figure1.5: stress varied with a

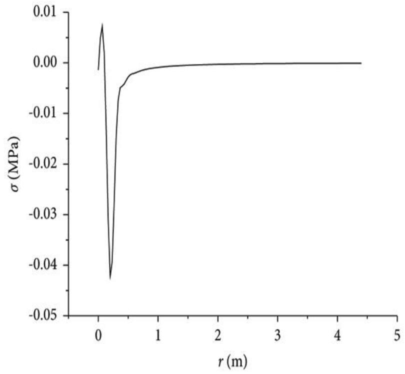


Figure: 1.6 stress varied with r and c Brittle materials are strong in compression and weak in tension. In this itself the Young's modulus is different for short time (eg: live load) and long time (eg: creep) lodings. Young's modulus is more in case of short time loading and less in

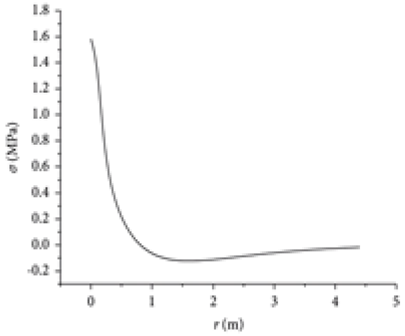
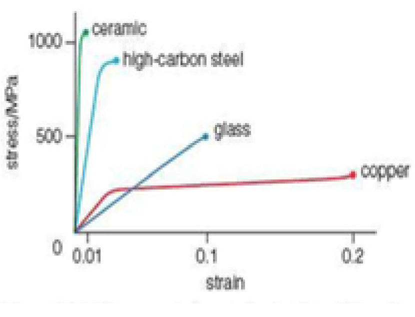


figure1. 7: stress varied with r and c

From Figure 4, the displacement at point a decreases rapidly with the increase of *r*, and the displacement at point a gradually tends to a constant value when the *r* is greater than 3 m. From Figures 5–7, it can be seen that the stresses at each point occurred great changing amplitude with the increase of *r*.

Stress-strain graphs allow us to describe the properties of materials, and also to predict the stresses at which changes in these properties might occur



This graph compares the stress-strain graphs for four different materials: ceramic, steel, glass and copper Ceramics are extremely strong and have very high UTS values. However, they show very little plastic behaviour before they fracture, so they are also very brittle Glasses have lower UTS values than ceramics and so are less strong, but they are also brittle, generally showing no plastic behavior before they break Steel is made by adding different elements to iron to form an alloy

Strain Enery Density: The strain energy density is the strain energy per unit volume of a sample Again, this does not depend on the dimensions of the material being tested (the result will be the same for all materials, regardless)⇒ From earlier we saw that: Strain energy = 1/2 F ▵ l⇒ If l is the original length of the wire, and A is its cross section, then the volume of the wire is Al. Therefore:

Result: In particular, the stress of the plate changes from compressive stress to tensile stress (Figure 5) or changes from tensile to compressive stresses (Figures 6and 7) with the increase of *r*. By variable transform, the general solution can be transformed into the South solution. According to the boundary conditions of the pavement and the characteristics of the general solution.

IV. Conclusions

The general solution provides a broader idea for analyzing the axisymmetric elastic space problem of cement concrete pavement. Using the general solution proposed by the author, the mechanical calculation of a cement concrete pavement slab on Winkler foundation as an illustration is carried out, the displacement curve at the top surface of the slab and the stress curves at the top, middle, and bottom surface of the slab were obtained, and the result showed that the method is feasible in the calculation space problem was derived in pav displacement . solution, the general solution is divided into the general solution for solving the composite boundary, the general solution for the stress problem, and the general solution for the displacement problem, these three types of problem are represented in the form of the Love general solution, the Southwell general solution, and the proposed general solution in the paper.

Data Availability
The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest: The authors declare that there are no conflicts of interest.

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