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ON FUZZY U-SPACES

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ABSTRACT
In this paper, the notions of fuzzy U-spaces and fuzzy ultranormal spaces are introduced and studied. Sufficient conditions for fuzzy strongly zero-dimensional spaces to become fuzzy U-spaces and fuzzy U-spaces to become fuzzy strongly zero-dimensional spaces are obtained. Certain fuzzy topological spaces which are not fuzzy U-spaces are identified. The inter-relations between fuzzy ultranormal spaces and fuzzy normal spaces are established and sufficient conditions under which fuzzy ultranormal become fuzzy U-spaces, are identified in this paper.
KEYWORDS : *Fuzzy F_σ-set, fuzzy G_δ-set, fuzzy clopen set, fuzzy simply* open set, fuzzy F' space, fuzzy P-space, fuzzy perfectly disconnected space, fuzzy normal space, fuzzy connected space, fuzzy Brown space, fuzzy weakly F_σ-complemented space, weak fuzzy Oz-space.*
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I. INTRODUCTION

The universe is a complex system filled with uncertainties. Problems of vagueness have probably always existed in human experience and vagueness is not regarded with suspicion, but is simply an acknowledged characteristic of the world around us. Mathematics equips us with the tools to quantify and manage these uncertainties. Human concepts have a graded structure in that whether or not a concept applies to a given object is a matter of degree, rather than a yes –or - no question and that people are capable of working with the degrees in a consistent way. In 1965, **L.A. Zadeh** [**40**] in his classic paper, called the concepts with a graded structure "*fuzzy concepts*" and proposed the notion of a " fuzzy set " that gave birth to the field of fuzzy logic . The potential of fuzzy notion was realized by researchers and has successfully been applied for new investigations in all branches of Science and Technology for more than last six decades. In 1968, **C.L. Chang** [**6**] introduced the concept of fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

In the recent years, there has been a growing trend to introduce and study different types of fuzzy topological spaces. Also a lot of fuzzy separation axioms have been defined that generalize the classical ones and **B.Hutton** [**13**] in 1980 studied hierarchy of separation axioms in fuzzy topological spaces and some hereditary properties of the separation axioms. In 2004, **M. Henriksen** and **R.G. Woods** [**12**] introduced the notion of cozero complemented space and **R. Levy** and **J. Shapiro** [**15**] studied cozero complemented spaces under the name "*z-good spaces*". In [**2**], **F. Azarpanah** and **M.Karavan** studied cozero complemented spaces in the name "*m-spaces*". In 2009, **Knox, M. L.,** *et al* [**14**] introduced the notion of weakly cozero complemented space. The notion of feebly F_σ-complemented spaces was introduced and studied by **T.Dube** and **O. Ighedo** [**7**]. Motivated on these lines , the notions of F_σ-complemented spaces, weakly F_σ- complemented spaces and feebly F_σ-complemented spaces in fuzzy setting were introduced and studied by **G. Thangaraj** and **L .Vikraman** in [**30,31,32**] **.**

In classical topology, the definition of U-space is due to **Gillman** and **Henriksen** [**11**]. In 2020 , **W.D.Burgess,** and **R. Raphael** [**5**] established that U-space is a topological space in which any two disjoint co-zero sets can be separated by a clopen set. Motivated on these lines, in Section **3,** the notion of fuzzy U-space is introduced and several characterizations of fuzzy U-spaces are established in this paper.

The notion of zero-dimensional space was defined by **Sierpinski** , in 1921. A topological space (X,τ) is called zero dimensional provided that X has a basis consisting of clopen sets. **N.Ajmal** and **J.K. Kohli** [**1**] extended the notions of connectedness to an arbitrary fuzzy set and defined the notions of *c*-zero dimensionality, strong *c*- zero dimensionality and total disconnectedness and studied their basic properties and interrelationships among them. In classical topology, **P. J. Nyikos [16]** introduced and studied the notion of strongly zero- dimensional spaces as Tychonoff spaces whose Čech-Stone compactification βX are totally disconnected. **William G. Bade** [**39**] defined a topological space X strongly zero -dimensional as given any two disjoint zero sets Z_1 and Z_2 in X, there exists a clopen set C such that $Z_1 \subseteq C,\ Z_2 \cap \mathrm{C} = \emptyset$. In continuation of their works, in Section **4,** the notion of fuzzy strong zero dimensional space is defined in terms of fuzzy G_δ -sets and sufficient conditions under which fuzzy strongly zero-dimensional spaces become fuzzy U-spaces are explored. In Section **5**, sufficient conditions under which fuzzy U-spaces become fuzzy strongly zero-dimensional spaces are obtained and some of the fuzzy topological spaces that are not fuzzy U-spaces , are also identified in this paper.

R. Staum [**17**] introduced the notion of ultranormality in classical topology . In 2013**, Joseph Van Name** [**38**] gave an overview of the notions of ultranormality and ultraparacompactness in the general topological and point-free contexts. Motivated on these lines, in Section **6,** the notion of fuzzy ultranormal space is introduced and inter-relations between fuzzy ultranormal spaces and fuzzy normal spaces are established and sufficient conditions under which fuzzy ultranormal spaces become fuzzy U-spaces , are obtained in this paper.

II. PRELIMINARIES

Some basic notions and results used in the sequel, are given in order to make the exposition self - contained. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$. For any fuzzy set λ in X and a family $(\lambda_i)_{i \in J}$ of fuzzy sets in X, the complement λ' , the union $\vee_{i\in j} \lambda_i$ and the intersection $\wedge_{i\in j} \lambda_i$ are defined respectively as follows : for each $x \in X$, $\lambda'(x) = 1 - \lambda(x)$, $\left(\vee_{i\in j} \lambda_i\right)(x) = \sup_{i\in j} \lambda_i(x)$, $\left(\wedge_{i\in j} \lambda_i\right)(x) = \inf_{i\in j} \lambda_i(x)$, where J is an index set.

Definition 2.1 [**6**]: A fuzzy topology on a set X is a family T of fuzzy sets in X which satisfies the following conditions **:**

(a). $0_X \in T$ and $1_X \in T$,
(b). If A, B ∈ T, then $A \wedge B \in T$,
(c). If $A_i \in T$ for each $i \in J$, then $\vee_{i\in J} A_i \in T$.

The pair (X, T) is called a fuzzy topological space (briefly, fts). Members of T are called fuzzy open sets in X and their complements are called fuzzy closed sets in X. A fuzzy set is considered fuzzy clopen if it is both fuzzy closed and fuzzy open.

Definition 2.2 [**6**] **:** Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior , the closure and the complement of λ are defined respectively as follows :

(i). $\mathrm{int}(\lambda) = \vee \{ \mu \in T : \mu \leq \lambda \}$,
(ii). $\mathrm{cl}(\lambda) = \wedge \{ \mu' : \lambda \leq \mu',\ \mu \in T \}$.

Lemma 2.1 [**3**] **:** For a fuzzy set λ of a fuzzy topological space X,
(i). $1 - \mathrm{int}(\lambda) = \mathrm{cl}(1 - \lambda)$ and (ii). $1 - \mathrm{cl}(\lambda) = \mathrm{int}(1 - \lambda)$.

Definition 2.3 : A fuzzy set λ in a fuzzy topological space (X,T) is called a

(i). fuzzy regular-open set in (X,T) if $\lambda = \mathrm{int\,cl}(\lambda)$ and

fuzzy regular-closed set in (X,T) if $\lambda = \mathrm{cl\,int}(\lambda)$ [**3**].

(ii). fuzzy G_δ -set in (X,T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where $\lambda_i \in T$ for $i \in I$;
fuzzy F_σ -set in (X,T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ [**4**].
(iii). fuzzy dense set in (X,T) if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $\mathrm{cl}(\lambda) = 1$, in (X,T) [**19**] .

(iv). fuzzy nowhere dense set in (X,T) if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\, cl(\lambda) = 0$, in (X,T) [**19**].
(v). fuzzy first category set in (X,T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category [**19**].
(vi). fuzzy somewhere dense set in (X,T) if there exists a non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\, cl(\lambda) \neq 0$, in (X,T) [**20**].
(vii) .fuzzy residual set in (X,T) if $1-\lambda$ is a fuzzy first category set in (X,T) [**21**].
(viii). fuzzy simply* open set in (X,T) if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in (X,T) and $1-\lambda$ is called a fuzzy simply* closed set in (X,T) [**24**].
(ix). fuzzy pseudo-open set in (X,T) if $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set and δ is a fuzzy first category set in (X,T) [**23**].

Definition 2.4 [ ] : A fuzzy topological space (X, T) is called a
(i). fuzzy normal space if for every pair of closed fuzzy sets P, Q such that $P \leq c(Q)$, there exist open fuzzy sets A, B such that $P \leq A$, $Q \leq B$ and $A \wedge B = O$ [**9**] .
(ii). fuzzy connected space if it has no proper fuzzy clopen (closed and open) set. [A fuzzy set λ in X is proper if $\lambda \neq 0$ and $\lambda \neq 1$] [**8**].
(iii). fuzzy extremally disconnected space if the closure of each fuzzy open set of (X,T) is fuzzy open in (X,T) [**10**].
(iv). fuzzy P-space if each fuzzy G_δ-set in (X,T) is fuzzy open in (X,T) [**18**].

(v). fuzzy D-Baire space if every fuzzy first category set in (X,T) is a fuzzy nowhere dense set in (X,T) [**22**].
(vi). fuzzy perfectly disconnected space if for any two non – zero fuzzy sets λ and μ defined on X with $\lambda \leq 1-\mu$, $cl(\lambda) \leq 1 - cl(\mu)$, in (X,T)[**25**].
(vii). fuzzy F'-space if $\lambda \leq 1-\mu$, where λ and μ are fuzzy F_σ -sets

in (X,T) , then $cl(\lambda) \leq 1 - cl(\mu)$, in (X,T)[**26**].

(viii). fuzzy F- space if for any two fuzzy F_σ- sets λ and μ in (X,T) with $\lambda \leq 1-\mu$, there exist fuzzy G_δ- sets α and β in (X,T) such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1-\beta$, in (X, T) [**27**].
(ix). fuzzy ∂^* space if each fuzzy G_δ - set in (X, T) is a fuzzy simply* open set in (X,T) [**28**].
(x). fuzzy Brown space if for any two non-zero fuzzy open sets λ and μ in (X,T), $cl(\lambda) \nleq 1 - cl(\mu)$, in (X, T) [**35**].
(xi). weak fuzzy Oz-space if for each fuzzy F_σ -set δ in (X,T), $cl(\delta)$ is a fuzzy G_δ -set in (X,T) [**34**].
(xii).fuzzy weakly F_σ-complemented space if for each pair of fuzzy F_σ- sets μ_1 and μ_2 with $\mu_1 \leq 1-\mu_2$ in (X,T), there exist fuzzy F_σ-sets λ_1 and λ_2 with $\lambda_1 \leq 1-\lambda_2$, in (X,T) such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$ [**31**].
(xiii). fuzzy feebly F_σ-complemented space if for each pair of fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1-\mu_2$ in (X,T), there exist fuzzy F_σ -sets λ_1 and λ_2 in (X,T) such that $\mu_1 \leq 1-\lambda_1$, $\mu_2 \leq 1-\lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$ [**32**].
(xiv). fuzzy strongly hyperconnected space if and only if each fuzzy somewhere dense set is a fuzzy open set in (X,T) [**33**].
(xv). fuzzy semi- P -space if each fuzzy G_δ-set in (X, T) is an fuzzy semi-open set in (X,T) [**36**].
(xvi). fuzzy S*N -space if for each pair of fuzzy closed sets μ_1 and μ_2 in (X,T) with $\mu_1 \leq 1-\mu_2$, there exist fuzzy simply* open sets λ_1 and λ_2 in (X,T) such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq \lambda_2$ and $\lambda_1 \leq 1-\lambda_2$ [**37**].

Theorem 2.1 [**35**] **:** If (X,T) is a fuzzy Brown space, then there is no fuzzy set λ which is both fuzzy open and fuzzy closed in (X,T).

Theorem 2.2 [**33**] **:** If (X,T) is a fuzzy strongly hyperconnected space, then there is no fuzzy set η which is both fuzzy open and fuzzy closed in (X,T).

Theorem 2.3 [**32**] **:** If a fuzzy topological space (X,T) is a fuzzy feebly F_σ-complemented space, then for each pair of fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$ in (X,T), there exist fuzzy G_δ-sets δ_1 and δ_2 in (X,T) such that $\mu_1 \leq \delta_1$, $\mu_2 \leq \delta_2$ and $\mathrm{int}(\delta_1) \leq 1 - \mathrm{int}(\delta_2)$.

Theorem 2.4 [**32**]**:** If a fuzzy topological space (X,T) is a fuzzy weakly F_σ-complemented space, then (X,T) is a fuzzy feebly F_σ-complemented space.

Theorem 2.5 [**37**]**:** If a fuzzy set λ is a fuzzy simply* open set in a fuzzy topological space (X,T), then $\mathrm{cl}(\lambda)$ is a fuzzy F_σ-set in (X,T).

Theorem 2.6 [**28**] **:** If a fuzzy set λ is a fuzzy G_δ-set in a fuzzy ∂^*space (X, T), then $\mathrm{cl}(\lambda)$ is a fuzzy F_σ-set in (X, T).

Theorem 2.7 [**29**] **:** If λ is a fuzzy pseudo-open set in a fuzzy D-Baire space (X, T), then λ is a fuzzy simply* -open set in (X, T).

Theorem 2.8 [**25**] **:** If (X,T) is a fuzzy perfectly disconnected space, then (X,T) is a fuzzy extremally disconnected space.

Theorem 2.9 [**36**] **:** If λ is a fuzzy G_δ-set in a fuzzy perfectly disconnected and fuzzy semi-P-space (X, T), then $\mathrm{cl}(\lambda)$ is a fuzzy open set in (X, T).

Theorem 2.10 [**36**] **:** If a fuzzy topological space (X,T) is a fuzzy P-space, then (X,T) is a fuzzy semi-P-space.

Theorem 2.11 [**37**] **:** If a fuzzy topological space (X, T) is a fuzzy perfectly disconnected and fuzzy S*N-space, then (X,T) is a fuzzy normal space.

III. FUZZY U-SPACES

Motivated by the works of **W.D. Burgess** and **R. Raphael** [**5**] on U-spaces in classical topology, the notion of fuzzy U-space shall be defined as follows :

Definition 3.1 : A fuzzy topological space (X,T) is called a fuzzy U-space if for each pair of fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$ in (X,T), there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda$, $\mu_2 \leq 1 - \lambda$.

Example 3.1 : Let $X = \{a, b, c, \}$. Let $I = [0, 1]$. The fuzzy sets α, β, γ, λ, δ and θ are defined on X as follows **:**

$\alpha : X \to I$ is defined by $\alpha(a) = 0.4$; $\alpha(b) = 0.5$; $\alpha(c) = 0.6$,
$\beta : X \to I$ is defined by $\beta(a) = 0.5$; $\beta(b) = 0.7$; $\beta(c) = 0.4$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.6$; $\gamma(b) = 0.4$; $\gamma(c) = 0.5$,
$\lambda : X \to I$ is defined by $\lambda(a) = 0.6$; $\lambda(b) = 0.5$; $\lambda(c) = 0.5$,
$\delta : X \to I$ is defined by $\delta(a) = 0.4$; $\delta(b) = 0.5$; $\delta(c) = 0.5$,
$\theta : X \to I$ is defined by $\theta(a) = 0.5$; $\theta(b) = 0.5$; $\theta(c) = 0.5$,
$\rho : X \to I$ is defined by $\rho(a) = 0.6$; $\rho(b) = 0.6$; $\rho(c) = 0.6$

Then, $T = \{ 0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma], \beta \vee [\alpha \wedge \gamma], \gamma \vee [\alpha \wedge \beta], \alpha \wedge [\beta \vee \gamma], \beta \wedge [\alpha \vee \gamma], \gamma \wedge [\alpha \vee \beta], \alpha \vee \beta \vee \gamma, 1 \}$ is a fuzzy topology on X. By computation, one shall find that

$$\rho = \{1 - \alpha\} \vee \{1 - \beta\} \vee \{1 - \gamma\};$$
$$\theta = \{1 - [\alpha \vee \beta]\} \vee \{1 - [\alpha \vee \gamma]\} \vee \{1 - [\beta \vee \gamma]\};$$
$$\delta = \{1 - [\alpha \vee \gamma]\} \vee \{1 - (\gamma \vee [\alpha \wedge \beta])\} \vee \{1 - [\alpha \vee \beta \vee \gamma]\}$$

$1 - (\alpha \wedge \beta) = \{1 - (\alpha \vee [\beta \wedge \gamma])\} \vee \{1 - (\alpha \wedge [\beta \vee \gamma])\} \vee \{1 - (\beta \wedge [\alpha \vee \gamma])$, are fuzzy F_σ-sets in X. One can see that $\lambda = \gamma \vee [\alpha \wedge \beta]$ and $\lambda = 1 - (\alpha \wedge [\beta \vee \gamma])$ and thus λ is an fuzzy clopen set λ in X. Also on computation one shall find that $\rho \nleq 1 - \theta$, $\rho \nleq 1 - [1 - (\alpha \wedge \beta)]$, $\rho \nleq 1 - \delta$, $\theta \nleq 1 - [1 - (\alpha \wedge \beta)]$, $\theta \leq 1 - \delta$ and $1 - (\alpha \wedge \beta) \nleq 1 - \delta$, in X. Thus, for the fuzzy F_σ-sets θ and δ in X with $\theta \leq 1 - \delta$, there exists a fuzzy clopen set λ in X such that $\theta \leq \lambda$ and $\delta \leq 1 - \lambda$. Hence (X,T) is an fuzzy U-space.

Example 3.2 : Let $X = \{a, b, c, \}$. Let $I = [0, 1]$. The fuzzy sets α, β, γ, λ, δ θ and ρ are defined on X as follows **:**

$\alpha : X \to I$ is defined by $\alpha(a) = 0.5$; $\alpha(b) = 0.4$; $\alpha(c) = 0.7$,
$\beta : X \to I$ is defined by $\beta(a) = 0.6$; $\beta(b) = 0.5$; $\beta(c) = 0.6$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.4$; $\gamma(b) = 0.6$; $\gamma(c) = 0.3$,
$\lambda : X \to I$ is defined by $\lambda(a) = 0.6$; $\lambda(b) = 0.5$; $\lambda(c) = 0.7$,

$\delta : X \to I$ is defined by $\delta(a) = 0.4$; $\delta(b) = 0.5$; $\delta(c) = 0.3$,
$\theta : X \to I$ is defined by $\theta(a) = 0.4$; $\theta(b) = 0.4$; $\theta(c) = 0.3$,
$\rho : X \to I$ is defined by $\rho(a) = 0.6$; $\rho(b) = 0.6$; $\rho(c) = 0.7$.

Then, $T = \{ 0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma], \gamma \vee [\alpha \wedge \beta], \beta \wedge [\alpha \vee \gamma], \alpha \vee \beta \vee \gamma, 1 \}$ is a fuzzy topology on X. By computation, one shall find that

$$\alpha \vee \beta \vee \gamma = \{1 - \alpha\} \vee \{1 - \beta\} \vee \{1 - \gamma\} ;$$
$$1 - (\beta \wedge [\alpha \vee \gamma] = \{1 - [\alpha \vee \beta]\} \vee \{1 - [\alpha \vee \gamma]\} \vee \{1 - [\beta \vee \gamma]\} ;$$
$$1 - \beta = \{1 - [\alpha \vee \beta]\} \vee \{1 - [\beta \vee \gamma]\} \vee \{1 - [\alpha \vee \beta \vee \gamma]\}$$

are fuzzy F_σ-sets in X.

One can see that $\lambda = \alpha \vee \beta$ and $\lambda = 1 - [\beta \wedge \gamma]$
$\delta = \beta \wedge \gamma$ and $\delta = 1 - [\alpha \vee \beta]$
$\theta = \alpha \wedge \gamma$ and $\theta = 1 - [\alpha \vee \beta \vee \gamma]$
$\rho = \alpha \vee \beta \vee \gamma$ and $\rho = 1 - [\alpha \wedge \gamma]$

Then, λ, δ, θ and ρ are fuzzy clopen sets in X. Also on computation one shall find that $(1 - \beta) \leq 1 - \{1 - (\beta \wedge [\alpha \vee \gamma]\}$ and $(1 - \beta) \nleq 1 - (\alpha \vee \beta \vee \gamma)$, $\{1 - (\beta \wedge [\alpha \vee \gamma]\} \nleq 1 - \{1 - (\alpha \vee \beta \vee \gamma)\}$ in X. Thus, for the fuzzy F_σ-sets $(1 - \beta)$ and $\{1 - (\beta \wedge [\alpha \vee \gamma]\}$, in X with $(1 - \beta) \leq 1 - \{1 - (\beta \wedge [\alpha \vee \gamma]\}$, there exists no fuzzy clopen set σ $(= \lambda, \delta, \theta, \rho)$ in X such that $(1 - \beta) \leq \sigma$ and $1 - (\beta \wedge [\alpha \vee \gamma] \leq 1 - \sigma$. Hence (X,T) is not an fuzzy U-space.

Proposition 3.1 : If μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$ in a fuzzy U-space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda \leq 1 - \mu_2$.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy U-space, there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda$, $\mu_2 \leq 1 - \lambda$. Now $\mu_2 \leq 1 - \lambda$, implies that $\lambda \leq 1 - \mu_2$ and thus $\mu_1 \leq \lambda \leq 1 - \mu_2$, in (X,T).

Proposition 3.2 : If μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$ in a fuzzy U-space (X,T), then $cl(\mu_1) \neq 1$ and $cl(\mu_2) \neq 1$, in (X,T).

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy U-space, by Proposition **3.1,** there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda \leq 1 - \mu_2$. Then, $cl(\mu_1) \leq cl(\lambda) \leq cl(1 - \mu_2)$ and $cl(\mu_1) \leq \lambda \leq 1 - int(\mu_2)$. Now, λ being a fuzzy closed set in (X,T), $cl(\lambda) = \lambda$ and then $cl(\mu_1) \leq \lambda \leq 1 - int(\mu_2)$. Thus, $cl(\mu_1) \neq 1$, in (X,T). Also $\mu_2 \leq 1 - \lambda$, implies that $cl(\mu_2) \leq cl(1 - \lambda) = 1 - int(\lambda) \leq 1$ and thus $cl(\mu_2) \neq 1$, in (X,T).

Proposition 3.3 : If δ_1 and δ_2 are fuzzy G_δ-sets with $1 - \delta_1 \leq \delta_2$ in a fuzzy U-space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $1 - \delta_1 \leq \lambda \leq \delta_2$.

Proof : Suppose that δ_1 and δ_2 are fuzzy G_δ-sets with $1 - \delta_1 \leq \delta_2$, in (X,T). Then, $1 - \delta_1 \leq 1 - (1 - \delta_2)$, in (X,T). Since (X,T) is a fuzzy U-space, for the fuzzy F_σ-sets $1 - \delta_1$ and $1 - \delta_2$, by Proposition **3.1,** there exists a fuzzy clopen set λ in (X,T) such that $1 - \delta_1 \leq \lambda \leq 1 - (1 - \delta_2)$. Then, $1 - \delta_1 \leq \lambda \leq \delta_2$, in (X,T).

Corollary 3.1 : If δ_1 and δ_2 are fuzzy G_δ-sets with $1 - \delta_1 \leq \delta_2$, in a fuzzy U-space (X,T), then $int(\delta_1) \neq 0$ and $int(\delta_2) \neq 0$, in (X,T).

Proof : Suppose that δ_1 and δ_2 are fuzzy G_δ-sets with $1 - \delta_1 \leq \delta_2$, in a fuzzy U-space (X,T). Then, by Proposition **3.3**, there exists a fuzzy clopen set λ in (X,T) such that $1 - \delta_1 \leq \lambda \leq \delta_2$. Now $\lambda \leq \delta_2$, implies that $int(\lambda) \leq int(\delta_2)$ and then $\lambda = int(\lambda) \leq int(\delta_2)$ shows that $int(\delta_2) \neq 0$, in (X,T). Also $1 - \delta_1 \leq \lambda$, implies that $cl(1 - \delta_1) \leq cl(\lambda)$. By Lemma **2.1,** $cl(1 - \delta_1) = 1 - int(\delta_1)$ and then $1 - int(\delta_1) \leq \lambda$ implies that $1 - \lambda \leq int(\delta_1)$ and thus $int(\delta_1) \neq 0$, in (X,T).

Proposition 3.4 : If μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$ in a fuzzy U-space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \wedge \mu_2 \leq Bd(\lambda)$.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy U-space, there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda$, $\mu_2 \leq 1 - \lambda$. Since λ is a fuzzy clopen set in (X,T), $cl(\lambda) = \lambda$ and $cl(1 - \lambda) = 1$

$-$ int $(\lambda) = 1 - \lambda$. Now $\mu_1 \wedge \mu_2 \leq \lambda \wedge [1 - \lambda] = \mathrm{cl}(\lambda) \wedge \mathrm{cl}(1-\lambda) = \mathrm{Bd}(\lambda)$ [**22**] and thus $\mu_1 \wedge \mu_2 \leq \mathrm{Bd}(\lambda)$, in (X,T).

Proposition 3.5 : If μ_1 and μ_2 are fuzzy simply* open sets with $\mathrm{cl}(\mu_1) \leq 1 - \mathrm{cl}(\mu_2)$ in a fuzzy U-space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda \leq 1 - \mu_2$.

Proof : Suppose that μ_1 and μ_2 are fuzzy simply* open sets in (X,T). By Theorem **2.5**, $\mathrm{cl}(\mu_1)$ and $\mathrm{cl}(\mu_2)$ are fuzzy F_σ-sets in (X, T). By hypothesis $\mathrm{cl}(\mu_1) \leq 1 - \mathrm{cl}(\mu_2)$, in (X,T). Since (X,T) is a fuzzy U-space, there exists a fuzzy clopen set λ in (X,T) such that $\mathrm{cl}(\mu_1) \leq \lambda$, $\mathrm{cl}(\mu_2) \leq 1 - \lambda$. Then, $\mu_1 \leq \mathrm{cl}(\mu_1) \leq \lambda$ and $\mu_2 \leq \mathrm{cl}(\mu_2) \leq 1 - \lambda$, in (X, T) and thus it follows that $\mu_1 \leq \lambda \leq 1 - \mathrm{cl}(\mu_2) \leq 1 - \mu_2$ and thus $\mu_1 \leq \lambda \leq 1 - \mu_2$, in (X,T).

Proposition 3.6 : If μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$ in a fuzzy U-space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $\mathrm{int}(\mu_1) \leq \lambda \leq 1 - \mathrm{int}(\mu_2)$.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy U-space, by Proposition **3.1,** there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda \leq 1 - \mu_2$. Then, $\mathrm{int\,cl}(\mu_1) \leq \mathrm{int\,cl}(\lambda) \leq \mathrm{int\,cl}(1 - \mu_2)$. Since λ is a fuzzy clopen set, $\mathrm{int\,cl}(\lambda) = \mathrm{int}(\lambda) = \lambda$ and then $\mathrm{int}(\mu_1) \leq \mathrm{int\,cl}(\mu_1) \leq \lambda \leq 1 - \mathrm{cl\,int}(\mu_2) \leq 1 - \mathrm{int}(\mu_2)$. Hence, it follows that $\mathrm{int}(\mu_1) \leq \lambda \leq 1 - \mathrm{int}(\mu_2)$.

Corollary 3.2 : If μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$ in a fuzzy U-space (X,T), then $\mathrm{int}(\mu_1) \leq 1 - \mathrm{int}(\mu_2)$, in (X,T).

Proposition 3.7 : If μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$ in a fuzzy U-space (X,T), then there exists a fuzzy regular open set λ in (X,T) such that $\mathrm{int}(\mu_1) \leq \lambda \leq 1 - \mathrm{int}(\mu_2)$.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy U-space, by Proposition **3.1,** there exists a fuzzy clopen set λ in (X,T) such tha $\mu_1 \leq \lambda \leq 1 - \mu_2$. Then, $\mathrm{int\,cl}(\mu_1) \leq \mathrm{int\,cl}(\lambda) \leq \mathrm{int\,cl}(1 - \mu_2)$ and $\mathrm{int\,cl}(\mu_1) \leq \mathrm{int\,cl}(\lambda) \leq 1 - \mathrm{cl\,int}(\mu_2)$ [by Lemma **2.1**]. Since λ is a fuzzy clopen set, $\mathrm{int\,cl}(\lambda) = \mathrm{int}(\lambda) = \lambda$ and then λ is a fuzzy regular open set in (X,T). Now $\mathrm{int}(\mu_1) \leq \mathrm{int\,cl}(\mu_1) \leq \lambda \leq 1 - \mathrm{cl\,int}(\mu_2) \leq 1 - \mathrm{int}(\mu_2)$. Hence, it follows that $\mathrm{int}(\mu_1) \leq \lambda \leq 1 - \mathrm{int}(\mu_2)$, in (X,T).

IV. FUZZY STRONGLY ZERO-DIMENSIONAL SPACES

Motivated by the works of **P.Nyikos** [**16**] and **William G. Bade** [**39**] on strongly zero-dimensional spaces in classical topology, the notion of fuzzy strongly zero-dimensional space shall be defined in terms of fuzzy G_δ-sets as follows :

Definition 4.1 : A fuzzy topological space (X,T) is called a fuzzy strongly zero-dimensional space if for each pair of fuzzy G_δ-sets δ_1 and δ_2 with $\delta_1 \leq 1 - \delta_2$ in (X,T), there exists a fuzzy clopen set λ in (X,T) such that $\delta_1 \leq \lambda$, $\delta_2 \leq 1 - \lambda$.

Example 4.1 : Let X = { a, b, c, }. Let I = [0, 1]. The fuzzy sets α, β, γ, λ, δ, θ, ρ, μ and σ are defined on X as follows **:**

$\alpha : X \to I$ is defined by $\alpha(a) = 0.4$; $\alpha(b) = 0.5$; $\alpha(c) = 0.6$,

$\beta : X \to I$ is defined by $\beta(a) = 0.5$; $\beta(b) = 0.7$; $\beta(c) = 0.4$,

$\gamma : X \to I$ is defined by $\gamma(a) = 0.6$; $\gamma(b) = 0.4$; $\gamma(c) = 0.5$,

$\lambda : X \to I$ is defined by $\lambda(a) = 0.6$; $\lambda(b) = 0.5$; $\lambda(c) = 0.6$,

$\delta : X \to I$ is defined by $\delta(a) = 0.5$; $\delta(b) = 0.5$; $\delta(c) = 0.6$,

$\theta : X \to I$ is defined by $\theta(a) = 0.4$; $\theta(b) = 0.5$; $\theta(c) = 0.4$,

$\rho : X \to I$ is defined by $\rho(a) = 0.5$; $\rho(b) = 0.5$; $\rho(c) = 0.4$,

$\mu : X \to I$ is defined by $\mu(a) = 0.4$; $\mu(b) = 0.4$; $\mu(c) = 0.4$,

$\sigma : X \to I$ is defined by $\sigma(a) = 0.5$; $\sigma(b) = 0.5$; $\sigma(c) = 0.5$.

Then, $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \vee [\beta \wedge \gamma], \beta \vee [\alpha \wedge \gamma], \gamma \vee [\alpha \wedge \beta], \alpha \wedge [\beta \vee \gamma], \beta \wedge [\alpha \vee \gamma], \gamma \wedge [\alpha \vee \beta], \alpha \vee \beta \vee \gamma, 1\}$ is a fuzzy topology on X. By computation, one shall find that

$\gamma = \gamma \wedge (\alpha \vee \gamma) \wedge (\beta \vee \gamma) \wedge (\gamma \vee [\alpha \wedge \beta]) \wedge (\alpha \vee \beta \vee \gamma)$,
$\mu = \alpha \wedge (\alpha \wedge \beta) \wedge (\alpha \wedge \gamma) \wedge (\alpha \wedge [\beta \vee \gamma])$,
$\beta \wedge \gamma = \beta \wedge (\beta \wedge \gamma) \wedge (\beta \vee [\alpha \wedge \gamma]) \wedge (\alpha \vee [\beta \wedge \gamma]) \wedge (\beta \wedge [\alpha \vee \gamma])$,
$\sigma = (\alpha \vee \beta) \wedge (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$, are fuzzy G_δ -sets in X.

Let $\delta_1 = \gamma$, $\delta_2 = \mu$, $\delta_3 = \beta \wedge \gamma$ and $\delta_4 = \sigma$. On computation, one shall find that $\delta_1 \leq 1 - \delta_2$, $\delta_1 \nleq 1 - \delta_3$, $\delta_1 \nleq 1 - \delta_4$, $\delta_2 \leq 1 - \delta_3$, $\delta_2 \leq 1 - \delta_4$ and $\delta_3 \leq 1 - \delta_4$ in (X,T).

Now one shall see that $\lambda = \alpha \vee \gamma$ and $\lambda = 1 - (\alpha \wedge \beta)$ in (X,T). Thus, λ is an fuzzy clopen set in (X,T) and for the fuzzy G_δ –sets δ_1, δ_2 with $\delta_1 \leq 1 - \delta_2$, $\delta_1 \leq \lambda$ and $\delta_2 \leq 1 - \lambda$, in (X,T).

Also $\delta = \alpha \vee [\beta \wedge \gamma]$ and $\delta = 1 - (\beta \wedge [\alpha \vee \gamma])$ in (X,T) . Thus, δ is a fuzzy clopen set in (X,T) and for the fuzzy G_δ -sets δ_2, δ_3 with $\delta_2 \leq 1 - \delta_3$, $\delta_2 \leq \delta$ and $\delta_3 \leq 1 - \delta$, in (X,T).

Now one shall see that $\theta = \alpha \wedge \beta$ and $\theta = 1 - (\alpha \vee \gamma)$ in (X,T) . Thus, θ is a fuzzy clopen set in (X,T) and for the fuzzy G_δ -sets δ_2, δ_4 with $\delta_2 \leq 1 - \delta_4$, $\delta_2 \leq \theta$ and $\delta_4 \leq 1 - \theta$, in (X,T).

Also $\rho = \beta \wedge [\alpha \vee \gamma]$ and $\rho = 1 - (\alpha \vee [\beta \wedge \gamma])$ in (X,T) . Thus, ρ is a fuzzy clopen set in (X,T) and for the fuzzy G_δ -sets δ_3, δ_4 with $\delta_3 \leq 1 - \delta_4$, $\delta_3 \leq \rho$ and $\delta_4 \leq 1 - \rho$, in (X,T) and hence (X,T) is a fuzzy strongly zero-dimensional space.

Proposition 4.1 : If $\delta \leq \gamma$, where δ is a fuzzy G_δ-set and γ is a fuzzy F_σ-set in a fuzzy strongly zero-dimensional space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $\delta \leq \lambda \leq \gamma$.

Proof : Suppose that δ is a fuzzy G_δ-set and γ is a fuzzy F_σ-set with $\delta \leq \gamma$, in (X,T). Then, $\delta \leq 1 - (1 - \gamma)$, where $1 - \gamma$ is a fuzzy G_δ-set in (X,T). Since (X,T) is a fuzzy strongly zero-dimensional space, there exists a fuzzy clopen set λ in (X,T) such that $\delta \leq \lambda$ and $1 - \gamma \leq 1 - \lambda$ and it follows that $\delta \leq \lambda \leq \gamma$, in (X,T).

Proposition 4.2 : If $1 - \mu_2 \leq \mu_1$, where μ_1 and μ_2 are fuzzy F_σ-sets in a fuzzy strongly zero-dimensional space (X,T), then there exists a fuzzy clopen set δ in (X,T) such that $1 - \mu_2 \leq \delta \leq \mu_1$.

Proof : Suppose that $1 - \mu_2 \leq \mu_1$, where μ_1 and μ_2 are fuzzy F_σ-sets in (X,T). Then, $1 - [1 - \mu_2] \geq 1 - \mu_1$ where $1 - \mu_1$ and $1 - \mu_2$ are fuzzy G_δ-sets in the fuzzy strongly zero-dimensional space (X,T). Then, there exists a fuzzy clopen set λ in (X,T) such that $1 - \mu_1 \leq \lambda$ and $1 - \mu_2 \leq 1 - \lambda$. This implies that $1 - \mu_2 \leq 1 - \lambda \leq \mu_1$. Let $\delta = 1 - \lambda$. Then, δ is a clopen set in (X,T) such that $1 - \mu_2 \leq \delta \leq \mu_1$.

Proposition 4.3 : If a fuzzy topological space (X,T) is a fuzzy perfectly disconnected and fuzzy semi- P-space, then (X,T) is a fuzzy strongly zero-dimensional space.

Proof : Suppose that δ_1 and δ_2 are fuzzy G_δ-sets with $\delta_1 \leq 1 - \delta_2$, in (X,T). Since (X,T) is a fuzzy perfectly disconnected space, for the fuzzy sets δ_1 and δ_2 with $\delta_1 \leq 1 - \delta_2$, one will have that $cl(\delta_1) \leq 1 - cl(\delta_2)$, in (X,T). Then, $\delta_1 \leq cl(\delta_1) \leq 1 - cl(\delta_2) \leq 1 - \delta_2$. This implies that $\delta_1 \leq cl(\delta_1)$ and $\delta_2 \leq 1 - cl(\delta_1)$. Since (X,T) is a fuzzy perfectly disconnected and fuzzy semi-P-space, by Theorem **2.9**, for the fuzzy G_δ-set δ_1, $cl(\delta_1)$ is a fuzzy open set in (X,T). Thus, $cl(\delta_1)$ is a fuzzy clopen set in (X,T). Let $\lambda = cl(\delta_1)$. Hence, for the fuzzy G_δ-sets δ_1 and δ_2 with $\delta_1 \leq 1 - \delta_2$, the existence of a fuzzy clopen set λ in (X,T) such that $\delta_1 \leq \lambda$ and $\delta_2 \leq 1 - \lambda$, proves that (X,T) is a fuzzy strongly zero-dimensional space.

Corollary 4.1 : If a fuzzy topological space (X,T) is a fuzzy perfectly disconnected and fuzzy P-space, then (X,T) is a fuzzy strongly zero-dimensional space.

Proof : The proof follows from Theorem **2.10** and Proposition **4.3**.

Remarks : It should be noted that fuzzy strongly zero-dimensional spaces need not be fuzzy U - Spaces. For, consider the following example:

Example 4.2 : Let X = { a, b, c, }. Let I = [0, 1]. The fuzzy sets α, β, γ, δ_1 and μ_1 are defined on X as follows **:**

α : X → I is defined by $\alpha(a) = 0.4$; $\alpha(b) = 0.5$; $\alpha(c) = 0.6$,
β : X → I is defined by $\beta(a) = 0.6$; $\beta(b) = 0.4$; $\beta(c) = 0.5$,
γ : X → I is defined by $\gamma(a) = 0.7$; $\gamma(b) = 0.6$; $\gamma(c) = 0.4$.,

$\delta_1 : X \to I$ is defined by $\delta_1(a) = 0.4$; $\delta_1(b) = 0.4$; $\delta_1(c) = 0.4$.
$\mu_1 : X \to I$ is defined by $\mu_1(a) = 0.6$; $\mu_1(b) = 0.6$; $\mu_1(c) = 0.6$.
Then, $T = \{ 0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, \alpha \wedge [\beta \vee \gamma], \gamma \wedge [\alpha \vee \beta], \beta \vee [\alpha \wedge \gamma], 1 \}$ is a fuzzy topology on X. By computation, one shall find that
$\alpha (= 1 - (\gamma \wedge [\alpha \vee \beta])$, $\alpha \wedge \gamma (= 1 - (\alpha \vee \beta))$, $\alpha \wedge [\beta \vee \gamma] (= 1 - (\beta \vee [\alpha \wedge \gamma])$ and $\gamma \wedge [\alpha \vee \beta] (= 1 - \alpha)$ are fuzzy clopen sets in (X,T).

By computation, one shall find that $\delta_1 = \alpha \wedge \beta \wedge \gamma$;
$\delta_2 = [\alpha \vee \beta] \wedge [\alpha \vee \gamma] \wedge [\beta \vee \gamma] \quad \{ = \beta \vee [\alpha \wedge \gamma] \}$
$\delta_3 = \alpha \wedge (\alpha \wedge [\beta \vee \gamma]) \wedge (\gamma \wedge [\alpha \vee \beta]) \quad \{ = \alpha \wedge \gamma \}$,
$\delta_4 = \beta \wedge [\alpha \vee \beta] \wedge [\beta \wedge \gamma] \wedge (\gamma \wedge [\alpha \vee \beta]) \wedge (\beta \vee [\alpha \wedge \gamma]) \quad \{ = \beta \wedge \gamma \}$, are fuzzy G_δ-sets with $\delta_1 \leq 1 - \delta_2$, $\delta_1 \leq 1 - \delta_3$ and $\delta_3 \leq 1 - \delta_2$, $\delta_1 \leq 1 - \delta_4$ and $\delta_3 \leq 1 - \delta_4$ in (X,T).

Now, (i) for the fuzzy G_δ-sets δ_1 and δ_2, there exists a fuzzy clopen set $\alpha \wedge \gamma$ in (X,T) such that $\delta_1 \leq \alpha \wedge \gamma$, $\delta_2 \leq 1 - [\alpha \wedge \gamma]$.
(ii). for the fuzzy G_δ-sets δ_1 and δ_3 , there exists a fuzzy clopen set $\gamma \wedge [\alpha \vee \beta]$ in (X,T) such that $\delta_1 \leq \gamma \wedge [\alpha \vee \beta]$, $\delta_2 \leq 1 - (\gamma \wedge [\alpha \vee \beta])$.
(iii). for the fuzzy G_δ-sets δ_3 and δ_2 , there exists a fuzzy clopen set $\alpha \wedge \gamma$ in (X,T) such that $\delta_3 \leq \alpha \wedge \gamma$, $\delta_2 \leq 1 - (\alpha \wedge \gamma)$.
(iv). for the fuzzy G_δ-sets δ_1 and δ_4 , there exists a fuzzy clopen set $\alpha \wedge \gamma$ in (X,T) such that $\delta_1 \leq \alpha \wedge \gamma$, $\delta_4 \leq 1 - (\alpha \wedge \gamma)$.
(v). for the fuzzy G_δ-sets δ_3 and δ_4 , there exists a fuzzy clopen set $\alpha \wedge [\beta \vee \gamma]$ in (X,T) such that $\delta_3 \leq \alpha \wedge [\beta \vee \gamma]$, $\delta_4 \leq 1 - (\alpha \wedge [\beta \vee \gamma])$.
Hence it follows that **(X,T) is a fuzzy strongly zero-dimensional space.**
Now $\mu_1 = 1 - \delta_1$, $\mu_2 = 1 - \delta_2$, $\mu_3 = 1 - \delta_3$, $\mu_4 = 1 - \delta_4$, are fuzzy F_σ-sets in (X,T). On computation, one shall find that $\mu_1 \nleq 1 - \mu_2$, $\mu_1 \nleq 1 - \mu_3$, $\mu_1 \nleq 1 - \mu_4$ and $\mu_2 \nleq 1 - \mu_3$, $\mu_2 \nleq 1 - \mu_4$ and $\mu_3 \nleq 1 - \mu_4$, in (X,T). Also $\mu_2 \leq \alpha$, but $\mu_1 \nleq 1 - \alpha$. Hence it follows that **(X,T) is not a fuzzy U-space.**

Proposition 4.4 : If α_1 and α_2 are fuzzy closed sets such that $\alpha_1 \leq 1 - \alpha_2$ in a fuzzy normal and fuzzy strongly zero-dimensional space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $\alpha_1 \leq \lambda$ and $\alpha_2 \leq 1 - \lambda$.
Proof : Suppose that α_1 and α_2 are fuzzy closed sets such that $\alpha_1 \leq 1 - \alpha_2$ in (X,T). Since (X,T) is a fuzzy normal space, there exist open fuzzy sets δ_1 and δ_2 such that $\alpha_1 \leq \delta_1$, $\alpha_2 \leq \delta_2$ and $\delta_1 \wedge \delta_2 = 0$. Then, $\delta_1 \leq 1 - \delta_2$, in (X,T). Now (X,T) being a fuzzy strongly zero-dimensional space and each fuzzy open set is a fuzzy G_δ-set in fuzzy topological spaces, for the fuzzy G_δ-sets δ_1 and δ_2 with $\delta_1 \leq 1 - \delta_2$ in (X,T), there exists a fuzzy clopen set λ in (X,T) such that $\delta_1 \leq \lambda$, $\delta_2 \leq 1 - \lambda$. Then, it follows that $\alpha_1 \leq \lambda$ and $\alpha_2 \leq 1 - \lambda$, in (X,T).
Corollary 4.2 : If α_1 and α_2 are fuzzy closed sets such that $\alpha_1 \leq 1 - \alpha_2$ in a fuzzy normal and fuzzy strongly zero-dimensional space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $\alpha_1 \leq \lambda \leq 1 - \alpha_2$.

V. FUZZY U-SPACES AND OTHER FUZZY TOPOLOGICAL SPACES

It should be noted that fuzzy strongly zero-dimensional spaces need not be fuzzy U-Spaces. For, consider the following example:
Example 5.1 : Let $X = \{ a, b, c, \}$. Let $I = [0, 1]$. The fuzzy sets α, β , γ, δ_1 and μ_1 are defined on X as follows **:**
$\alpha : X \to I$ is defined by $\alpha(a) = 0.4$; $\alpha(b) = 0.5$; $\alpha(c) = 0.6$,
$\beta : X \to I$ is defined by $\beta(a) = 0.6$; $\beta(b) = 0.4$; $\beta(c) = 0.5$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.7$; $\gamma(b) = 0.6$; $\gamma(c) = 0.4$.,
$\delta_1 : X \to I$ is defined by $\delta_1(a) = 0.4$; $\delta_1(b) = 0.4$; $\delta_1(c) = 0.4$.
$\mu_1 : X \to I$ is defined by $\mu_1(a) = 0.6$; $\mu_1(b) = 0.6$; $\mu_1(c) = 0.6$.

Then, T = { 0, α, β, γ, $\alpha \vee \beta$, $\alpha \vee \gamma$, $\beta \vee \gamma$, $\alpha \wedge \beta$, $\alpha \wedge \gamma$, $\beta \wedge \gamma$, $\alpha \wedge [\beta \vee \gamma]$, $\gamma \wedge [\alpha \vee \beta]$, $\beta \vee [\alpha \wedge \gamma]$, 1 } is a fuzzy topology on X. By computation, one shall find that α ($= 1 - (\gamma \wedge [\alpha \vee \beta])$), $\alpha \wedge \gamma$ ($= 1 - (\alpha \vee \beta)$), $\alpha \wedge [\beta \vee \gamma]$ ($= 1 - (\beta \vee [\alpha \wedge \gamma])$) and $\gamma \wedge [\alpha \vee \beta]$ ($= 1 - \alpha$) are fuzzy clopen sets in (X,T).

By computation, one shall find that $\delta_1 = \alpha \wedge \beta \wedge \gamma$;
$\delta_2 = [\alpha \vee \beta] \wedge [\alpha \vee \gamma] \wedge [\beta \vee \gamma]$ { $= \beta \vee [\alpha \wedge \gamma]$ }
$\delta_3 = \alpha \wedge (\alpha \wedge [\beta \vee \gamma]) \wedge (\gamma \wedge [\alpha \vee \beta])$ { $= \alpha \wedge \gamma$ },
$\delta_4 = \beta \wedge [\alpha \vee \beta] \wedge [\beta \wedge \gamma] \wedge (\gamma \wedge [\alpha \vee \beta]) \wedge (\beta \vee [\alpha \wedge \gamma])$ { $= \beta \wedge \gamma$ }, are fuzzy G_δ-sets with $\delta_1 \leq 1 - \delta_2$, $\delta_1 \leq 1 - \delta_3$ and $\delta_3 \leq 1 - \delta_2$, $\delta_1 \leq 1 - \delta_4$ and $\delta_3 \leq 1 - \delta_4$ in (X,T).

Now, (i) for the fuzzy G_δ-sets δ_1 and δ_2, there exists a fuzzy clopen set $\alpha \wedge \gamma$ in (X,T) such that $\delta_1 \leq \alpha \wedge \gamma$, $\delta_2 \leq 1 - [\alpha \wedge \gamma]$.

(ii). for the fuzzy G_δ-sets δ_1 and δ_3, there exists a fuzzy clopen set $\gamma \wedge [\alpha \vee \beta]$ in (X,T) such that $\delta_1 \leq \gamma \wedge [\alpha \vee \beta]$, $\delta_2 \leq 1 - (\gamma \wedge [\alpha \vee \beta])$.

(iii). for the fuzzy G_δ-sets δ_3 and δ_2, there exists a fuzzy clopen set $\alpha \wedge \gamma$ in (X,T) such that $\delta_3 \leq \alpha \wedge \gamma$, $\delta_2 \leq 1 - (\alpha \wedge \gamma)$.

(iv). for the fuzzy G_δ-sets δ_1 and δ_4, there exists a fuzzy clopen set $\alpha \wedge \gamma$ in (X,T) such that $\delta_1 \leq \alpha \wedge \gamma$, $\delta_4 \leq 1 - (\alpha \wedge \gamma)$.

(v). for the fuzzy G_δ-sets δ_3 and δ_4, there exists a fuzzy clopen set $\alpha \wedge [\beta \vee \gamma]$ in (X,T) such that $\delta_3 \leq \alpha \wedge [\beta \vee \gamma]$, $\delta_4 \leq 1 - (\alpha \wedge [\beta \vee \gamma])$.

Hence it follows that *(X,T) is a fuzzy strongly zero-dimensional space.*

Now $\mu_1 = 1 - \delta_1$, $\mu_2 = 1 - \delta_2$, $\mu_3 = 1 - \delta_3$, $\mu_4 = 1 - \delta_4$, are fuzzy F_σ-sets in (X,T). On computation, one shall find that $\mu_1 \nleq 1 - \mu_2$, $\mu_1 \nleq 1 - \mu_3$, $\mu_1 \nleq 1 - \mu_4$ and $\mu_2 \nleq 1 - \mu_3$, $\mu_2 \nleq 1 - \mu_4$ and $\mu_3 \nleq 1 - \mu_4$, in (X,T). Also $\mu_2 \leq \alpha$, but $\mu_1 \nleq 1 - \alpha$. Hence it follows that ***(X,T) is not a fuzzy U-space.***

The following proposition gives a condition under which fuzzy strongly zero-dimensional spaces become fuzzy U-spaces.

Proposition 5.1 : If a fuzzy topological space (X,T) is a fuzzy F-space and fuzzy strongly zero-dimensional space, then (X,T) is a fuzzy U-space.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy F- space, for the fuzzy F_σ-sets μ_1 and μ_2, there exist fuzzy G_δ- sets α and β in (X,T) such that $\mu_1 \leq \alpha$, $\mu_2 \leq \beta$ and $\alpha \leq 1 - \beta$. Now (X,T) being a fuzzy strongly zero-dimensional space, for the fuzzy G_δ-sets α and β with $\alpha \leq 1 - \beta$, there exists a fuzzy clopen set λ in (X,T) such that $\alpha \leq \lambda$, $\beta \leq 1 - \lambda$. Then, $\alpha \leq \lambda \leq 1 - \beta$ in (X,T). Thus $\mu_1 \leq \alpha \leq \lambda$ and $\mu_2 \leq \beta \leq 1 - \lambda$. Hence, for the fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$ in (X,T), the existence of a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda$, $\mu_2 \leq 1 - \lambda$, shows that (X,T) is a fuzzy U-space.

Corollary 5.1 : If a fuzzy topological space (X,T) is a fuzzy F-space, fuzzy perfectly disconnected and fuzzy semi- P-space, then (X,T) is a fuzzy U-space.

Proof : The proof follows from Proposition **5.1** and Proposition **4.3**.

The following proposition gives conditions under which fuzzy U–spaces become fuzzy strongly zero-dimensional spaces.

Proposition 5.2 : If a fuzzy topological space (X,T) is a fuzzy perfectly disconnected and fuzzy U-space which is also a fuzzy ∂^*space, then (X,T) is a fuzzy strongly zero-dimensional space.

Proof : Suppose that δ_1 and δ_2 are fuzzy G_δ-sets with $\delta_1 \leq 1 - \delta_2$, in (X,T). Since (X,T) is a fuzzy ∂^*space, by Theorem **2.6**, cl(δ_1) and cl(δ_2) are F_σ-sets in (X,T). Also since (X,T) is a fuzzy perfectly disconnected space, for the fuzzy sets δ_1 and δ_2 with $\delta_1 \leq 1 - \delta_2$, one will have that cl(δ_1) $\leq 1 -$ cl(δ_2), in (X,T). Now (X,T) being a fuzzy U-space, there exists a fuzzy clopen set λ in (X,T) such that cl(δ_1) $\leq \lambda$ and cl(δ_2) $\leq 1 - \lambda$. Then, it follows that $\delta_1 \leq$ cl(δ_1) $\leq \lambda$ and $\delta_2 \leq$ cl(δ_2) $\leq 1 - \lambda$. Thus, for a pair of fuzzy G_δ-sets δ_1 and δ_2 with $\delta_1 \leq 1 - \delta_2$ in (X,T), there exists a fuzzy clopen set λ in (X,T) such that $\delta_1 \leq \lambda$, $\delta_2 \leq 1 - \lambda$. Hence, it follows that (X,T) is a fuzzy strongly zero-dimensional space.

The following proposition gives conditions under which fuzzy F'-spaces become fuzzy U-spaces.

Proposition 5.3 : If a fuzzy topological space (X,T) is a fuzzy strongly zero-dimensional fuzzy F'- space which is also a weak fuzzy Oz-space, then (X,T) is a fuzzy U-space.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy F'- space, $cl(\mu_1) \leq 1 - cl(\mu_2)$, in (X,T). Now (X,T) being a fuzzy weak fuzzy Oz-space, for the fuzzy F_σ -sets μ_1 and μ_2 , $cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy G_δ -sets in (X,T). Thus, $cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy G_δ -sets such that $cl(\mu_1) \leq 1 - cl(\mu_2)$ in the fuzzy strongly zero-dimensional space (X,T). Then, there exists a fuzzy clopen set λ in (X,T) such that $cl(\mu_1) \leq \lambda$, $cl(\mu_2) \leq 1 - \lambda$. This implies that $\mu_1 \leq cl(\mu_1) \leq \lambda$ and $\mu_2 \leq cl(\mu_2) \leq 1 - \lambda$. Thus, for the fuzzy F_σ -sets that μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$, the existence of a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda$, $\mu_2 \leq 1 - \lambda$ shows that (X,T) is a fuzzy U-space.

The following proposition gives conditions under which fuzzy feebly F_σ-complemented spaces become fuzzy U-spaces.

Proposition 5.4 : If a fuzzy topological space (X,T) is a fuzzy feebly F_σ-complemented, fuzzy P-space which is also a fuzzy strongly zero-dimensional space, then (X,T) is a fuzzy U-space.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy feebly F_σ-complemented space, by Theorem **2.3** , for the fuzzy F_σ-sets μ_1 and μ_2 , there exist fuzzy G_δ -sets δ_1 and δ_2 in (X,T) such that $\mu_1 \leq \delta_1$, $\mu_2 \leq \delta_2$ and $int(\delta_1) \leq 1 - int(\delta_2)$. Since (X,T) is a fuzzy P-space, the fuzzy G_δ -sets δ_1 and δ_2 are fuzzy open in (X,T) and thus $int(\delta_1) = \delta_1$ and $int(\delta_2) = \delta_2$. Thus, for the fuzzy F_σ-sets μ_1 and μ_2 , there exist fuzzy G_δ -sets δ_1 and δ_2 in (X,T) such that $\mu_1 \leq \delta_1$, $\mu_2 \leq \delta_2$ and $\delta_1 \leq 1 - \delta_2$. Now (X,T) being a fuzzy strongly zero-dimensional space, for the fuzzy G_δ-sets δ_1 and δ_2 with $\delta_1 \leq 1 - \delta_2$, there exists a fuzzy clopen set λ in (X,T) such that $\delta_1 \leq \lambda$, $\delta_2 \leq 1 - \lambda$. Thus $\mu_1 \leq \delta_1 \leq \lambda$, $\mu_2 \leq \delta_2 \leq 1 - \lambda$, in (X,T). Hence, for the fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$ in (X,T), the existence of a fuzzy clopen set λ in (X,T) such that $\mu_1 \leq \lambda$, $\mu_2 \leq 1 - \lambda$, shows that (X,T) is a fuzzy U-space.

Corollary 5.2 : If a fuzzy topological space (X,T) is a fuzzy weakly F_σ-complemented, fuzzy P-space and fuzzy strongly zero-dimensional space, then (X,T) is a fuzzy U-space.

Proof : The proof follows from Theorem **2.4** and Proposition **5.4**.

The following proposition gives conditions under which fuzzy weakly F_σ-complemented spaces become fuzzy U-spaces.

Proposition 5.5 : If a fuzzy topological space (X,T) is a fuzzy weakly F_σ-complemented and fuzzy perfectly disconnected space, then (X,T) is a fuzzy U-space.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1 - \mu_2$, in (X,T).
Since (X,T) is a fuzzy weakly F_σ-complemented space, for the fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$ in (X,T), there exist fuzzy F_σ -sets λ_1 and λ_2 with $\lambda_1 \leq 1 - \lambda_2$, in (X,T) such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Then , $\mu_1 \leq \lambda_1 \leq cl(\lambda_1)$, $\mu_2 \leq \lambda_2 \leq cl(\lambda_2)$ and $1 - cl(\lambda_1 \vee \lambda_2) = 0$, in (X,T). By Lemma **2.1**, $1 - cl(\lambda_1 \vee \lambda_2) = int(1 - [\lambda_1 \vee \lambda_2])$ and then, $int([1 - \lambda_1] \wedge [1 - \lambda_2]) = 0$. This implies that $int(1 - \lambda_1) \wedge int(1 - \lambda_2) = 0$, in (X,T). Then, $int(1 - \lambda_2) \leq 1 - int(1 - \lambda_1)$ and $1 - cl(\lambda_2) \leq 1 - [1 - cl(\lambda_1)]$. Thus, $1 - cl(\lambda_2) \leq cl(\lambda_1)$, in (X,T).

Now (X,T) being a fuzzy perfectly disconnected space, $\lambda_1 \leq 1 - \lambda_2$, in (X,T) implies that $cl(\lambda_1) \leq 1 - cl(\lambda_2)$. Then, $cl(\lambda_1) \leq 1 - cl(\lambda_2) \leq cl(\lambda_1)$ and thus $cl(\lambda_1) = 1 - cl(\lambda_2)$. This implies that $cl(\lambda_1)$ is a fuzzy clopen set in (X,T). Now $\mu_1 \leq cl(\lambda_1)$, $\mu_2 \leq cl(\lambda_2) = 1 - cl(\lambda_1)$ and hence, for the fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$, the existence of a fuzzy clopen set $cl(\lambda_1)$ in (X,T) such that $\mu_1 \leq cl(\lambda_1)$ and $\mu_2 \leq 1 - cl(\lambda_1)$ shows that (X,T) is a fuzzy U-space.

The following proposition shows that fuzzy U-spaces are neither fuzzy Brown spaces nor fuzzy connected spaces

Proposition 5.6 : If a fuzzy topological space (X,T) is a fuzzy U-space, then (X,T) is not a fuzzy Brown space.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \le 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy U-space, by Proposition **3.1,** there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \le \lambda \le 1 - \mu_2$. Thus, the existence of a fuzzy clopen set λ in (X,T) shows, by Theorem **2.1**, that (X,T) is not a fuzzy Brown space.

Proposition 5.7 If a fuzzy topological space (X,T) is a fuzzy U-space, then (X,T) is not a fuzzy connected space.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \le 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy U-space, by Proposition **3.1,** there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \le \lambda \le 1 - \mu_2$. Thus, the existence of a proper fuzzy clopen set λ in (X,T) establishes that (X,T) is not a fuzzy connected space.

The following proposition shows that fuzzy U-spaces are not fuzzy strongly hyperconnected spaces.

Proposition 5.8 : If a fuzzy topological space (X,T) is a fuzzy U-space then (X,T) is not a fuzzy strongly hyperconnected space.

Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \le 1 - \mu_2$, in (X,T). Since (X,T) is a fuzzy U-space, by Proposition **3.1,** there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \le \lambda \le 1 - \mu_2$. Thus, the existence of a proper fuzzy clopen set λ in (X,T) establishes, by Theorem **2.2**, that (X,T) is not a fuzzy strongly hyperconnected space.

Proposition 5.9 : If μ_1 and μ_2 are fuzzy simply* open sets with $\mu_1 \le 1 - \mu_2$ in a fuzzy perfectly disconnected and fuzzy U-space (X,T), then there exists a fuzzy clopen set λ in (X,T) such that $\mu_1 \le \lambda \le 1 - \mu_2$.

Proof : Suppose that μ_1 and μ_2 are fuzzy simply* open sets in (X,T) with $\mu_1 \le 1 - \mu_2$. Since (X,T) is a fuzzy perfectly disconnected space, for the fuzzy sets μ_1 and μ_2 with $\mu_1 \le 1 - \mu_2$, $cl(\mu_1) \le 1 - cl(\mu_2)$, in (X,T). By Theorem **2.5**, $cl(\mu_1)$ and $cl(\mu_2)$ are F_σ-sets in (X,T). Now (X,T) being a fuzzy U-space, by Proposition **3.1**, there exists a fuzzy clopen set λ in (X,T) such that $cl(\mu_1) \le \lambda \le 1 - cl(\mu_2)$ and thus $\mu_1 \le cl(\mu_1) \le \lambda \le 1 - cl(\mu_2) \le 1 - \mu_2$. Hence, it follows that $\mu_1 \le \lambda \le 1 - \mu_2$, in (X,T).

Proposition 5.10 : If μ_1 and μ_2 are fuzzy pseudo-open sets with the property that $cl(\mu_1) \le 1 - cl(\mu_2)$ in a fuzzy D-Baire and fuzzy U-space (X,T), then there exists a fuzzy regular open set λ in (X,T) such that $int(\mu_1) \le \lambda \le 1 - int(\mu_2)$.

Proof : Suppose that μ_1 and μ_2 are fuzzy pseudo-open sets with $cl(\mu_1) \le 1 - cl(\mu_2)$, in (X,T). Since (X,T) is a fuzzy D-Baire space, by Theorem **2.7**, μ_1 and μ_2 are fuzzy simply* open sets in (X,T) and by Theorem **2.5**, $cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy F_σ-sets in (X, T). Then, $cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy F_σ-sets with $cl(\mu_1) \le 1 - cl(\mu_2)$, in (X,T). Now (X,T) being a fuzzy U-space, by Proposition **3.1**, there exists a fuzzy clopen set λ in (X,T) such that $cl(\mu_1) \le \lambda \le 1 - cl(\mu_2)$ and thus $\mu_1 \le cl(\mu_1) \le \lambda \le 1 - cl(\mu_2) \le 1 - \mu_2$. Hence, it follows that $\mu_1 \le \lambda \le 1 - \mu_2$, in (X,T). Then, $int\,cl(\mu_1) \le int\,cl(\lambda) \le int\,cl(1 - \mu_2)$ and $int\,cl(\mu_1) \le int\,cl(\lambda) \le 1 - cl\,int(\mu_2)$ [by Lemma **2.1**]. Since λ is a fuzzy clopen set, λ is a fuzzy regular open set in (X,T). Now $int(\mu_1) \le int\,cl(\mu_1) \le \lambda \le 1 - cl\,int(\mu_2) \le 1 - int(\mu_2)$. Hence, it follows that $int(\mu_1) \le \lambda \le 1 - int(\mu_2)$.

VI. FUZZY ULTRANORMAL SPACES

Motivated by the works of **R. Staum** [**17**] and **John Van Name** [**38**] on ultranormal spaces in classical topology, the notion of fuzzy ultranormal space shall be defined as follows :

Definition 6.1: A fuzzy topological space (X,T) is called a fuzzy ultranormal space if for each pair of fuzzy closed sets μ_1 and μ_2 with $\mu_1 \le 1 - \mu_2$ in (X,T), there exist fuzzy clopen sets λ_1 and λ_2 with $\lambda_1 \le 1 - \lambda_2$ in (X,T) such that $\mu_1 \le \lambda_1$, $\mu_2 \le \lambda_2$.

Example 6.1 : Let X = { a, b, c, }. Let I = [0, 1]. The fuzzy sets α, β and γ are defined on X as follows **:**

$\alpha : X \to I$ is defined by $\alpha(a) = 0.5$; $\alpha(b) = 0.6$; $\alpha(c) = 0.4$,
$\beta : X \to I$ is defined by $\beta(a) = 0.4$; $\beta(b) = 0.5$; $\beta(c) = 0.6$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.6$; $\gamma(b) = 0.4$; $\gamma(c) = 0.5$.
Then, $T = \{ 0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \alpha\wedge\gamma, \beta\wedge\gamma, \alpha\vee[\beta\wedge\gamma], \beta\vee[\alpha\wedge\gamma], \gamma\vee[\alpha\wedge\beta], \alpha\wedge[\beta\vee\gamma], \beta\wedge[\alpha\vee\gamma], \gamma\wedge[\alpha\vee\beta], \alpha\vee\beta\vee\gamma, 1 \}$ is a fuzzy topology on X. On computation, one shall see that $\alpha\vee\beta$, $\alpha\vee\gamma$, $\beta\vee\gamma$, $\alpha\wedge\beta$, $\alpha\wedge\gamma$, $\beta\wedge\gamma$, $\alpha\vee[\beta\wedge\gamma]$, $\beta\vee[\alpha\wedge\gamma]$, $\gamma\vee[\alpha\wedge\beta]$, $\alpha\wedge[\beta\vee\gamma]$, $\beta\wedge[\alpha\vee\gamma]$, $\gamma\wedge[\alpha\vee\beta]$ and $\alpha\vee\beta\vee\gamma$, are fuzzy clopen sets in (X,T).

On computation, one shall see that for the fuzzy closed sets $1-\alpha$ and $1-(\alpha\vee\beta)$ with $1-\alpha \le 1-[1-(\alpha\vee\beta)]$ in (X,T), there exist fuzzy clopen sets $\beta\vee[\alpha\wedge\gamma]$ $(= 1-(\alpha\wedge[\beta\vee\gamma])$ and with $\alpha\wedge[\beta\vee\gamma]$ $(= 1-(\beta\vee[\alpha\wedge\gamma])$ such that $1-\alpha \le \beta\vee[\alpha\wedge\gamma]$ and $1-(\alpha\vee\beta) \le \alpha\wedge[\beta\vee\gamma]$ with $\beta\vee[\alpha\wedge\gamma] \le 1-(\alpha\wedge[\beta\vee\gamma])$, in (X,T).

Also, for the fuzzy closed sets $1-\alpha$ and $1-(\beta\vee\gamma)$ with $1-\alpha \le 1-[1-(\beta\vee\gamma)]$ in (X,T), there exist fuzzy clopen sets $\beta\vee[\alpha\wedge\gamma]$ $(= 1-(\alpha\wedge[\beta\vee\gamma])$ and $\alpha\wedge\beta$ $(= 1-(\beta\vee\gamma))$ such that $1-\alpha \le \beta\vee[\alpha\wedge\gamma]$ and $1-(\beta\vee\gamma) \le \alpha\wedge\beta$ with $\beta\vee[\alpha\wedge\gamma] \le 1-[\alpha\wedge\beta])$, in (X,T).

For the fuzzy closed sets $1-\alpha$ and $1-(\alpha\vee\beta\vee\gamma)$ with $1-\alpha \le 1-[1-(\alpha\vee\beta\vee\gamma)]$ in (X,T), there exist fuzzy clopen sets $\beta\vee[\alpha\wedge\gamma]$ $(= 1-(\alpha\wedge[\beta\vee\gamma])$ and $\alpha\wedge\gamma$ $(= 1-(\alpha\vee\beta))$ such that $1-\alpha \le \beta\vee[\alpha\wedge\gamma]$ and $1-(\alpha\vee\beta\vee\gamma) \le \alpha\wedge\gamma$ with $\beta\vee[\alpha\wedge\gamma] \le 1-[\alpha\wedge\gamma])$, in (X,T).

For the fuzzy closed sets $1-\alpha$ and $1-(\beta\vee[\alpha\wedge\gamma])$ with $1-\alpha \le 1-[1-(\beta\vee[\alpha\wedge\gamma])]$ in (X,T), there exist fuzzy clopen sets $\beta\vee[\alpha\wedge\gamma]$ $(= 1-(\alpha\wedge[\beta\vee\gamma])$ and $\alpha\wedge[\beta\vee\gamma]$ $(= 1-(\beta\vee[\alpha\wedge\gamma]))$ such that $1-\alpha \le \beta\vee[\alpha\wedge\gamma]$ and $1-(\beta\vee[\alpha\wedge\gamma]) \le \alpha\wedge[\beta\vee\gamma]$ with $\beta\vee[\alpha\wedge\gamma] \le 1-(\alpha\wedge[\beta\vee\gamma]$, in (X,T).

On computation, one shall verify that for each pair of fuzzy closed μ_1 and μ_2 with $\mu_1 \le 1-\mu_2$, [where μ_1, $\mu_2 = 1-\beta$, $1-\gamma$, $1-(\alpha\vee\beta)$, $1-(\alpha\vee\gamma)$, $1-(\beta\vee\gamma)$, $1-(\alpha\wedge\beta)$, $1-(\alpha\wedge\gamma)$, $1-(\beta\wedge\gamma)$, $1-(\alpha\vee[\beta\wedge\gamma])$, $(1-\beta\vee[\alpha\wedge\gamma])$, $1-(\gamma\vee[\alpha\wedge\beta])$, $1-(\alpha\wedge[\beta\vee\gamma])$, $1-(\beta\wedge[\alpha\vee\gamma])$, $1-(\gamma\wedge[\alpha\vee\beta])$, $1-(\alpha\vee\beta\vee\gamma)$, in (X,T), there exist fuzzy clopen sets λ_1 and λ_2 with $\lambda_1 \le 1-\lambda_2$ [where λ_1, $\lambda_2 = \alpha\vee\beta$, $\alpha\vee\gamma$, $\beta\vee\gamma$, $\alpha\wedge\beta$, $\alpha\wedge\gamma$, $\beta\wedge\gamma$, $\alpha\vee[\beta\wedge\gamma]$, $\beta\vee[\alpha\wedge\gamma]$, $\gamma\vee[\alpha\wedge\beta]$, $\alpha\wedge[\beta\vee\gamma]$, $\beta\wedge[\alpha\vee\gamma]$, $\gamma\wedge[\alpha\vee\beta]$ and $\alpha\vee\beta\vee\gamma$] in (X,T) such tha $\mu_1 \le \lambda_1$, $\mu_2 \le \lambda_2$ and hence (X,T) is a fuzzy ultranormal space.

Example 6.2 : Let $X = \{ a, b, c, \}$. Let $I = [0, 1]$. The fuzzy sets α, β, γ, δ_2, δ_3 and δ_4 are defined on X as follows **:**
$\alpha : X \to I$ is defined by $\alpha(a) = 0.4$; $\alpha(b) = 0.5$; $\alpha(c) = 0.6$,
$\beta : X \to I$ is defined by $\beta(a) = 0.6$; $\beta(b) = 0.4$; $\beta(c) = 0.5$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.7$; $\gamma(b) = 0.6$; $\gamma(c) = 0.4$,
$\delta_2 : X \to I$ is defined by $\delta_2(a) = 0.4$; $\delta_2(b) = 0.5$; $\delta_2(c) = 0.4$,
$\delta_3 : X \to I$ is defined by $\delta_3(a) = 0.6$; $\delta_3(b) = 0.5$; $\delta_3(c) = 0.4$,
$\delta_4 : X \to I$ is defined by $\delta_4(a) = 0.4$; $\delta_4(b) = 0.5$; $\delta_4(c) = 0.5$.
Then, $T = \{ 0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \alpha\wedge\gamma, \beta\wedge\gamma, \beta\vee[\alpha\wedge\gamma], \alpha\wedge[\beta\vee\gamma], \gamma\wedge[\alpha\vee\beta], 1 \}$ is a fuzzy topology on X. On computation, one shall see that

δ_1 $(= \alpha, 1-\gamma\wedge[\alpha\vee\beta])$, δ_2 $(= \alpha\wedge\gamma, 1-(\alpha\vee\beta))$, δ_3 $(= \gamma\wedge[\beta\vee\gamma], 1-[\beta\vee[\alpha\wedge\gamma]])$, δ_4 $(= \gamma\wedge[\alpha\vee\beta], 1-\alpha,)$ are fuzzy clopen sets in (X,T). Now $1-\beta$ and $1-(\alpha\vee\gamma)$ are fuzzy closed sets in (X,T) such that $(1-\beta) \le 1-[1-(\alpha\vee\gamma)]$ and $1-(\alpha\vee\gamma) \le \delta_i$ (i = 1,2,3,4) and there are no fuzzy clopen sets δ_i in (X,T) such that $1-\beta \le \delta_i$. Hence (X,T) is not a fuzzy ultranormal space.

Remarks : It is to be noted that fuzzy ultranormal spaces are fuzzy normal spaces whereas fuzzy normal spaces need not be fuzzy ultranormal spaces.

The following proposition gives a condition for fuzzy normal spaces to become fuzzy ultranormal spaces.

Proposition 6.1 : If a fuzzy topological space (X,T) is a fuzzy normal and fuzzy perfectly disconnected space, then (X,T) is a fuzzy ultranormal space.

Proof : Suppose that δ_1 and δ_2 are fuzzy closed sets in (X,T) with $\delta_1 \le 1-\delta_2$. Since (X,T) is a fuzzy normal space, for the fuzzy closed sets δ_1 and δ_2, there exist fuzzy open sets λ_1 and λ_2 with $\lambda_1 \leq 1-\lambda_2$ in (X,T) such that $\delta_1 \leq \lambda_1$ and $\delta_2 \leq \lambda_2$. Then, $\delta_1 \leq \lambda_1 \leq$ cl (λ_1) and $\delta_2 \leq \lambda_2 \leq$ cl (λ_2). Now (X,T) being a fuzzy perfectly disconnected space, for the fuzzy sets λ_1 and λ_2 with $\lambda_1 \leq 1-\lambda_2$, cl $(\lambda_1) \leq 1-$ cl (λ_2), in (X,T). By Theorem **2.8**, the fuzzy perfectly disconnected space (X,T) is a fuzzy extremally disconnected space and thus, for the fuzzy open sets λ_1 and λ_2, cl (λ_1) and cl (λ_2) are fuzzy open sets in (X,T). Thus, cl (λ_1) and cl (λ_2) are fuzzy clopen sets in (X,T). Thus, for each pair of fuzzy closed sets δ_1 and δ_2 with $\delta_1 \leq 1-\delta_2$ in (X,T), there exist fuzzy clopen sets cl (λ_1) and cl (λ_2) with cl $(\lambda_1) \leq 1-$ cl (λ_2), in (X,T) such that $\mu_1 \leq$ cl(λ_1) and $\mu_2 \leq$ cl(λ_2) .Hence (X,T) is a fuzzy ultranormal space.

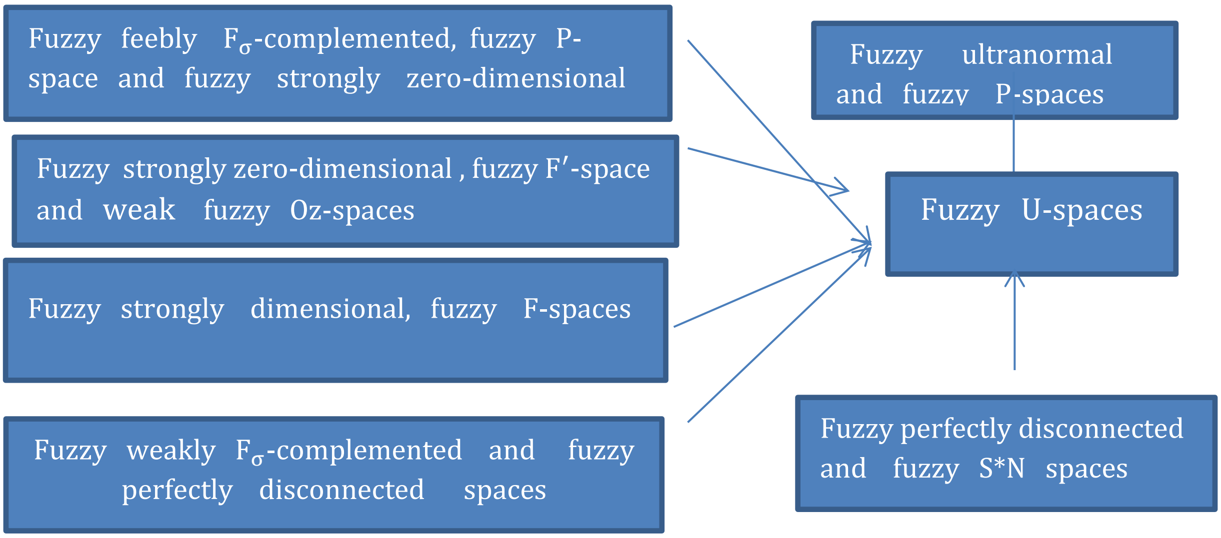
The following proposition gives a condition for fuzzy ultranormal spaces to become fuzzy U- spaces.

Proposition 6. 2 : If a fuzzy topological space (X,T) is a fuzzy ultranormal and fuzzy P-space, then (X,T) is a fuzzy U- space.
Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1-\mu_2$, in (X,T). Since (X,T) is a fuzzy P-space, the fuzzy F_σ-sets μ_1 and μ_2 are fuzzy closed sets in (X,T). Now (X,T) being a fuzzy ultranormal space, for the fuzzy closed sets μ_1 and μ_2, there exist fuzzy clopen sets λ_1 and λ_2 with $\lambda_1 \leq 1-\lambda_2$ in (X,T) such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq \lambda_2$. Now $\lambda_1 \leq 1-\lambda_2$ implies that $\lambda_2 \leq 1-\lambda_1$. Then, $\mu_1 \leq \lambda_1$ and $\mu_2 \leq \lambda_2 \leq 1-\lambda_1$ in (X,T). Hence, for the fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1-\mu_2$ in (X,T), the existence of a fuzzy clopen set λ_1 in (X,T) such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq 1-\lambda_1$, shows that (X,T) is a fuzzy U-space.
Corollary 6.1 : If a fuzzy topological space (X,T) is a fuzzy normal, fuzzy perfectly disconnected and fuzzy P-space, then (X,T) is a fuzzy U- space.
Proof : The proof follows from Propositions **6.1** and **6. 2.**
Corollary 6.2 : If a fuzzy topological space (X,T) is a fuzzy normal, fuzzy perfectly disconnected and fuzzy P-space, then (X,T) is a fuzzy strongly zero-dimensional fuzzy U-space.
Proof : The proof follows from **Corollary 4.1** and **Corollary 6.1 .**

From Corollary 6.2 , one will infer that those fuzzy P-spaces having fuzzy normality and fuzzy perfectly disconnectedness are fuzzy U- spaces with fuzzy strongly zero-dimensionality .

The following proposition gives a condition for fuzzy S*N- spaces to become fuzzy U- spaces.
Proposition 6.3 : If a fuzzy topological space (X,T) is a fuzzy perfectly disconnected, fuzzy S*N-space , then (X,T) is a fuzzy U- space.
Proof : Suppose that μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \leq 1-\mu_2$, in (X,T). Now (X,T) being a fuzzy perfectly disconnected space, for the fuzzy sets μ_1 and μ_2 with $\mu_1 \leq 1-\mu_2$, cl $(\mu_1) \leq 1-$ cl (μ_2), in (X,T). By Theorem **2.11,** the fuzzy perfectly disconnected and fuzzy S*N-space (X,T) is a fuzzy normal space and thus (X,T) is a fuzzy normal and fuzzy perfectly disconnected space. By proposition **6.1,** (X,T) is a fuzzy ultranormal space . Then, for the fuzzy closed sets cl (μ_1) and cl (μ_2) with cl $(\mu_1) \leq 1-$ cl (μ_2), there exist fuzzy clopen sets λ_1 and λ_2 with $\lambda_1 \leq 1-\lambda_2$ in (X,T) such that cl $(\mu_1) \leq \lambda_1$, cl $(\mu_2) \leq \lambda_2$. Then, cl $(\mu_1) \leq \lambda_1$ and cl $(\mu_2) \leq \lambda_2 \leq 1-\lambda_1$. This implies that $\mu_1 \leq$ cl $(\mu_1) \leq \lambda_1$ and $\mu_2 \leq$ cl $(\mu_2) \leq 1-\lambda_1$. Hence, for the fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1-\mu_2$, the existence of a fuzzy clopen set λ_1 in (X,T) such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq 1-\lambda_1$, shows that (X,T) is a fuzzy U-space.

Remarks : The inter-relations between fuzzy U-spaces and other fuzzy topological spaces can be summarized as follows :



Remarks : It is to be noted that the converses of the above implications need not be true.

(i). *Fuzzy U–spaces need not be fuzzy ultranormal and fuzzy P-spaces.* For, in example **3.1,** (X,T) is not a fuzzy P-space, since the fuzzy G_δ-set $[\alpha \vee \beta] \wedge [\alpha \vee \gamma] \wedge [\beta \vee \gamma]$, is not a fuzzy open set in (X,T). Also , $1-\gamma$ and $1-[\alpha \vee \beta]$ are fuzzy closed sets such that $1-\gamma \leq 1-(1-[\alpha \vee \beta])$. It is found that there exists no fuzzy clopen set λ_1 in (X,T) such that $1-\gamma \leq \lambda_1$ and thus (X,T) is not a fuzzy ultranormal space.

(ii). *Fuzzy U–spaces need not be fuzzy F'-spaces and fuzzy perfectly disconnected spaces .* For, in example **3.1,** (X,T) is not a fuzzy F'-space, since for the fuzzy F_σ-sets θ and δ with $\theta \leq 1-\delta$, $cl(\theta) \nleq 1-cl(\delta)$ where $cl(\theta) = 1-(\beta \wedge [\alpha \vee \gamma])$ an $cl(\delta) = 1-\gamma$. Also (X,T) is not a fuzzy perfectly disconnected space , since $cl(\theta) \nleq 1-cl(\delta)$ for the fuzzy sets θ and δ with $\theta \leq 1-\delta$, in (X,T).

VII. CONCLUSION

In this paper, the notion of fuzzy U-space is introduced and several characterizations of fuzzy U-spaces are established. Also the notion of fuzzy strongly zero-dimensional space is defined in terms of fuzzy G_δ-sets and studied. It is established that fuzzy F-spaces which are either fuzzy strongly zero-dimensional or fuzzy perfectly disconnected , fuzzy semi -P- spaces , are fuzzy U-spaces and those fuzzy U-spaces which are fuzzy perfectly disconnected and fuzzy ∂^*spaces, are fuzzy strongly zero-dimensional spaces. Also it is obtained that fuzzy strongly zero-dimensional fuzzy F'- spaces which are weak fuzzy Oz-spaces and fuzzy feebly F_σ-complemented, fuzzy P-spaces which are fuzzy strongly zero-dimensional spaces, are fuzzy U-spaces. It is found that fuzzy weakly F_σ-complemented and fuzzy perfectly disconnected spaces are fuzzy U-spaces. It is proved that fuzzy U-spaces are neither Brown spaces nor fuzzy connected spaces. It is also found that fuzzy U-spaces failed to be fuzzy strongly hyperconnected spaces. The notion of fuzzy ultranormal space is introduced and the inter-relations between fuzzy ultranormal spaces and fuzzy normal spaces are established in this paper. It is obtained that fuzzy normal spaces which are possessing fuzzy perfectly disconnectedness are fuzzy ultranormal spaces. Also it is established that fuzzy ultranormal and fuzzy P-spaces are fuzzy U-spaces and fuzzy P-spaces having fuzzy normality and fuzzy perfectly disconnectedness are fuzzy U-spaces with fuzzy strongly zero-dimensionality . It is also established that those fuzzy S*N-spaces possessing fuzz perfectly disconnectedness, are fuzzy U- spaces.

REFERENCES

[1]. N.Ajmal, J.K.Kohli, Zero dimensional and strongly zero dimensional fuzzy topological spaces, Fuzzy Sets and Systems, 61 (1994), 231 - 237.

[2]. F. Azarpanah and M. Karavan, On non regular ideals and z°-ideals in C(X), Czech. Math. J. 55 (130) (June 2005), 397 – 407.

[3]. K .K. Azad, On Fuzzy Semi Continuity, Fuzzy Almost Continuity and Fuzzy Weakly Contiunity, J. Math. Anal. Appl, 82 (1981), 14 - 32.
[4]. G. Balasubramanian, Maximal Fuzzy Topologies, Kybernetika, Vol .31, No. 5 (1995), 459 – 464.
[5]. W. D. Burgess, and R. Raphael, Tychonoff spaces and a ring theoretic order on C(X), Topol. Appl. 279, (2020) 107250.
[6]. C. L .Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl., 24, (1968),182 - 190.
[7]. T. Dube and O. Ighedo, On Lattices of Z-Ideals of Function Rings, Math. Slovaca , Vol. 68, No. 2 (2018), 271 – 284.
[8]. U. V. Fatteh and D. S. Bassan , Fuzzy Connectedness and Its Stronger Forms, J. Math. Anal. Appl., 111 (1985), 449 -464..
[9]. A. A. Fora, Separation axioms for fuzzy spaces, Fuzzy Sets and Sys., 33 (1989) , 59 – 70.
[10]. B. Ghosh, Fuzzy Extremally Disconnected Spaces , Fuzzy Sets and Sys., Vol. 46, No. 2 (1992), 245 – 250.
[11]. Gillman, L. and Henriksen, M., 1956. Rings of continuous functions in which every finitely generated ideal is principal, Trans. Amer. Math. Soc,(82), 366 - 391.
[12]. M. Henriksen and R.G. Woods , Cozero -complemented spaces ; when the space of minimal prime ideals of a C(X) is compact, Topology Appl. 141 (2004), 147 – 170.
[13]. Hutton B., 1975, Normality in Fuzzy topological Spaces, J. Math. Anal. Appl., 50, 74 - 79.
[14]. M. L. Knox, R. Levy, W. Wm. Mcgovern, and J. Shapiro, Generalizations of complemented rings with applications to rings of functions , J. Algebra Appl. 8 (2009), 17 - 40.
[15]. R. Levy and J. Shapiro, Rings of quotients of rings of functions, Topology Appl., Vol. 146-147 (2005), 253 - 265.
[16]. P. J. Nyikos, Not every zero-dimensional realcompact space is N-compact, Bull. Amer. Math. Soc., 77 (1971) 392 - 396.
[17]. R. Staum, The algebra of bounded continuous functions into nonarchimedean field, Pacific J. Math., 50 (1974), 169 - 185.
[18]. G. Thangaraj and G. Balasubramanian , On Fuzzy Basically Disconnected Spaces, J. Fuzzy Math., Vol. 9 , No.1 (2001), 103 – 110.
[19]. G. Thangaraj and G. Balasubramanian, On Somewhat Fuzzy Continuous Functions, J. Fuzzy Math, Vol. 11, No. 2 (2003), 725 - 736.
[20]. G. Thangaraj, Resolvability and irresolvability in fuzzy topological spaces, News Bull. Cal. Math. Soc., 31 (4 - 6) (2008), 11 – 14.
[21]. G. Thangaraj and S. Anjalmose, On fuzzy Baire Spaces, J. Fuzzy Math. Vol. 21, No.3 (2013), 667 - 676.
[22]. G. Thangaraj and S. Anjalmose, On fuzzy D-Baire Spaces, Ann. Fuzzy Math. Inform., Vol. 7, No. 1 (2014), 99 – 108.
[23]. G.Thangaraj and K. Dinakaran , On Fuzzy Pseudo- Continuous Functions, IOSR. J. Math., Vol. 13, No. 5 (2017), 12 - 20.
[24]. G.Thangaraj and K. Dinakaran , On fuzzy simply* continuous functions, Adv. Fuzzy Math., Vol. 11 , No.2 (2016) , 245 - 264.
[25]. G.Thangaraj and S. Muruganantham, On Fuzzy Perfectly Disconnected Spaces, Inter. J . Adv. Math., 2017, No. 5 (2017) , 12 - 21.
[26]. G. Thangaraj and S. Muruganantham, On Fuzzy F'-spaces, J. Phy. Sci., Vol. 24 (2019), 97 – 106.
[27]. G. Thangaraj and S. Muruganantham, Fuzzy F- Spaces, Ann. Fuzzy Math. Inform., Vol. 25, No. 2 (2023), 99 – 110.
[28]. G.Thangaraj and J.Premkumar, On fuzzy ∂^*-spaces, Adv. Fuzzy Sets & Sys., Vol. 27, No. 2 (2022), 169 -188.
[29]. G. Thangaraj and N.Raji , On Fuzzy Baire Dense Sets and Fuzzy Pseudo-Open Sets, Adv. Fuzzy Sets and Sys., Vol. 27, No. 1 (June 2023), 31 – 64.
[30]. G.Thangaraj and L . Vikraman , On Fuzzy 〖F〗_σ-Complemented Spaces, Adv. Appl. Math. Sci., Vol. 22, No . 9 (2023), 1991 – 2008.
[31]. G.Thangaraj and L. Vikraman , On fuzzy weakly 〖F〗_σ –Complemented spaces, (Accepted and to be published in Ann. Fuzzy Math. Inform , South Korea).
[32]. G.Thangaraj and L. Vikraman , Fuzzy feebly 〖F〗_σ –Complemented spaces, Universal J . Math. & Math. Sci., Vol. 21, No. 2 (2025), 67 - 89.
[33]. G.Thangaraj and L. Vikraman, Some remarks on fuzzy strongly hyper- connected spaces, High Tech. Letters, Vol. 29, No.12, (2023), 735 - 753.
[34]. G. Thangaraj and M. Ponnusamy, On fuzzy Oz-Spaces, Adv. Appl. Math. Sci., Vol 22, No. 5 (2023), 939 - 954.
[35]. G.Thangaraj and M. Ponnusamy, Some Remarks on Fuzzy Brown Spaces, High Tech. Letters, Vol. 30, No.5, (2024), 653 - 672.
[36]. G.Thangaraj and A.Vinothkumar, On Fuzzy Semi-P-spaces and Related Concepts, Pure & Applied Math. Jour ., Vol. 11 , No. 1 (2022), 20 - 27.
[37]. G.Thangaraj and A.Vinothkumar , On Fuzzy S*N –spaces, Ann. Fuzzy Math. Inform., Vol. 30, No.1, (August 2025), 97 - 112.
[38]. J. Van Name, Ultraparacompactness and Ultranormality, Preprint, available as arXiv:1306.6086, [math.GN] 2013. https://doi.org/10.48550/arXiv.1306.6086
[39]. William G. Bade , Two properties of the sorgenfrey plane, Pacific J. Math., Vol. 51, No. 2 (1974), 349 - 354.
[40]. L A. Zadeh , Fuzzy Sets, Inform. and Control, Vol. 8 (1965), 338 - 353.