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Double Logic And Paraconsistent Existential Graphs GET, GET4 And GEG (4.516)

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Abstract:
The paraconsistent propositional logic GT is presented, along with its semantic characterization. It is shown that GT's set of theorems corresponds to the set of valid existential graphs, GET, which turns out to be an extension of Peirce's Gamma system, without becoming Zeman's gamma-4 system. This result is amplified by constructing the paraconsistent system of existential graphs GET4, and its semantic-deductive characterization. The paraconsistent propositional logic LG is presented, along with its semantic characterization. It is shown that the set of theorems of LG corresponds to the set of valid existential graphs of Charles Sanders Peirce's Gamma system (1903). All evidence is presented in a complete, rigorous, and detailed manner. Finally, Zeman's Gamma-4, Gamma-4.2, and Gamma-5 existential graph systems are proven to be paraconsistent.

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I. Introduction

Existential graphs, alpha, beta, and gamma, were created by Charles Sanders Peirce in the late 19th century, see Roberts [10] and Peirce [8]. Alpha graphs correspond to classical propositional calculus, beta graphs correspond to classical logic of first-order relations. Gamma charts were introduced by Peirce and later extended by Jay Zeman, constructing existential graphs for modal logics S4, S4.2 and S5 in Zeman [14]. On the other hand, Brade and Trymble [2] have proposed categorical models for alpha existential graphs. Recently, existential graphs were presented for intuitionistic propositional calculus in Oostra [4, 7], for intuitionistic relationship calculus in Oostra [5], and for modal logics S4, S4.2, and S5, intuitionist versions, in Oostra [6]. Finally, Sierra [11] presents the Gamma-LD system of existential graphs, and Sierra [12] presents the first system of paraconsistent existential graphs.

The paraconsistent propositional logic GT is presented, along with its semantic characterization. It is shown that GT's set of theorems corresponds to the set of valid existential graphs, GET, which turns out to be an extension of Peirce's Gamma system, without becoming Zeman's gamma-4 system. This result is amplified by constructing the paraconsistent system of existential graphs GET4, and its semantic-deductive characterization.

The paraconsistent propositional logic LG is presented, along with its semantic characterization. It is shown that the set of theorems of LG corresponds to the set of valid existential graphs of Charles Sanders Peirce's Gamma system. All evidence is presented in a complete, rigorous, and detailed manner. Finally, Zeman's Gamma-4, Gamma-4.2, and Gamma-5 existential graph systems are proven to be paraconsistent. These results were presented at SALOME1: 1st South American LOgic MEeting, in Cusco, Peru, January 12-15, 2024.

The deductive system for *double paracomplete (LD) propositional logic* and *gamma-LD existential graphs* are presented in Sierra [11]. LD has 2 negations, the classical ($\sim$) and a paracomplete ($\neg$). Gamma-LD has 2 cuts, continuous and continuous-thick. The theorems of LD correspond exactly to the valid existential graphs of gamma-LD. LD can be seen as an extension of the modal propositional logic S4, by extending the language with a set of strong atomic formulas. LD is characterized by a semantics of possible worlds, where the relation of accessibility is reflexive and transitive.

When the language of LD is restricted to the language of *classical propositional logic* (LC), the constraint associated with gamma-LD coincides with the valid existential graphs of Charles Sanders Peirce's *alpha* system [8] where Peirce's continuous cut corresponds to the continuous cut of Gamma-LD. When the language of LD is restricted to the language of intuitionistic *propositional logic* (LI) van Dalen [13], the constraint associated with gamma-LD coincides with the valid existential graphs of the intuitionistic alpha system (alpha-I) presented by Oostra [4, 7], and when LD is restricted to LI, the paracomplete negation turns out to be the *intuitionistic* negation, where Peirce's continuous cut corresponds to the continuous-thick cut of Gamma-LD.

Definition 1.1 *Properly combining the negations of LD yields a paraconsistent negation, and redefining LD and Gamma-LD in terms of the classical and paraconsistent negations yields the double logic paraconsistent LD'. The theorems of LD' correspond to the valid existential graphs of the existential graph system Gamma-LD', this system has 2 cuts: the continuous and the broken {X}=([(X)]).*

When the strong atomic formulas are removed from the LD' language, the resulting theorems correspond to the valid existential graphs of the GET4 existential graph system.

Proposition 1.2 *GET4 existential graph system coincide with the gamma-4 system presented by Zeman[14].*

Proof. In [11] LD is deductively characterized by the propositional logic S4 (adding alternate atomic formulas), and characterized by a semantics of possible worlds, whose accessibility relation is reflexive and transitive. By translating into the LD' existential graph system (and also eliminating the alternate atomic graphs), the deductive system (GT4) results, which is equivalent to S4 (only that the language remains in terms of the paraconsistent negation), but the semantics of possible worlds do not change, it is the same as S4, so the LD' graph system is restricted, is the graphics system associated with S4, i.e. Gamma-4.

When LD' language is restricted to LC language, the constraint associated with gamma-4 coincides with valid existential alpha graphs; where the classical negation corresponds to the continuous cut. Considering all the possible combinations of 3 cuts in Gamma-LD and Gamma-LD', The diagrams in Figure 1 are obtained.

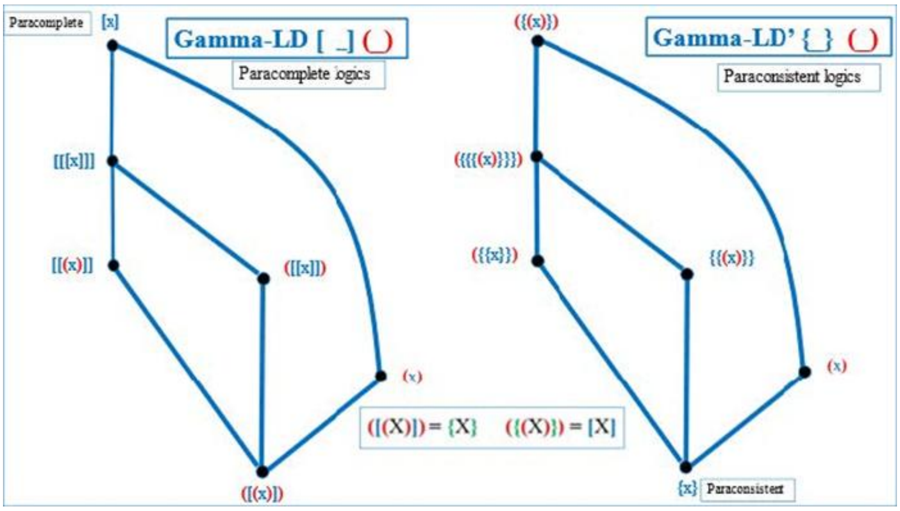

Figure 1: Combining 3 cuts in LD and LD'

This diagram with the rules of Gamma-LD', corresponds to the semantics of possible worlds Reflexive and transitive (GET4 graphs), omitting the transitivity resulting in the GET graphs (sections 8 to 13). If, in addition, (sections 2 to 7), the axioms are properly restricted, so that there is no semantics of possible worlds, the deductive system (LG) associated with the Gamma existential graphs proposed by Peirce in CP 4.516 [8] is constructed.

II. Deductive System LG

In this section, the deductive system of propositional logic, LG (Gamma Logic), is presented, and its connections with classical propositional calculus.

Definition 2.1 *The set of formulas, FL, of the deductive system, LG, is constructed from a set FA of atomic formulas, from the constant* $\perp$*, the unary connective weak negation,* $\{-\}$*, and the binary connective conditional,* $\{\supset\}$*, as follows. 1)* $P\in FA$ *implies* $P\in FL$*. 2)* $\perp\in FL$*. 3)* $X\in FL$ *implies* $-X\in FL$*. 4)* $X,Y\in FL$ *implies* $X\supset Y, X\cap Y\in FL$*.*

Classical negation, strong affirmation, weak affirmation, disjunction, lambda and biconditional are defined as: 1) $\sim X = X\supset\perp$. 2) $+X = \sim -X$. 3) $\otimes X = -\sim X$. 4) $X\cup Y = \sim X\supset Y$. 5) $\lambda = \sim\perp$. 6) $X\equiv Y = (X\supset Y)\cap(Y\supset X)$.

Definition 2.2 *The LG deductive system consists of the axioms*

(where X,Y,Z∈FL): Ax1) $\perp\supset X$. Ax2) $X\supset(Y\supset X)$. Ax3) $[X\supset(Y\supset Z)]\supset[(X\supset Y)\supset(X\supset Z)]$. Ax4) $[(X\supset Y)\supset X]\supset X$. Ax5) $(X\supset\perp)\supset -X$. Ax6) $-X\supset -(X\cap Y)$. Ax7) $(X\cap Y)\supset X$. Ax8) $(X\cap Y)\supset Y$. Ax9) $(X\supset Y)\supset[(X\supset Z)\supset(X\supset\{Y\cap Z\})]$. Ax10) $-(Y\cap -X)\supset -(Y\cap(X\supset\perp))$. Ax11) $-(Y\cap -(Z\cap(X\supset\perp)))\supset -(Y\cap -(Z\cap -X))$. The only rule of inference is the *modus ponens* Mp: Z is inferred from X and $X\supset Z$.

Definition 2.3 *Let X,* $X_1, , X_n \in FL$*. X is a theorem of LG, denoted X∈TL, if there is a proof of X from the axioms using the rule Mp, i.e., X is the last row of a finite sequence of lines, in which, each of the lines is an axiom, or is inferred from two preceding rows, using the inference rule Mp. The number of lines in the sequence is referenced as the length of the X proof. Y is a theorem (or consequence) of* $\{X_1, \dots, X_n\}$*, denoted* $\{X_1, \dots, X_n\} >> Y$*, if there is a proof of Y, from the axioms and assumptions* $\{X_1, \dots, X_n\}$*.*

Proposition 2.4 *Let them be X,Y,*$X_1,\ldots,X_n$ *∈FL. If* $\{X_1,\ldots,X_n, X\}$ *>>Y then LG, then* $\{X_1,\ldots,X_n\}$ *>>X⊃Y.*

Proof. Axioms 2, 3 and 4, with the single inference rule Mp, determine the calculus for the classical implication CIC Rasiowa [9], in which the deduction theorem applies.

Proposition 2.5 *For X,Y∈FL. The following formulas are LG theorems:*

1) (X⊃∼Y)⊃(Y⊃∼X). 2) ∼(X⊃X)⊃Y. 3) X∪∼X. 4) X⊃∼∼X. 5) ∼∼X⊃X. 6) (X⊃Y)⊃(∼Y⊃∼X).
7) (∼Y⊃∼X)⊃(X⊃Y).

Proof. 1) Suppose X⊃(Y⊃⊥), Y, X. By Mp is derived Y⊃⊥, again by Mp is inferred ⊥. Applying proposition 2.4, 3 times and using the definition of ∼, we conclude (X⊃∼Y)⊃(Y⊃∼X). 2) Suppose ∼(X⊃X), i.e. (X⊃X)⊃ ⊥, but X⊃X is a CIC theorem, resulting in ⊥, using Ax1 follows Y. Applying proposition 2.4 concludes ∼(X⊃X)⊃Y. 3) By the principle of identity of the CIC we have ∼X ⊃∼X, by the definition we conclude X∪ ∼X. 4) Suppose X, X⊃⊥. By Mp it follows ⊥, applying proposition 2.4, 2 times and definition concludes X⊃∼ ∼X. 5) Suppose ∼∼X, i.e., ∼X⊃⊥, by Ax1 we have ⊥⊃X, as (∼X⊃⊥)⊃[(⊥⊃X)⊃(∼X⊃X)] is a theorem of the CIC deduces ∼X⊃X, i.e. (X⊃⊥)⊃X, using Ax4 implies X. Proposition 2.4 concludes ∼∼X⊃X. 6) and 7). Direct consequences for 1), 4) and 5).

Proposition 2.6 *Let them be X,Y,Z∈FL. The following formulas are theorems of L:*

1) X⊃(X∪Y). 2) X⊃(Y∪X). 3) (X⊃Y)⊃[(Z⊃Y)⊃({X∪Z} ⊃Y)].

Proof. 1) Suppose X, ∼X, i.e., X⊃⊥, by Mp we obtain ⊥, according to Ax1 we derive Y. Applying proposition 2.4, 2 times we conclude X⊃(∼X⊃Y), i.e., X⊃(X∪Y). 2) By first part we conclude X⊃(∼X⊃Y), using proposition 2.5, it can be said that X⊃(∼Y⊃X), i.e., X⊃(Y∪X). 3) Suppose X⊃Y, Z⊃Y, X∪Z, i.e., ∼X⊃Z, by CIC we infer ∼X⊃Y, by proposition 2.5 we derive ∼Y⊃X, by CIC we infer ∼Y⊃Y, i.e. (Y⊃–λ)⊃Y, by Ax4 we get Y. Applying proposition 2.4, 3 times we get (X⊃Y)⊃[(Z⊃Y)⊃({X∪Z} ⊃Y)].

Proposition 2.7 *For X,Y∈FL. X⊃[Y⊃(X∩Y)]∈TL.*

Proof. Suppose X, Y. Ax2 results in Ax1⊃X, Ax1⊃Y, Ax8 results in Ax1⊃(X∩Y), Mp results in X∩Y. Applying proposition 2.4, 2 times concludes X⊃[Y⊃(X∩Y)].

Proposition 2.8 *The classical propositional calculus CPC with the language* {⊃, ∩, ∪, ≡, ∼} *is included in the propositional calculus LG.*

Proof. Axioms 2, 3, 4, 7, 8 and 9 along with propositions 2.5, 2.6, 2.7 and the inference rule Mp determine CPC Rasiowa [9].

Proposition 2.9 *Sean X,Y∈FL. The following formulas are theorems of L:*

1) – –λ ⊃ λ*. 2) X∪–X. 3) ∼X⊃–X. 4) −X≡∼+X. 5) +X⊃X.*

Proof. 1) By Ax2 we have λ ⊃(– –λ ⊃ λ), in addition to Ax1 of has ⊥⊃⊥, i.e., ∼⊥, which means λ, applying Mp we conclude that – –λ ⊃ λ. 2) By definition in Ax5, result X∪–X. 3) Ax5. 4) By definition we have ∼ −X≡+X, applying proposition 2.5 we conclude −X≡∼+X. 5) By Ax5 we have ∼X⊃ −X, applying CPC we deduce ∼ −X⊃X, i.e., +X⊃X.

Proposition 2.10 *Sean X,Y∈FL. The following formulas are theorems of L:*

1) ∼+∼X≡⊗X. 2) +∼X≡∼⊗X. 3) ∼+X≡ −X. 4) X⊃⊗X. 5) −X⊃⊗∼X.

Proof. 1) By proposition 2.9 we have −∼X≡∼+∼X, by definition it results ⊗X≡∼+∼X. 2) By CPC we conclude ∼⊗X≡+∼X. 3) By definition you have ∼ −X≡+X, by CPC you get −X≡∼+X. 4) By proposition 2.9 we have +∼X⊃∼X, using CPC we deduce X⊃∼+∼X, by part 1 we conclude X⊃⊗X. 5) By definition we have −∼X⊃⊗∼∼X, by CPC we conclude −X⊃⊗∼X.

III. Semantics LG

In this section, the semantics for the LG system are presented, in proposition 3.6 it is proved that the theorems of the LG system are valid formulas in the proposed semantics. This semantics follows the ideas presented by Batens & De Clercq [1].

Definition 3.1 *M=(*V_M*, v) is a model for LG, it means that,* V_M *is a function of FL in* {*0,1*}*, v is a function of −FL in* {*0, 1*}*, where −FL =* {*−X : X∈FL*}*.*

Definition 3.2 *In the model M=(*V_M*, v), with X,Y∈FL.*

V_M(X)=1 is abbreviated as M(X)=1, and means that in the M model, the formula X is *true*. V_M(X)=0 is abbreviated as M(X)=0, and means that in the M model, the formula X is *false*.

The V_M function satisfies the following rules: 1) V⊥. M(⊥)=0. 2) V⊃. M(X⊃Y)=1 means M(X)=1 implies M(Y)=1. 3) V−. M(−X)=1 means M(X)=0 or v(−X)=1.

4) V− ∩. M(−X)=1 implies M(−(X∩Y))=1. 5) V∩. M(X∩Y)=1 means M(X)=M(Y)=1.

6) V–∼. M(−(Y∩ −X))=1 implies M(−(Y∩∼X))=1.

7) V−−∼. M(−(Y∩ −(Z∩∼X)))=1 implies M(−(Y∩ −(Z∩ −X)))=1.

Proposition 3.3 *For X,Y∈FL. 1) V∼. M(∼X)=1 means M(X)=0.*

2) V∪. M(X∪Y)=1 means M(X)=1 o M(Y)=1. 3) V≡. M(X≡Y)=1 means M(X)=M(Y). 4) V+. M(+X)=1

means M(–X)=0. 5) V⊗. M(⊗X)=1 means M(–∼X)=1.
6) M(+X)=1 implies M(X)=1. 7) M(−X)=0 implies M(X)=1. 8) Vλ. M(λ)=1

Proof. 1), 2) y 3), resulting from CPC semantics. 4) M(+X)=1, means M(∼ −X)=1), for part a means M(−X)=0). 5) M(⊗X)=1 by definition means M(−∼X)=1. 6) If M(+X)=1, for the part 4), we have M(–X)=0, applying V− we infer M(X)=1. 7) Direct consequence of V−. 8) If M(λ)=0, by V∼ we say M(∼ λ)=1, this means M(⊥)=1, which is not the case. ■

Definition 3.4 *For X,X_1, ... , X_n ∈FL. A formula X is said to be valid, denoted X∈VL, if and only if X is true in all models for LG. It is said that {X_1,... , X_n} validates Y if and only if (X_1 ∩X_2 ∩ ... ∩X_n)⊃Y∈VL.*

Proposition 3.5 *Let X∈FL. If X is an axiom of LG, then X∈VL.*

Proof. Ax1) ⊥⊃X. Suppose ⊥⊃X∉VL, so there exists a model M, such that M(⊥⊃X)=0, by V⊃ we have M(⊥)=1, which contradicts V⊥. Hence, Ax1∈VL.

Ax2, Ax3, Ax4) If X is one of the axioms Ax2, Ax3, Ax4, using the rule V⊃ and proceeding as usual for the validity of CPC in van Dalen [13], it is concluded that X∈VL, i.e., Ax2, Ax3, Ax4∈VL.

Ax5) (X⊃⊥)⊃ −X. Suppose that (X⊃⊥)⊃ −X∉VL, so there is a model, M such that M((X⊃⊥)⊃ −X)=0, by V⊃ results M(X⊃⊥)=1 and M(−X)=0, according to V–, M(X)=1 and v(−X)=0, are derived, applying V⊃ it follows that M(⊥)=1, which contradicts V⊥. Hence, Ax5∈VL.

Ax6) −X⊃ −(X∩Y). Suppose that −X⊃ −(X∩Y)∉VL, so there is a model M, such that M((−X⊃ −(X∩Y))=0, by V⊃ results M(−X)=1 and M(−(X∩Y))=0, according to V–∩ it is derived, M(−X)=0, which is not the case. Hence, Ax6∈VL.

Ax7) (X∩Y)⊃X and Ax8) (X∩Y)⊃X. From V∩ it follows that M(X∩Y)=1 implies M(X)=M(Y)=1. Hence, Ax7∈VL and Ax8∈VL.

Ax9) (X ⊃ Y) ⊃ [(X ⊃ Z) ⊃ (X ⊃ { Y ∩ Z })]. Suppose that Ax9 ∉ VL, so there is a model M, such that M[(X⊃Y)⊃(X⊃Z)⊃(X⊃ {Y∩Z})])=0, by V⊃ results M(X⊃Y)=1, M(X⊃Z)=1, M(X)=1, M(Y∩Z)=0, applying V⊃ we derive M(Y)=1, M(Z)=1, which by V∩ means M(Y∩Z)=1, which is not the case. Hence, Ax9∈VL.

Ax10) −(Y∩ −X)⊃ −(Y∩(X⊃⊥)). It is satisfied by the rule V–∼: M(−(Y∩ −X))=1 implies M(−(Y∩∼X))=1. Hence, Ax10∈VL.

Ax11) −(Y∩ −(Z∩(X⊃⊥)))⊃ −(Y∩ −(Z∩ −X)). It is satisfied by rule V−−∼: M(−(Y∩ −(Z∩(X⊃⊥))))=1 implies M(−(Y∩ −(Z∩ −X)))=1. Hence, Ax11∈VL.

Proposition 3.6 *Sean X,Y∈FL. 1) If X∈TL then X∈VL.*
2) If {X_1,… , X_n} >>Y then {X_1, ..., X_n} *validates* Y.

Proof. 1) Suppose X∈TL. X∈VL is proved by induction over the length, L, of the proof of X. Base step L=1. It means that X is an axiom, which from proposition 3.5 it follows that X∈VL.

Induction step. As an inductive hypothesis, we have that for every formula Y, if Y∈TL and the length of the proof of Y is less than L (where L>1) then Y∈VL. If X∈TL and the length of the proof of X is L, then X is an axiom or X is a consequence of applying Mp in earlier steps of the proof. In the first case, we proceed as in the base step. In the second case, we have for some formula Y, proofs of Y and Y⊃X, where the length of both proofs is less than L, using the inductive hypothesis it is inferred that Y∈VL and Y⊃X∈VL, so that, in any model M, we have M(Y)=1 and M(Y⊃X)=1, by V⊃ it turns out that M(X)=1, consequently, X∈VL. Using the principle of mathematical induction, it has been proven that, for every X∈FL, X∈TL implies X∈VL.

2) Suppose that {X_1,…, X_n} >>Y, applying CPC, we have (X_1 ∩X_2 ∩ ... ∩X_n)⊃Y∈TL, from the part 1 is inferred, (X_1 ∩X_2 ∩ ... ∩X_n)⊃Y∈VL, which by definition means that {X_1,… , X_n} *validates* Y.

IV. Semantic-Deductive Characterization LG

In this section, the characterization of LG with the semantics of the previous section is presented. In proposition 4.5 we have completeness and in proposition 4.6 we have semantic-deductive characterization.

Definition 4.1 *An extension of a set of formulas C of LG, denoted C∈EXT(LG), is obtained by altering the set of formulas of C in such a way that the theorems of C are preserved, and that the language of the extension matches the language of LG. An extension is locally consistent if there is no X∈FL such that both X and ∼X are extension theorems. A set of formulas is locally inconsistent if a contradiction Z∩∼Z for some Z∈FL is derived from them. An extension is locally complete if for all X∈FL, either X is an extension theorem or ∼X is an extension theorem.*

Proposition 4.2 *For X∈FL. 1) LT is locally consistent. 2) If E∈EXT(LG), X∉TL-E, and E_x ∈EXT(LG) are obtained by adding ∼X as a new formula to E, then E_x is locally consistent.*

Proof. 1) Suppose that LG is not *locally* consistent, so that there must be Z∈FL such that Z∩∼Z∈TL, i.e. Z∩(Z⊃⊥)∈TL, by CPC results ⊥∈TL, by the validity theorem it is concluded that ⊥∈VL, i.e., for every model M, M(⊥)=1, which contradicts rule V⊥. Therefore, LG is *locally* consistent.

2) Let X∉TL-E, and let E_x the extension obtained by adding X as a new formula to E. Suppose that E_x is locally inconsistent, so that, for some Z∈FL, we have Z,∼Z∈TL-E_x, by CPC we get ⊥∈TL-E_x, by Ax1 we

derive X∈TL-E $_x$. But E $_x$ differs from E only in that it has ∼X as an additional axiom, so 'X is a theorem of E $_x$' is equivalent to 'X is a theorem of E from the set {∼X}'. By proposition 2.4 it follows that ∼X⊃X∈TL-E, and by CPC it is inferred that X∈TT-E, which is not the case, therefore E $_x$ is locally consistent.

Proposition 4.3 *If E∈EXT(LG) is locally consistent, then there is* E'∈EXT(LG) that is *locally* consistent and complete.

Proof. Let be X $_0$, X $_1$, X $_2$, . . . an enumeration of all LG formulas. A sequence E' $_0$, E' $_1$, E' $_2$, . . . of extensions of E as follows: Let E' $_0$ = E. If X $_0$ ∈TL-E' $_0$, is E' $_1$ = E' $_0$, otherwise add ∼X $_0$ as a new formula to get E' $_1$ from E' $_0$. In general, given t ≥ 1, to construct E' $_t$ from E' $_{t-1}$, we proceed as follows: if X $_{t-1}$ ∈TL-E' $_{t-1}$, then E' $_t$ = E' $_{t-1}$, otherwise let E' $_t$ be the extension of E' $_{t-1}$ obtained by adding ∼X $_{t-1}$ as a new formula. The proof is widely known, details in [12].

Proposition 4.4 *If E'EXT∈(LG) is locally consistent, then there is a model in which all X∈TL-E' is true.*

Proof. The model MF=(V $_{M\ F}$, v) is defined as follows: each extension F is associated with an MF model. For each MF and for each X∈FL, V $_{MF}$(X)=1 if X∈F; and V $_{MF}$(X)=0 if ∼X∈F; v(–X)=1 if and only if −X∈F, where F is the *locally* consistent and complete extension associated with MF. Note that V $_{MF}$ is functional because F is *locally* consistent and complete. To claim that MF is a model, rules 1 through 7 of the model definitions must be guaranteed.

1. By CPC we have ⊥⊃⊥∈TL, so ∼⊥∈F, i.e. V $_{MF}$(⊥)=0. Therefore, V⊥ is satisfied.
2. Using CPC we have the following chain of equivalences: V $_{MF}$(X⊃Y)=0, i.e. ∼(X⊃Y)∈F, by CPC we follow X∩∼Y∈F, resulting by CPC that X∈F and ∼Y∈F, which means that V $_{MF}$(X)=1 y V $_{MF}$(Y)=0, so V⊃ is satisfied.
3. Suppose that V $_{MF}$(−Z)=1, so −Z∈F, from which v(–Z)=1, and then V $_{MF}$(Z)=0 o v(–Z)=1.

To prove the reciprocal, suppose V $_{MF}$(Z)=0 or v(–Z)=1. For the case V $_{MF}$(Z)=0, this means that Z∉F, since F is complete, it is inferred that ∼Z∈F, using Ax5 can be assured that −Z∈F, i.e. V $_{MF}$(−Z)=1. For the case v(–Z)=1, this means V $_{MF}$(−Z)=1. So, if V $_{MF}$(Z)=0 o v(–Z)=1 then V $_{MF}$(−Z)=1. Since the reciprocal was initially proved, it is concluded that V− is satisfied.

4. Suppose V $_{MF}$(−X)=1, so −X∈F, using Ax6, is derived −(X∩Y)∈F, i.e. V $_{MF}$(−(X∩Y))=1. Therefore, V∩ is satisfied.
5. Suppose that V $_{MF}$(X∩Y)=1, so X∩Y∈F, applying Ax6 and Ax7 derive X∈F and Y∈F, i.e. V $_{MF}$(X)=1 and V $_{MF($Y)=1. To prove the reciprocal, suppose V $_{MF}$(X)=1 and V $_{MF}$(Y)=1, which means that X∈F and Y∈F, using Ax8 results in X∩Y∈F, consequently, V $_{MF}$(X∩Y)=1, Since the reciprocal was initially proved, it is concluded that V∩ is satisfied.
6. Suppose that V $_{MF}$(−(Y∩ −X))=1, i.e., −(Y∩ −X)∈F, using Ax10 infers −(Y∩∼X)∈F, which means V $_{MF}$(−(Y∩ ∼X))=1. Therefore, V−∼ is satisfied.
7. Suppose that V $_{MF}$(−(Y∩ −(Z∩∼X)))=1, i.e −(Y∩ −(Z∩∼X)))∈F, using Ax11 infers −(Y∩ −(Z∩ −X))∈F, which means V $_{MF}$(−(Y∩ −(Z∩ −X)))=1. Therefore, V−−∼ is satisfied.

Based on the above analysis, it is inferred that M is an LG model. To conclude the proof, let X be a theorem of E', so X∈E'. Therefore, using the definition of V $_{ME}$, it turns out that V $_{ME}$(X)=1, i.e., X is true in the model ME=(V $_{ME}$,v).

Proposition 4.5 *For X,X $_1$,..., X $_n$ ∈FL. 1) If X∈VL then X∈TL. 2) If* {X_1,… , X_n} *validates Y then* {X_1, ..., X_n} *>>Y.*

Proof. By proposition 4.2, the extension E', obtained by adding ∼X as a new formula, is locally consistent. Thus, according to proposition 4.4, there is a model ME such that every theorem of E' is true in ME, and since ∼X∈TL-E', then ∼X is true in ME, i.e., X is false in ME, hence X∉VL. It has been proven that X∉TL implies X∉VL, i.e., X∈VL implies X∈TL.

2) Suppose {X $_1$,… , X $_n$} *validates* Y, i.e., (X $_1$ ∩X $_2$ ∩ ... ∩X $_n$)⊃Y∈VL, by part 1, it follows that, (X $_1$ ∩X $_2$ ∩ ... ∩X $_n$)⊃Y∈TL. If {X $_1$,… , X $_n$} are assumed, by CPC we infer Y, hence {X $_1$, ..., X $_n$} >>Y.

Proposition 4.6 *For X,Y,X $_1$,...,X $_n$ ∈FL. 1) X∈VL if and only if X∈TL.*

2) {X $_1$,… , X $_n$} *validates* Y if and only if {X $_1$, ..., X $_n$} >>Y.

Proof. Direct consequence of propositions 3.6 and 4.5.

V. Existential Graphs GEG

This section presents the original gamma existential graphs, GEG, proposed in 4.516 of Peirce's *Collected Papers* [8]. For the construction of existential graphs, a variant of notation is used, proposed by Peirce in 4.378 of [8].

Definition 5.1 *The set of graphs, GG, of original existential gamma graphs, GEG, is constructed from a set of atomic graphs, GA, and the constant λ (empty graph,* **λ***='_'), as follows. 1) P∈GA implies P∈GG. 2) λ ∈GG. 3) X∈GG implies* {*X*}*,(X)∈GG. 4) X,Y∈GG implies (X(Y)),XY∈GG.*

Definition 5.2 *The graph (X(Y)) it is called a conditional graph. The outer parentheses determine the external* *cut of the conditional, and the internal parentheses determine the internal cut of the conditional. X is called antecedent and Y consequent. Conditional cuts are called continuous cuts. In the {Z} graph, the keys determine the broken cut. The part where Z is located is called the inner region of the broken cut or simply the region of the broken cut.*

Definition 5.3 *Let them be X,Y,Z∈GG. A graph X is said to be in an even region, denoted X_p, if X is surrounded by an even number of cuts (continuous and/or broken). X is in an odd region, denoted X_i, if X is surrounded by an odd number of cuts (continuous and/or broken). X_{nc} means that the graph X is in a region surrounded by n continuous and/or broken cuts (n = 0, 1, 2, 3, ...), where n can be odd or even. X_{pc} indicates an even number of continuous cuts. X_{1c} indicates an odd number of continuous cuts. X_{ncc} means that X is in a region of continuous cuts only, i.e., no broken cuts appear. X_{1cq} means that X is in a region with at least one broken cut.*

Definition 5.4 *Let be X∈GG. Lambda is defined as the assertion sheet λ= '_'. Strong graph is defined as *X = ({X}). Total falsehood is defined as ⊥={_}.*

Definition 5.5 *The system consists of the following RTRA primitive transformation rules:*

R1) Alpha Rules. The primitive transformation rules of Pierce's Alpha existential graph system are primitive transformation rules of the GEG system. These rules are: Erase and Write, Iteration and Deiteration in regions of continuous cuts only or no cut, Write and Erase the double cut. The assertion sheet, λ, is the only axiom of the Alpha system.

R2) Writing graphs in broken cut region. On a broken cut that is written on the assertion sheet, any graph can be written. EG{}. {X}|⇒ {XY}.

R3) Writing and erasing in the cuts. A continuous cut can be *partially erased* (generating a broken cut) when it is in an *even* region. 3a. Ecq. $(X)_p$|⇒ {X}.

A broken cut can be *completed* (generating a continuous cut) when it is in an *odd* region. 3b. Bcc. $\{X\}_i$|⇒(X) .

In addition to the primitive rules, you have the following implicit rules:

RI1) Concatenation. Two graphs that are in the same region can be concatenated. Conversely, two graphs that are concatenated can be separated in the same region. Conc. X, Y ⇔ YX, in any region.

RI2) Commutativity. Two concatenated graphs can be rewritten by changing the order. Com. XY⇔YX, in any region.

RI3) Associativity. In three graphs that are concatenated, *the order* in which they were concatenated is irrelevant. Initially, the first is concatenated with the second and this result is concatenated with the third, or the first is concatenated with the result of concatenating the second with the third. Aso. XY, Z ⇔ X, YZ ⇔ XYZ, in any region.

Remark 5.6 *Rules RI1, RI2 and RI3 are called implicit rules, since, given their obviousness and graphic naturalness, they may not be referenced, but they are applied.*

Definition 5.7 *For X∈GG. X is a graphical theorem of GEG, denoted X∈TG, if there is a proof of X from the graph λ, using the graph transformation rules, i.e., X is the last row of a finite sequence of lines, in which each of the lines is λ, or is inferred from previous rows, using the transformation rules. Or to put it briefly, X∈TG if and only if λ >>X. The number of lines, of the finite sequence, is referenced as the length of the proof of X. Y> >X, means that X is obtained from Y using a finite number of transformation rules.*

Proposition 5.8 *For X,Y∈GG. Let be R∈RTRA and R≠R2. If $X_p \overset{R}{\Longrightarrow}$ Y then there exists R'∈RTRA such that $Y_i \overset{R'}{\Longrightarrow}$ X. Proof by simple inspection of the primitive rules.*

Proposition 5.9 *For X,Y,Z∈GG. When you have an inference, in every even region of the antecedent you infer the consequent, provided you don't use rule R2, then X>>Z implies X_p >>Z.*

When an inference is made, in every *odd* region of the consequent the antecedent is inferred, if rule R2 is not used, then X>>Z implies Z_i >>X.

Proof. Suppose X>>Z , it must be proved that X_p >>Z and Z_i >>X.

If X >> Z then there are R_1,… , R_n ∈ RTRA, and there are X_1,… , X_{n-1} ∈ GG, such that $XR_1X_1R_2X_2 ...X_{n-1}R_nZ$, and the *length* of the transformation of X>>Z is said to be n and denoted by X> $>_n$Z. The proof is performed by induction on the length of the transformation.

Base step. n=1. It means that only one of the primitive rules was applied, and since X is in an even region, then R must be of the form $X_p \overset{R}{\Longrightarrow}$ Z with R∈RTRA. From proposition 5.8 it is inferred that there is R', $Z_1 \overset{R'}{\Longrightarrow}$ X with R'∈RTRA. Inductive step. Inductive hypothesis (∀n>1)[W>> $_n$K ⇒ {W_p >>K and K_i >>W}]. If X> $>_{n+1}$Z, then $XR_1X_1R_2X_2 ... X_{n-1}R_nX_nR_{n+1}Z$, i.e.,

$XR_1X_1R_2X_2 ...X_{n-1}R_nX_n$ and $X_nR_{n+1}Z$, so X>> $_nX_n$ and $X_nR_{n+1}Z$. Applying the inductive hypothesis and proposition 5.8 we get $X_p >>X_n$ y $X_nR_{n+1}Z$, X_{ni} >>X and $Z_iR'_{n+1}X_n$. So, $X_p >>Z_p$ and Z_i >>X.

By the principle of mathematical induction, the truth of the proposition is concluded. ■

Proposition 5.10 *For X,Y,Z∈GG. A conditional graph can be written when the consequent is inferred from the antecedent, if the R2 rule is not used.*
I.e., X>>Z |⇒ (X(Z)).

Proof. Suppose X>>Z. $\lambda \overset{R1}{\Rightarrow}$ ((_)) $\overset{R1}{\Rightarrow}$ (X(_)) $\overset{R1}{\Rightarrow}$ (X(X)) $\overset{X\gg Z \text{ y proposición } 5.9}{\Rightarrow}$ (X(Z)).
Hence, X>>Z ⇒ (X(Z)).

VI. Equivalence LG And GEG

In this section, the equivalence between LG and GEG is presented, initially, in proposition 6.3, it is proved that LG's theorems are graphical theorems of, in proposition 6.7, it is proved that the graphical theorems of are valid in the semantics of possible worlds, in proposition 6.13, it is proved that LEG's theorems are exactly the graph theorems.

Definition 6.1 *FA=GA Translation function [_]':FL→GG. Let be X,Y∈FL and P∈FA.*
1) P'=P. 2) [X⊃Y]'=(X'(Y)). 3) [X∨Y]'=((X')(Y')). 4) (−X)'={X'}. 5) λ'=λ. 6) (X∩Y)'=X'Y'.
7) (∼X)'=(X'). 8) ⊥'=(λ).

Proposition 6.2 *Let X∈FL be. If X is LG's axiom, then X'∈TG.*

Proof. Ax1) ⊥⊃X. By R1 we have ((_)), according to R1 we have ((X')(_)), i.e. (⊥⊃X)'. Therefore, (Ax1)'∈TG.
Ax2, Ax3, Ax4, Ax7, Ax8 and Ax9. Their translations are valid thanks to R1, since these are axioms of CPC, which is validated by the Alpha system. Ax5) (X⊃–λ)⊃–X. (X')>> {X'} is satisfied by rule R3. It is concluded that (Ax5)'∈TG.
Ax6) −X⊃ −(X∩Y). {X'} >> {X'Y'} is satisfied by rule R2. It is concluded that (Ax6)'∈TG.
Ax10) −(Y∩ −X)⊃ −(Y∩(⊥⊃X)). By R1 we have the sequence ((_)), so ({Y{X'}}(_)), we derive ({Y{X'}}({Y{X'}})), applying R3 we infer ({Y{X'}}({Y(X')})). It is concluded that (Ax10)'∈TG.
Ax11) −(Y∩ −(Z∩(X⊃⊥)))⊃ −(Y∩ −(Z∩ −X)). By R1 we have the sequence ((_)), so ({Y{Z(X')}}(_)), is derived ({Y{Z(X')}}({Y{Z(X')}})), applying R3 infers ({Y{Z(X')}} ({Y{Z{X'}} })). It is concluded that (Ax11)'∈TG.

Proposition 6.3 *For X∈FL. 1) If X∈TL then X'∈TG.* 2) If X>>Y then X'>>Y'.

Proof. 1) Induction about the length of the X demonstration in LG. Base step. If the length of the proof is 1, then X is an axiom, by the proposition 6.2 X'∈TG.
Induction step. The inductive hypothesis is: if Y∈TG and the length of the proof of Y is less than L, then Y'∈TG. Suppose X∈TG and that the length of the proof of X is L, so X is an axiom or obtained from previous steps using Mp. In the first case, proceed as in the base step. In the second case, Y and Y⊃X are taken in previous steps of the proof of X, i.e., the lengths of the proofs of Y and Y⊃X are less than L, by the inductive hypothesis it turns out that Y'∈TG and (Y'(X'))∈TG, applying R1 infers ((X'))∈TG, using R1 concludes X'∈TG. By the principle of mathematical induction, LG's theorems are proved to be graphical theorems.
2) If X>>Y, then X⊃Y∈TL, by the part 1, (X'(Y'))∈TG, i.e., λ >>(X'(Y'))), if X' is assumed, by R1 follows ((Y')), applying R1 results in Y', so X'>>Y'.

Definition 6.4 *Translation function, (_)":GG→FG. For X,Y∈FL and P∈GA. 1) P"=P. 2) λ"=⊥⊃⊥. 3) (X(Y))"=X"⊃Y". 4) ((X)(Y))"=X"∪Y".* 5) {X}"=−X". 6) [XY]"= X"∩Y". 7) (X)"=∼X".

Proposition 6.5 *Rules R1 and R2 are valid rules in LG semantics*

Proof. R1) Peirce's Alpha system rules are validated by CPC. Therefore, R1" is valid. R2) {X}|⇒ {XY}. Consider an arbitrary model M=(V $_M$,v). By V− ∩ M(−X")=1 implies M(−(X"∩Y"))=1. Therefore, R2", is a valid rule in LG.

Proposition 6.6 *The R3. (X) $_p$|⇒ {X} and {X} $_i$|⇒(X) are valid in LG's semantics.*

Proof. Induction in the number, *n*, of negations surrounding X.
Base step. *n*=1. (X)|⇒ {X}. Let M=(V $_M$,v) be any model. Suppose that V $_M$(∼X)=1, by V∼ it turns out that V $_M$(X)=0, applying V− we infer V $_M$(−X")=1. Therefore, V $_M$(∼X")=1 implies V $_M$(−X")= 1, so R3 is satisfied for n=1.
n=2. There are 2 possibilities, {Y{X}} ⇒ {Y(X)} and (Y{X})⇒(Y(X)). For the first case, by the rule V–∼ we have M(−(Y"∩ −X"))=1 implies M(−(Y"∩∼X"))=1, so the rule is satisfied. For the second case, let M=(V $_M$,v) be any model, suppose that V $_M$(∼(Y"∩ −X"))=1, i.e. V $_M$(Y"∩ −X")=0, resulting in V $_M$(Y")=0 or V $_M$(−X")=0, using the result when n=1, deduces V $_M$(Y")=0 or V $_M$(∼X")=0, which means that it is not the case that V $_M$(Y"∩∼X")=1, and then V $_M$(∼(Y"∩∼X"))=1, has been tested, V $_M$(∼(Y"∩ −X"))=1 implies V $_M$(∼(Y"∩∼X"))=1, so the rule is satisfied. Therefore, R3 is satisfied for n=2.
n=3. There are 2 possibilities, {Y{Z(X)}} ⇒ {Y{Z{X}}} and (Y{Z(X)})⇒(Y{Z{X}}). For the first case, by rule V−−∼ we have M(−(Y"∩ −(Z"∩∼X")))=1 implies M(−(Y"∩ −(Z"∩ −X")))=1, so the rule is satisfied. For the second case, let M=(VM,v) be any model, suppose that V $_M$((∼(Y"∩ −(Z"∩∼X")))=1, i.e. V $_M$(Y"∩ −(Z"∩

~X”))=0, resulting in V $_M$(Y”)=0 o V $_M$(−(Z”∩~X”))=0, using the result when n=2, we deduce V $_M$(Y”)=0 o V $_M$(−(Z”∩ −X”))=0, which means that it is not the case that V $_M$(Y”∩ −(Z”∩ −X”))=1, and then V $_M$(~(Y”∩ −(Z”∩ −X”)))=1, has been tested, V $_M$(~(Y”∩ −(Z”∩ ~X”)))=1 implies V $_M$(~(Y”∩ −(Z”∩ −X”)))=1, so the rule is satisfied. Therefore, R3 is satisfied for n=3.

Inductive step. Rule 3a. As an inductive hypothesis we have that, if (X”) is surrounded by $2n$ negations, then R3a is satisfied.

{Y”{Z”(X”)}} ⇒ {Y”{Z”{X”}}} and (Y”{Z”(X”)})⇒(Y”{Z”{X”}}), are the only non-trivial cases in which two other negations can be added to X, and they result in valid rules, as proved in the base step when n=3. Therefore, if X is surrounded by 2n+2 slices, i.e., by 2(n+1) slices, then R3a is satisfied.

Rule 3b. As an inductive hypothesis it is that, if (X) is surrounded by 2n+1 negations, then R3b is satisfied. {Y”{Z”(X”)}} ⇒ {Y”{Z”{X”}}} and (Y”{Z” (X”)})⇒(Y”{Z”{X”}}), are the only non-trivial cases in which, to X, two other negations can be added, and they result in valid rules, as proved in the base step when n=3. Therefore, if X is surrounded by 2n+3 cuts, i.e. by 2($n+1$)+1 cuts, then R3b is satisfied.

By the principle of mathematical induction, the validity of R3 has been tested.

Proposition 6.7 *For X∈GG. 1) The primitive rules of G are valid rules in the semantics of LG. 2) If X∈TG then X”∈VL.*

Proof. 1) Direct consequence of propositions 6.5 and 6.6.

2) If X∈TG then λ >>X’, so there are R $_1$,… , R $_n$ ∈RTRA, and there are X $_1$,… , X $_{n-1}$ ∈GE, such that λR $_1$X $_1$R $_2$X $_{2…}$X $_{n-1}$R $_n$X (Proof length is n).

The proof is performed by induction over the length L of the demonstration. Base step. L=1. It means that only one of the primitive rules was applied, then X”∈VG.

Inductive step. Inductive hypothesis: The proposition is valid if L ≤ n with n > 0. Let L=n+1, so λR $_1$X $_1$R $_2$X $_2$ ...X $_{n-1}$R $_n$X $_n$R $_{n+1}$X, i.e., λR $_1$X $_1$R $_2$X $_2$ ...X $_{n-1}$R $_n$X $_n$ and X $_n$R $_{n+1}$X, both demonstrations with length less than n+1. Applying the inductive hypothesis is it turns out that X” $_n$ ∈VG and *from* X” $_n$ *is validly inferred* X”, hence X”∈VG. By the principle of mathematical induction, the truth of the proposition is concluded.

Proposition 6.8 *For X,Y∈GG. 1) If X∈TG then X”∈TL.*

2) If X>>Y, then X”>>Y”.

Proof. 1) By Proposition 4.6 we have that, X”∈VL if and only if X”∈TL, and by Proposition 6.7 we have that, if X∈TG then X”∈VL. Therefore, if X∈TG then X”∈TL. 2) Direct consequence of part 1 and proposition 3.6.

Definition 6.9 *Be T1=[_]’:FG→GG and T2=[_]”:GG→FG, be the translation functions presented in definitions 6.1 and 6.4. They are defined: the composite function T1oT2:GG → GG such that (T1oT2)[X]=T1[T2[X]]. The composite function, T2oT1:FG→FG such that (T2oT1)[X]=T2[T1[X]]. The identity function in FG, IdFK:FG→FG such that (IdFG)[X]=X. The identity function in G IdGG:GG→GG such that (IdGG)[X]=X.*

Definition 6.10 *For P∈GA, X,Y∈GET. The function, C , complexity of a graph, assigns each graph a non-negative integer, as follows: 1) C[P] = C[λ] = 0. 2) C[{X}] = 1+C[X].*

3) C[XY] = 1+max {C[X], C[Y]}. 4) C[((X)(Y))] = 2+max {C[X], C[Y]}. 5) C[(X(Y))] = 1+max {C[X], C[Y]+1}.

Definition 6.11 *Sean P∈ FA; X,Y∈ FT. The function, K, complexity of a formula, assigns each formula a non-negative integer, as follows: 1) K[P] = K[λ] = 0.*

2) K[−X] = 1+K[X]. 3) K[X∩Y] = K[X∪Y] = K[X⊃Y] = 1+max{K[X], K[Y]}.

Proposition 6.12 *The translations presented in definitions 6.1 and 6.4 are inverse functions. For G∈GG and X∈FL. 1) [X’]”=X. 2) [G”]’=G.*

Proof. For T1=( )’ and T2=( )”. Proof part 1. Induction on the complexity, C, of graph G. Base step. C[G]=0, then there are 2 cases. Case 1: G=P. (T1oT2)[P]=T1[T2[P]]=T1[P]=P.

Case 2: G=λ. (T1oT2)[λ]=T1[T2[λ]]=T1[λ]=λ. Inductive step. C[G]≥1. As an inductive hypothesis we have that (T1oT2)[G1]=G1, (T1oT2)[G2]=G2. There are 4 cases.

Case 3: G={G1}. (T1oT2)[{G1}]=T1[T2[{G1}]]= T1[-T2[G1]]={T1[T2[G1]]}={G1}.

Case 4: G=G1G2. (T1oT2)[G1G2] = T1[T2[G1G2]] = T1[T2[G1] T2[G2]]
=T1[T2[G1]] T1[T2[G2]]=G1G2.

Case 5: G=(G1(G2)). (T1oT2) [(G1(G2))]=T1[T2[(G1(G2))]] = T1(T2[G1] ⊃ T2[G2])
=(T1[T2[G1]](T1[T2[G2]])) =(G1(G2)).

Case 6: G = ((G1)(G2)). (T1oT2) [((G1)(G2))] = T1[T2[((G1)(G2)]] = T1[T2(G1)∪ T2(G2)]
= ((T1[T2[G1])(T1[T2[G2])) = ((G1)(G2)).

By the principle of mathematical induction it has been proved that (T1oT2)=IdGG.

Proof part 2. Induction on the complexity, K, of the formula X. Base step. K[X)=0, then there are 2 cases. Case 1: X=P. (T2oT1)[P] = T2[T1[P]] = T2[P] = P.

Case 2: X=λ. (T2oT1)[λ] = T2[T1[λ]] = T2[λ] = λ. Inductive step. K[X]≥ 1. As an inductive hypothesis we

have that (T2oT1)[X1]=X1, (T2oT1)[X2]=X2. There are 4 cases.
Case 3. X=−X1. (T2oT1) [−X1]=T2[T1[−X1]]=T2[{ T1[X1]}]=−(T2oT1)[X1]=−X1.
Case 4. X = X1∩X2. (T2oT1)X1∩X2] = T2[T1[X1∩X2]] = T2[T1[X1]T1[X2]] = T2[T1[X1]]∩[T1[X2]] = X1∩X2.
Case 5. X=X1⊃X2. (T2oT1)[X1⊃X2] = T2[T1[X1⊃X2]] = T2[(T1[X1](T1[X2]))] = T2[T1[X1]]⊃T2[T1[X2]] = X1⊃X2.
Case 6. X=X1 ∪ X2. (T2oT1)[X1 ∪ X2] = T2[T1[X1 ∪ X2]] = T2[((T1[X1])(T1[X2]))] = T2[T1[X1]]∪T2[T1[X2]] = X1∪ X2. By the principle of mathematical induction, (T2oT1)=IdFG.

Proposition 6.13 *For G,H∈GG and X,Y∈FL. 1) G∈TG if and only if G"∈TL. 2) X'∈TG if and only if X∈TL. 3) G>>H if and only if G">>H". 4) X'>>Y' if and only if X>>Y.*

Proof. 1) By proposition 6.8 we have that, if G∈TG then G"∈TL, in addition, by proposition 6.3 we have that, if G"∈TL then (G")'∈TG, but by proposition 6.12 we know that, (G")'=G, resulting that, if G"∈TL then G∈TG, and since we have the reciprocal, we conclude that, G∈TG if and only if G"∈TL.
2) By proposition 6.3 we have that, if X∈TL then X'∈TG, in addition, by proposition 6.8 we have that, if X'∈TG then (X')"∈TL, but by proposition 6.12 we know that, (X')"=X, resulting that, if X'∈TG then X∈TL, and since we have the reciprocal, we conclude that, X'∈TG if and only if X∈TL.
3) By proposition 6.3 we have, if G">>H" then [G"]'>>[H"]', by proposition 6.12 we have [G"]'=G and [H"]'=H, so if G">>H" then G>>H, in addition by proposition 6.8 we have the reciprocal. Therefore, G>>H if and only if G">>H".
4) by proposition 6.8 we have, if X'>>Y' then [X']">>[Y']", by proposition 6.12 we have [X']"=X and [Y']"=Y, so if X'>>Y' then X>>Y, by proposition 6.3 we have the reciprocal. Therefore, X'>>Y' if and only if X>>Y.

VII. Partial Remarks

In this section, in proposition 7.2, it is proved that the Original Gamma existential graphs are paraconsistent. Finally, in proposition 7.3, it is proven that Gamma-4, Gamma-4.2 and Gamma-5 systems are paraconsistent.

Definition 7.1 *Let SD be a deductive system with a negation operator N and let X be a formula for SD. SD is said to be paraconsistent when SD does not derive all SD formulas from X and NX.*

Proposition 7.2 *For G,H,K∈GG. 1) G"⊃(−G"⊃H")∉VG. 2) GEG is paraconsistent. 3) LG is paraconsistent.*

Proof. Consider a model M=(V_M,v), such that V_M(G")=1, V_M(H")=0 and v(−G")=1. As v(−G")=1, then V_M(−G")=1, and as V_M(H")=0, then V_M(−G" ⊃ H")=0, but also V_M(G")=1, consequently V_M(G"⊃(−G"⊃H"))=0. Therefore, G"⊃(−G"⊃H")∉VG. 2) Applying propositions 4.6 and 6.13 yields (G(({G}(H))))∉TG, which implies that this is *not* the case: G{G} >>H. Therefore, GEG is paraconsistent. 3) Using proposition 6.13 it turns out that LG is paraconsistent.

Proposition 7.3 *The Gamma-4, Gamma-4.2 and Gamma-5 systems* presented by Zeman [14] are paraconsistent.

Proof. Gamma-4, Gamma-4.2 and Gamma-5 correspond to the modal logic systems S4, S4.2 and S5 which are characterized by semantics of possible worlds, in which the broken cut corresponds to the possibility of the classical negation, so the rule {X} ⇒(X), corresponds to the modal formula ⊗∼X⊃∼X, which by CPC is equivalent to X⊃∼⊗∼X, i.e., X⊃+X (where is the operator of necessity of such systems), and in such systems the reciprocal is valid, so we would have X≡+X, if ⊗∼X⊃∼X were valid, but the formula X≡+X, in fact, is not valid in such semantics (and should not be, since, in that case, the modalities would make no difference with the statement of classical logic, S4, S4.2 and S5 would collapse into CPC), for details see Hughes and Cresswell [3], consequently, ⊗∼X⊃∼X is not valid, neither in Gamma-4, nor in Gamma-4.2 nor in Gamma-5. Therefore, the 3 systems of existential graphs are paraconsistent.

VIII. Deductive System GT

In this section, the deductive system of propositional logic GT is presented, its connections with classical propositional calculus, and some of its theorems.

Definition 8.1 *The FT set of GT formulas is constructed from a set FA of atomic formulas, from the constant λ, the unary connective weak negation {−}, and the binary connective conditional {⊃} as follows. 1) P∈FA implies P∈FT. 2) λ ∈FT. 3) X∈FT implies −X∈FT. 4) X,Y∈FT implies X⊃∈FT.*

Classical negation, strong affirmation, weak affirmation, disjunction, conjunction and biconditional are defined as: a) ∼X = X⊃ −λ. b) +X = ∼–X. c) ⊗X = –∼X.
d) X∪Y =∼X⊃Y. e) X∩Y = ∼(X⊃∼Y). f) X≡Y = (X⊃Y)∩(Y⊃X).

Definition 8.2 *The GT system consists of the axioms (where X,Y,Z∈FT):*
Ax1. λ. Ax2. X⊃(Y⊃X). Ax3. [X⊃(Y⊃Z)]⊃[(X⊃Y)⊃(X⊃Z)]. Ax4. [(X⊃Y) ⊃X]⊃X. Ax5. –λ ⊃Z. Ax6. (X⊃–λ)⊃–X. Ax7. [–(X⊃Y)⊃–λ]⊃[(–X⊃–λ)⊃(–Y⊃–λ)]. Ax+. If X∈ {Ax1, ..., Ax7, Ax+} then –X⊃–λ is an

axiom. Only rule of inference is modus ponens Mp: from X and X⊃Z we infer Z.

Definition 8.3 *For X, X_1, ..., X_n ∈FT. X is a theorem of GT, denoted X∈TT, is defined similar to definition 2.3.*

Proposition 8.4 *For X,Y,X_1,..., X_n ∈FT. If {X_1, ..., X_n, X} implies Y in GT, then {X_1, ..., X_n} implies X⊃Y.*

Proof. Axioms 2, 3 and 4, with the single inference rule Mp, determine the calculus for the classical implication CIC, Rasiowa [9], in which the deduction theorem, TD, applies.

Proposition 8.5 *For X,Y∈FT. +(X⊃Y)⊃(+X⊃+Y)∈TT.*

Proof by Ax7 and definition.

Proposition 8.6 *For X∈FT. If X∈TT then +X∈TT.*

Proof. Suppose X∈TT, +X∈TT will be tested, by induction over the length of the proof of X. Base step. The length of the proof of X is 1, i.e., X is an axiom. If X is one of axioms 1 to 7 or Ax+, Ax+ gives +X.

Induction step. As an inductive hypothesis, if the length of the proof of Y is less than L, then +Y is a theorem. Suppose that the proof of X has length L greater than 1. It follows that X is an axiom or X is a consequence of previous steps using the inference rule Mp. In the first case, proceed as in the base step. In the second case, we have, for some formula Z, proofs of Z⊃X and Z, both of which are shorter in length than L. From the inductive hypothesis we infer +(Z⊃X),+Z∈TT. By proposition 8.5 we have +(Z⊃X)⊃(+Z⊃+X)∈TT, applying the rule Mp twice we get that +X∈TT.

So, according to the principle of mathematical induction, it has been proved X∈TT implies +X∈TT. ■

Proposition 8.7 *For X,Y∈FT. GT theorems are: a) (X⊃∼Y)⊃(Y⊃∼X).* b) ∼(X⊃X)⊃Y. c) X∪∼X.
d) X⊃∼∼X. e) ∼∼X⊃X. f) (X⊃Y)⊃(∼Y⊃∼X)∩ (∼Y⊃∼X) ⊃(X⊃Y).

Proof. Part a. Suppose X⊃(Y⊃–λ), Y, X. By Mp is derived Y⊃ $-\lambda$, again by Mp is inferred $-\lambda$. Applying TD 3 times and using the definition of ∼, concludes (X⊃∼Y)⊃(Y⊃∼X).

Part b. Suppose ∼(X⊃X), i.e. (X⊃X)⊃–λ, but X⊃X is a theorem of CIC, resulting in –λ, using Ax5 follows Y. Applying TD concludes ∼(X⊃X)⊃Y.

Part c. By the principle of identity of the CIC we have ∼X⊃∼X, by the definition of ∪ we conclude X∪∼X.

Part d. Suppose X, X⊃–λ. By Mp we follow –λ, applying TD 2 times and definition of ∼ we conclude X⊃∼∼X.

Part e. Suppose ∼∼X, i.e., ∼X⊃–λ, by Ax5 we have –λ ⊃X, by CIC we deduce ∼X⊃X, i.e., (X⊃ λ–)⊃X, using Ax4 implies X. By TD we conclude ∼∼X ⊃X.

Part f. Direct consequence of parts a, d and e.

Proposition 8.8 *Sean X,Y,Z∈FT. GT theorems are: a) X⊃(X∪Y).* b) X⊃(Y∪X).
c) (X⊃Y)⊃[(Z⊃Y)⊃({X∪Z} ⊃Y)].

Proof. Part a. Suppose X, X⊃–λ, i.e., ∼X, by Mp we get –λ, according to Ax5 we derive Y. Applying TD 2 times concludes X⊃(∼X⊃Y), i.e. X⊃(X∪Y).

Part b. By part a we conclude X⊃(∼X⊃Y), using proposition 8.7, it can be said that X⊃(∼Y⊃X), i.e., X⊃(Y∪X).

Part c. Suppose X⊃Y, Z⊃Y, X∪Z, i.e. ∼X⊃Z, by CPC we infer ∼X⊃Y, by proposition 8.7 we derive ∼Y⊃X, by CIC we infer ∼Y⊃Y, i.e. (Y⊃–λ)⊃Y, by Ax4 we get Y. Applying TD 3 times we get (X⊃Y)⊃[(Z⊃Y)⊃ ({X∪Z} ⊃Y)].

Proposition 8.9 *For X,Y∈FT. GT theorems are: a) (X∩Y)⊃X. b) (X∩Y)⊃Y.*
c) (X⊃Y)⊃[(X⊃Z)⊃(X⊃ {Y∩Z})]. d) X⊃[Y⊃(X∩Y)]. e) +(X∩Y)≡(+X∩+Y).

Proof. Part a. Suppose X∩Y, i.e., ∼(X⊃∼Y), so (X⊃∼Y)⊃–λ, by Ax5 we have –λ ⊃X, by CIC we infer (X⊃ ∼Y)⊃X, using Ax4 results X. By TD we conclude (X∩Y)⊃X.

Part b. Suppose X∩Y, i.e., ∼(X⊃∼Y), so that (X⊃∼Y)⊃ $-\lambda$, using proposition 8.7 we deduce (Y⊃∼X)⊃–λ, by Ax5 we have –λ ⊃Y, by CIC we infer (Y⊃∼X)⊃Y and, using Ax4 we get Y. By TD we conclude (X∩Y)⊃Y.

Part c. Suppose X⊃Y, X⊃Z, ∼Y∪∼Z. By proposition 8.7 are derived ∼Y⊃∼X, ∼Z⊃∼X, applying proposition 8.8 is inferred ∼X, by TD results (∼Y∪∼Z)⊃∼X, by proposition 8.7 we can affirm X⊃∼(∼Y∪∼Z), i.e. X⊃ ∼(∼∼Y⊃∼Z),
so X⊃∼(Y⊃∼Z), and this means X⊃ {Y∩Z}. Applying TD 2 times concludes (X⊃Y)⊃[(X⊃Z)⊃(X⊃ {Y∩Z})].

Part d. Suppose X, Y. Ax2 results λ ⊃X, λ ⊃Y, by part c derives λ ⊃(X∩Y), using Ax1 infers X∩Y. Applying TD 2 times concludes X⊃ [Y⊃(X∩Y)].

Part e. Parts a and b gives (X∩Y)⊃X and (X∩Y)⊃Y, using proposition 8.6 we derive +[(X∩Y)⊃X] and +[(X∩Y)⊃Y], by proposition 8.5 we get +(X∩Y)⊃+X and +(X∩Y)⊃+Y, according to part c we conclude +(X∩Y)⊃(+X∩+Y). To prove the reciprocal, by the part d we have X⊃[Y⊃(X∩Y)]∈FT, using proposition 8.4 results +{X⊃[Y⊃(X∩Y)]}, by proposition 8.5 we derive +X⊃+[Y⊃(X∩Y)], again by proposition 8.5 and CIC we affirm +X⊃[+Y⊃+(X∩Y)]. Suppose +X∩+Y, applying parts a and b infer +X and +Y, by Mp 2 times derives +(X∩Y), using the deduction theorem follows (+X∩+Y)⊃+(X∩Y). Finally, applying part d and the definition of ≡, we conclude +(X∩Y)≡(+X∩+Y).

Proposition 8.10 *The classical propositional calculus CPC with the language*

{⊃, ∩, ∪, ≡, ~} *is included in the propositional calculus GT.*
Proof. Axioms 2, 3 and 4 along with propositions 8.7, 8.8 and 8.9, with the inference rule Mp determine CPC Rasiowa [9].
Proposition 8.11 *For X,Y∈FT. So, GT theorems: a)* $--\lambda \supset \lambda$. *b) X∪–X.* c) ~X⊃–X. d) −X≡~+X.
e) +X⊃X.
Proof. Part a. By Ax2 we have $\lambda \supset(- -\lambda \supset \lambda)$, in addition to by Ax$\lambda$ of has λ, applying Mp we conclude that $--\lambda \supset \lambda$.
Part b. By Ax6 we have ~X⊃ −X, by definition it means X∪–X.
Part c. By definition in Ax6.
Part d. By definition we have ~ −X≡+X, applying CPC we conclude −X≡~+X.
Part e. By Ax6 we have ~X⊃ −X, applying CPC we deduce ~ −X⊃X, i.e., +X⊃X.
Proposition 8.12 *For X,Y∈FT. So, GT theorems:* a) ~+~X≡⊗X, +~X≡~⊗X, ~+X≡ −X. b) X⊃⊗X.
c) −X⊃⊗~X. d) ~(Z $_1$ ∩ ∩Z $_k$ ∩Y)∈TT implies ~(+Z $_1$ ∩∩+Z $_k$ ∩⊗Y)∈TT.
Proof. Part a. By proposition 8.11 we have −~X≡~+~X, by definition it results ⊗X≡~+~X. By CPC we conclude ~⊗X≡+~X. By definition you have ~ −X≡+X, by CPC you get −X≡~+X.
Part b. By proposition 8.11 we have +~X⊃~X, using CPC we deduce X⊃~+~X, according to part a we conclude X⊃⊗X.
Part c. By definition we have −~~X⊃⊗~X, by CPC we conclude −X⊃⊗~X. Part d. Suppose ~(Z $_1$ ∩ … ∩Z $_k$ ∩Y)∈TT, which by CPC means,
(Z $_1$ ∩ ... ∩Z $_k$)⊃~Y∈TT. Using proposition 8.6 it turns out that +((Z $_1$ ∩ ... ∩ Z $_k$)⊃~Y)∈TT, from proposition 8.5 we infer +(Z $_1$ ∩ … ∩Z $_k$)⊃+~Y∈TT, by proposition 8.9 we get (+Z $_1$ ∩ … ∩+Z $_k$)⊃+~Y∈TT, which, by CPC implies ~(+Z $_1$ ∩ … ∩+Z $_k$ ∩~+~Y)∈TT, and for the part a, equivalent to ~(+Z $_1$ ∩ ... ∩+Z $_k$ ∩⊗Y)∈TT.

IX. Semantics GT

In this section, the semantics of possible worlds for the GT system are presented, in proposition 9.6, it is proved that the theorems of the GT system are valid formulas in the proposed semantics.
Definition 9.1 *(S, Ma, <, V) is a model for GT, it means that, S is a non-empty set of possible worlds, Ma is a possible world, called the actual world, < is a binary relation in S, V is a valuation of S×FT at {0, 1}. The relationship, <, satisfies the following constraints. Reflexivity of <. RR: (∀M∈S)(M<M).*
Definition 9.2 *In the model Mo=(S, Ma, <, V), with X,Y∈FT.*
V(M, X)=1 is abbreviated as M(X)=1, and means that in the possible world M, the formula X is *true*. V(M, X)=0 is abbreviated as M(X)=0, and means that in the possible world M, the formula X is *false*. X is true in Mo means that V(Ma, X)=1.
Valuation V satisfies the following rules: 1) Vλ. M(λ)=1.
2) V⊃. M(X⊃Y)=1 equivalent to M(X)=1 implies M(Y)=1.
3) V−. M(−X)=1 equivalent to (∃P∈S)(M<P and P(X)=0).
Proposition 9.3 *For X,Y∈FT. a) V~. M(~X)=1 equivalent to M(X)=0.*
b) V∪. M(X∪Y)=1 equivalent to M(X)=1 or M(Y)=1. c) V∩. M(X∩Y)=1 equivalent to M(X)=M(Y)=1.
d) V≡. M(X≡Y)=1 equivalent to M(X)=M(Y). e) V+. M(+X)=1 equivalent to (∀N∈S)(M<N implies N(X)=1).
f) V⊗. M(⊗X)=1 equivalent to (∃P∈S)(M<P)(P(X)=1). g) M(+X)=1 implies M(X)=1.
h) M(−X)=0 implies M(X)=1. i) M(−λ)=0
Proof. Parts a, b, c, and d. By CPC.
Part e. If M(+X)=1 equivalent to M(~ −X)=1, by part a equivalent to M(−X)=0, by V− equivalent to (∀N∈S)(M<N implies N(X)=1).
Part f. If M(⊗X)=1, then M(~+~X)=1, i.e., M(+~X)=0, by V+, follows (∃P∈S)(M<P and P(X)=1).
Part g. If M(+X)=1, for the part e, we have (∀N∈S)(M<N implies N(X)=1), as M<M then M(X)=1.
Part h. If M(−X)=0, by V− results (∀P∈S)(M<P implies P(X)=1), but M<M, so M(X)=1.
Part i. If M(−λ)=1, by V− affirms the existence of a world N, M<N and N(λ)=0, which contradicts Vλ.
Definition 9.4 *For X,X_1,..., X_n ∈FT, a formula X is said to be valid, denoted X∈VT, if and only if X is true in all models for GT, i.e., X is true in the actual world of all models for GT. It is said that {X_1, ..., X_n} validates Y if and only if (X_1 ∩X_2 ∩...∩X_n)⊃Y∈VT.*
Proposition 9.5 *For X∈FT. If X is an axiom of GT, then X∈VT.*
Proof. Ax1. By Vλ we have for all M∈S, V(λ)=1. Hence, Axλ ∈VT.
Ax2, Ax3, Ax4. Using the rule V⊃ and proceeding as usual for the validity of the intuitionistic propositional calculus in van Dalen [13], it is concluded that X∈VT, i.e., Ax2, Ax3, Ax14∈VT.
Ax5. Suppose that –λ ⊃Z∉VT, so there is a model, such that in the actual world M, M(–λ ⊃Z)=0 by V⊃ results M(−λ)=1, contradiction. Hence, Ax5∈VT.
Ax6 Suppose that (X⊃–λ)⊃–X∉VT, so there is a model, such that in the actual world M, M((X⊃–λ)⊃–X)=0,

by V⊃ results M(X⊃–λ)=1 y M(–X)=0, applying V– follows (∀P∈S)(M<P implies P(X)=1), as M<M follows M(X)=1, by V⊃ derives M(–λ)=1, contradiction. Hence, Ax6∈VT.

Ax7 Suppose that [–(X⊃Y)⊃–λ]⊃[(–X⊃–λ)⊃(–Y⊃–λ)]∉VT, so there is a model, such that in the actual world M, M([–(X⊃Y)⊃–λ]⊃[(–X⊃–λ)⊃(–Y⊃–λ)])=0, by V⊃ results M(–(X⊃Y)⊃–λ)=1, M(–X⊃–λ)=1 y M(–Y⊃–λ)=0, by V⊃ we get M(–Y)=1 y M(–λ)=0, applying Vλ we derive M(–(X⊃Y))=0. Since M(–Y)=1 according to V– means (∃P∈S)(M<P y P(Y)=0), since M(–(X⊃Y))=0, according to V– we obtain (∀P∈S)(M<P implies P(X⊃Y)=1), by V⊃ we get P(X)=0, in addition, as M(–X⊃–λ)=1 y M(–λ)=0, by V⊃ we infer M(−X)=0, by V− follows (∀P∈S)(M<P implies P(X)=1), in particular P(X)=1, which is impossible. Therefore, Ax7∈VT.

Ax+ If X∈ {Ax1, ... , Ax7} then –X⊃–λ is an axiom. Suppose that X∈ {Ax1, ..., Ax7} and –X⊃–λ ∉VT, so there is a model, such that in the present world M, M(–X⊃–λ)=0, by V⊃ results M(–X)=1, by V– equivalent to (∃P∈S)(M<P y P(X)=0), resulting in X∉VT, but as X∈ {Ax1, ... , Ax8}, the opposite has already been proved above. Hence, Ax+∈VT.

Proposition 9.6 *For X,Y∈FT. 1) X∈TT then X∈VT. 2) If {X1, ..., X_n} implies Y then {X1, ..., X_n} valid to Y.*

Proof. Part 1. Suppose X∈TΓ, X∈VT is proved by induction over the length, L, of the proof of X. Base step L = 1. It means that X is an axiom, which from proposition 9.5 follows that X∈VT.

Induction step. As an inductive hypothesis, we have that for every formula Y, if∈ Y∈TT and the length of the proof of Y is less than L (where L>1) then Y∈VT. If X∈TT and the length of the proof of X is L, then X is an axiom or X is a consequence of applying Mp in earlier steps of the proof. In the first case, we proceed as in the base case. In the second case, we have for some formula Y, proofs of Y and Y⊃X, where the length of both proofs is less than L, using the inductive hypothesis it is inferred that Y∈VT and Y⊃X∈VT, so that, in the current world, M, of any model we have M(Y)=1 and M(Y⊃X)=1, by V⊃ it turns out that M(X)=1, consequently, X∈VΓ. Using the principle of mathematical induction, it has been proved that, for every X∈FT, X∈TT implies X∈VT.

Part 2. Suppose that {X_1, ..., X_n} implies Y, applying CPC, we have (X_1 ∩... ∩X_n)⊃Y∈TT, from the part a is inferred, (X_1 ∩X_2 ∩...∩X_n)⊃Y∈VT, which means that {X_1, ..., X_n} *validates* Y.

X. Semantic-Deductive Characterization GT

In this section, we present the characterization of GT with the semantics of the previous section. Completeness is proved in proposition 10.9 (valid formulas in semantics are theorems of GT), and characterization is achieved in proposition 10.10 (theorems of GT are the valid formulas of semantics and only they).

Definition 10.1 *Extension locally consistent and extension locally complete similar to definition 4.1.*

Proposition 10.2 *For X∈FT. a) GT is locally consistent. b) If E∈EXT(GT), X∉TT-E, and Ex∈EXT(GT) are obtained by adding ~X as a new formula to E, then Ex is locally consistent. The proof is similar to proposition 4.2.*

Proposition 10.3 *If E∈EXT(GT) is locally consistent, then there exists*
E'∈*EXT(GT)* which is locally consistent and complete. The proof is similar to proposition 4.3.

Proposition 10.4 *For Y, Z_1, ..., Z_k ∈FT. If {+Z_1, ..., +Z_k, ⊗Y} is locally consistent then {Z_1, ..., Z_k, Y} is locally consistent.*

Proof. Suppose {Z_1, ..., Z_k, Y} is locally inconsistent in GT, so there exists a formula W∈FT such that, from {Z_1, ..., Z_k, Y}, W∩~W is inferred in GT, using CPC it turns out that ~(Z_1 ∩ ... ∩Z_k ∩Y)∈TΓ, by proposition 8.12 ~(+Z_1 ∩ ... ∩+Z_k ∩⊗Y)∈TT so {+Z_1, … , +Z_k, ⊗Y} is locally inconsistent in GT. It has been proved that {Z_1, … , Z_k, Y} locally inconsistent implies that {+Z_1, … , +Z_k, ⊗Y} locally inconsistent, i.e., {+Z_1, …, +Z_k, ⊗Y} locally consistent implies {Z_1, … , Z_k, Y} locally consistent.

Definition 10.5 *Be locally consistent and complete E,F∈EXT(GT). F is said to be subordinate to E if and only if there is Y∈FT, such that Y∈E, and furthermore for every Z∈FT, such that +Z∈E, we have to Y,Z∈F.*

Proposition 10.6 *For E∈EXT(GT), X∈FT. If E is locally consistent and complete and X∈E, then there exists F∈EXT(GT) locally consistent and complete such that X∈F and F is subordinate to E.*

Proof. Suppose X∈E. For E_X ={X}∪{Z: +Z∈E}, since E is locally consis- tent, then for proposition 10.4, E_X is also locally consistent. By adding to E_X the axioms of GT and all their consequences, we get an extension of GT that includes E_X , using proposition 10.3, we construct a locally consistent and lo- cally complete extension F of GT which includes E_X . Like X∈E_X , also X∈F. If +W∈E, by definition, W∈E_X , so W∈F. Therefore, F is subordinate to E. ■

Proposition 10.7 *For locally consistent and complete E,F,G∈EXT(GT).*

RR. Reflexivity. F is subordinate to F.

Proof. For X be the axiom Ax1, so X∈TT, and since by CPC we have X⊃X, it follows that X∈TT, then X∈F. Suppose that +W∈F, by proposition 8.11 it follows that W∈F. Hence, F subordinate to F.

Proposition 10.8 *If E'∈EXT(GT) is locally consistent and complete, then there exists a model in which all*

X∈TT-E' is true.

Proof. The model (S, ME $_a$, <, V) is defined as follows: For E, F, G, ..., be *locally* consistent and complete extensions of E' (E $_{to}$ the initial and the other subordinates), presented in the preceding propositions. To each extension F, a possible world MF is associated, for S the set of such possible worlds and ME $_{to}$ the actual world. The accessibility relation, <, is constructed as follows: MF<MG if and only if G is subordinate to F.

For each MF∈S and for each X∈F, V(MF,X)=1 if X∈F and V(MF,X)=0 if ~X∈F, where F is the locally consistent and complete extension associated with MF. Note that V is functional because F is *locally* consistent and complete. To claim that M is a model, rules 1 to 3 of definition 9.2 must be guaranteed.

1. By Axλ You have λ ∈TT, so λ ∈F, i.e., V(MF,λ)=1. Therefore, V$\boldsymbol{\lambda}$ is satisfied.

2. In the case of the conditional X ⊃ Y. Using CPC we have the following chain of equivalences: V(MF,X⊃Y)=0, i.e. ~(X⊃Y)∈F, by CPC we follow X∩~Y∈F, resulting in CPC that X∈F y ~Y∈F, which means that V(MF,X)=1 and V(MF,Y)=0, so V⊃ is satisfied.

3. In the case of rule V−. Sea MF is a world associated with F, MG is a world associated with G and Z∈FK. Suppose that V(MF,−Z)=1, so −Z∈F, and by proposition 8.12 ⊗~Z∈F, by proposition 10.6, there exists G subordinate to F, such that ~Z∈G, resulting that, (∃MG∈S)(MF<MG y MG(Z)=0).

To prove the reciprocal, suppose (∃MG∈S)(MF<MG y MG(Z)=0). If V(MF,−Z)=0, then it follows that ~ −Z∈F, i.e. +Z∈F, and since MF<MG, i.e., G is subordinate to F, then Z∈G, i.e., MG(Z)=1, result, by the hypothesis, that G is locally inconsistent, which is not the case. Therefore, V(MF, −Z)=1. Since the reciprocal has already been proved, then definition V− is satisfied.

Based on the above analysis, it is inferred that V is a valuation, and since the constraint RR is guaranteed by proposition 10.7, it is finally concluded that M is a model.

To conclude the proof, for X a theorem of E', so X is in E'. Therefore, using the definition of V, it turns out that V(MEa,X)=1, i.e., X is true in the model M=(S, MEa, <, V).

Proposition 10.9 *For X,X1, ..., X $_n$ ∈FT. a) If X∈VT then X∈TT.*

b) If {X $_1$,… , X $_n$} validates Y then Y is a consequence of {X $_1$,… , X $_n$}.

Proof. Part a. If X∉TT, then, by proposition 10.2, the extension E', obtained by adding ~X as a new formula, is locally consistent. Thus, according to proposition 10.8, there is a model M such that every theorem of E' is true in M, and since ~X∈TT-E', then ~X is true in M, i.e., X is false in M, hence X∉VT. It has been proved that X∉TT implies X∉VT, i.e., X∈VT implies X∈TT.

Part b. Suppose {X $_1$,… , X $_n$} validates Y, i.e., (X $_1$ ∩X $_2$ ∩ ... ∩X $_n$)⊃Y∈VT, by part a, follows that, (X $_1$ ∩X $_2$ ∩ ... ∩X $_n$)⊃Y∈TT. If {X $_1$,… , X $_n$} are assumed, by CPC Y is inferred, therefore Y is a consequence of {X $_1$, ..., X $_n$}.

Proposition 10.10 *For X,Y,X1,..., X $_n$ ∈FT. a) X∈VT if and only if X∈TT.*

b) {X $_1$, ..., X $_n$} validates Y if and only if {X $_1$, ..., X $_n$} implies Y.

Proof. Direct consequence of propositions 9.6 and 10.9.

XI. Existential Graphs GET

In this section, we present the primitive existential graphs for the GT system. For the construction of existential graphs, a variant of the notation proposed by Peirce in 4.378 of Peirce's Collected Papers [8] is used.

Definition 11.1 *The set, GET, of existential graphs for the GT system, is constructed from a set of atomic graphs, GA, and the constant (λ empty graph, ='_'), as follows.*

1) P∈GA implies P∈GET. 2) λ ∈GET. 3) X∈GET implies {X} ∈GET. 4) X,Y∈GET implies (X(Y))∈GET.

Definition 11.2 *The graph (X(Y)) it is called a conditional graph. In (X(Y))), X is called antecedent and Y consequent. Conditional cuts, (...(...)), are called continuous cuts. In the {Z} graph, the cut {...}, it's called a broken cut.*

Definition 11.3 *For X∈GET. Lambda is defined as the assertion sheet*

λ = '_'. Strong graph as *X = ({X}). Total falsehood as ⊥ = { }.

Definition 11.4 *RTRA. Primitive Transformation Rules*

R1. Strong double cut writing. The strong double cut is a graphical theorem, i. e., ({_}), ({λ}).

R2. 1) Graphs erasure. A graph can be *deleted* when it is in *an even* region. XY $_p$|⇒ X $_p$. 2) Writing graphs. In an odd region, any graph *can be* written. X $_i$|⇒ XY $_i$

R3. Unrestricted iteration and de-iteration of graphs in continuous cuts-only region. A *chart* can *be iterated* or *unrotated* in any region, odd or even, if the region is only surrounded by zero or more continuous cuts.)

Y(X) $_{ncc}$ ⇔ Y(XY) $_{ncc.}$ X ⇔ XX

R4. 1) Erased in a cut. A continuous cut can be *partially erased* (generating a broken cut) when it is in an *even* region. (X) $_{p|}$ ⇒ {X} $_p$. 2) Writing on a cut. A broken cut can be *completed* (generating a continuous cut) when it is in an *odd* region. {X} $_i$|⇒ (X) $_i$

R5. Erasure of continuous double cutting. A continuous double cut can be *erased* in *an even* region. ((X $_p$))|⇒

X_p

Continuous double cut writing. A continuous double cut can be *written around* a graph that is in *an odd* region.X_i|⇒((X_i))

Rules RI1, RI2 and RI3 presented in definition 5.5.

Definition 11.5 *For X∈GET. X is a graphical theorem of GET, denoted X∈ TGET, if there is a proof of X from the graph, using the graph λ transformation rules, i.e. X is the last row of a finite sequence of lines, in which each of the lines is , or is inferred from previous rows, using the transformation rules. Or to put it briefly, X∈TGET if and only if λ >>X. The number of lines, of the finite sequence, is referenced as the length of the proof of X.*

Y>>X, means that X is obtained from Y using a finite number of transformation rules.

Proposition 11.6 *For X,Y∈GET. (∀R∈RTRA)[$X_p \overset{R}{\Longrightarrow}$ Y]*(∃R'∈RTRA)[$Y_i \overset{R'}{\Longrightarrow}$X]. Proof by simple inspection of the primitive rules.

Proposition 11.7 *For X,Z∈GET. a)X>>Z |⇒ X_p >>Z. b) X>>Z |⇒ Z_i >>X.*

Proof. Suppose X>>Z, it must be proved that [X_p >>Z and Z_i >>X].

If X>>Z then there are R_1, … , R_n ∈ RTRA, and there are X_1, … , X_{n-1} ∈ GET, such that X$R_1X_1R_2X_2$ ... $X_{n-1}R_n$Z, and the length of the X>>Z transformation is said to be n and X>> $_n$Z is denoted. The proof is performed by induction on the length of the transformation. Base step. n=1. It means that only one of the primitive rules was applied, and since X is in an even region, then R must be of the form $X_p \overset{\mathbf{R}}{\Longrightarrow}$ Z with R∈RTRA. From proposition 11.6 it is inferred that $Z_1 \overset{R'}{\Longrightarrow}$ X with R'∈RTRA.

Inductive step. Inductive hypothesis (∀n>1)[W>> $_n$K ⇒ {W_p >>K, K_i >>W}, so X>> $_nX_n$ and X_nR_{n+1}Z. Applying the inductive hypothesis and proposition 11.6 we get X_p >>X_n and X_nR_{n+1}Z, X_{ni} > >X and Z_iR' $_{n+1}X_n$… . So, X_p >>Z_p y Z_i >>X. By the principle of mathematical induction, the truth of the proposition is concluded.

Proposition 11.8 *TDG. Graphic deduction theorem. For X,Z∈GET. X>>Z |⇒(X(Z)). Proof similar to proposition 5.10.*

Proposition 11.9 *For X∈GET. a) ((_)), *λ, {(_)}, ⊗ λ, λ. b) X_p |⇒((X_p)). c) ((X)) ⇔ X.*

Proof. Part a. By R1 we have ({_}), i.e. ({λ}), which means *λ. By having ({_}), by R4 we infer ((_)), again by R4 we derive {(_)}, i.e. {(λ)}, which means ⊗ λ. Since we already have ((_)), i.e. ((λ)), Using R5 we conclude λ.

Part b. Suppose X, for part a, we have ((_)), applying R3 we conclude ((X)). X⇒((X)) has been tested.

Part c results from parts b together with rule R5.

Proposition 11.10 *For X∈GET. a) *X_p|⇒ X_p. i.e. ({X_p}) |⇒ X_p.* b) X_i |⇒*X_i, *i.e.* X_i |⇒({X_i}).

Proof. Part a. Suppose ({Xp}), by R4 we get ((Xp)), according to R5 we derive Xp. Therefore, ({Xp})|⇒ Xp.

Part b. Using proposition 11.7 concludes X_i |⇒ ({X_i}).

Proposition 11.11 *For X∈GET. {_}$_p$ |⇒X_p, for everything X∈GET*

Proof. By R1 we have ({_}), using R2 we infer ({_}(X)), Suppose {_}, by R3 we ensure ((X)), according to R5 we conclude X, we have proved, {_} >>X. By proposition 11.7 we conclude {_}$_p$ |⇒ X_p.

Proposition 11.12 *For X∈GET. X∈TGET implies X_p |⇒*X_p. i.e.* X_p |⇒({X})$_p$

Proof. If X∈TGET then λ >>X, applying proposition 11.7 we derive λp>>Xp, and by R1 we have ({λp}), then we conclude ({Xp}). Therefore, Xp⇒({X}), i.e., Xp ⇒ *X.

Proposition 11.13 *For X∈GET. a) X>> {_} implies {X}*. b) {X} >> {_} implies X.

Proof. Part a. Suppose X>> {_}, by TD results (X({_})), by R1 we have ({_}), applying R3 we deduce (X), according to R4 we conclude {X}.

Part b. Suppose {X} >> {_}, by TD results ({X}({_})), by R1 we have ({_}), applying R3 we deduce ({X}), according to proposition 11.10 we conclude X.

XII. Equivalence Between GT And GET

In this section, the equivalence between GT and GET is presented, initially, in proposition 12.3, it is proved that the theorems of GT are graphical theorems of GET, in proposition 12.8, it is proved that the graphical theorems of GET are valid in the semantics of possible worlds, in proposition 12.12, it is proved that the theorems of GT are exactly the graphic theorems.

Definition 12.1 *Translation function [_]' of FT in GET. Be X,Y∈FT and P∈FA.*

1) P' = P. 2) [X⊃Y]' =(X'(Y')). 3) [X∪Y]' = ((X')(Y')). 4) [−X]' = {X'}. 5) [X∩Y]' = X'Y'.

6) [~X]' = (X'). 7) λ' = λ.

Translation function, (_)" of GET in FT. For X,Y∈GET, P∈GA. 1) P"=P. 2) λ"=λ.

3) (X(Y))"=X"⊃Y". 4) ((X)(Y))" = X"∪Y". 5) {X}"= −X'. 6) (XY)"=X"∩Y".

Proposition 12.2 *For X∈FT. If X is an axiom of GT then X'∈TGET.*
Proof. Using primitive rules, you have:
Ax1. λ. Since λ'=λ, then for proposition 11.9, λ ∈TGET.
Ax2. X⊃(Z⊃X). By proposition 11.9 we have ((_)) and ((_)), by R3 we derive ((((_)))), using the rule R2 follows (X'(((_))), applying R3 results (X'(((X')))), again by R2 we derive (X'((Z'(X')))), i.e. (Ax2)'∈TGET.
Ax3. (X⊃(Y⊃Z))⊃((X ⊃Y)⊃(X⊃Z)). Suppose (X'((Y'(Z')))), (X'(Y')) and X'. By R3 we can deduce ((Y'(Z')))), ((Y')), applying R5 we follow (Y'(Z')), Y', using R3 we infer ((Z')), by R5 we conclude Z'. By TDG (proposition 11.8) 2 times, it has been proved that (Ax3)'∈TGET.
Ax4. [(X⊃Y)⊃ X]⊃X. Suppose ((X'(Y'))(X')), by R2 we get ((X')(X')), as R3 results ((X')), applying R2 derives X'. Using TDG it is concluded that (Ax4)'∈TGET.
Ax5. –λ ⊃Z. By proposition 11.11 we have {_} >>Z, applying TDG we conclude that (Ax5)'∈TGET.
Ax6. (X⊃–λ)⊃–X. Suppose (X'({_})), whence X'>> {_}, by proposition 11.13 is derived {X'}. Using TDG it is concluded that (Ax6)'∈TGET.
Ax7. [–(X⊃Y)⊃–λ]⊃[(–X⊃–λ)⊃(–Y⊃–λ)]. Suppose ({(X'(Y'))}({_})),
({X'}({_})). i.e. *(X'(Y')), *X', by propositions 11.7 and 11.10 follows *(*X'(Y')), using R3 we deduce *((Y')), applying R5 we affirm *Y'. Using TDG it is concluded that (Ax7)'∈TGET.
Ax+. If X∈ {Ax1, , Ax7} then –X⊃–λ is an axiom. Consequence of proposition 11.12: X∈TGET implies $X_p \Rightarrow {*X_p}$.
Proposition 12.3 *For X∈FT. a) If X∈TT then X'∈TGET. b) If X⟩⟩Y then X'>>Y'.*
Proof. Part a. Induction on the length of the X demonstration in GT.
Base step. If the length of the proof is 1, then X is an axiom, by proposition 12.2 X'∈TGET.
Induction step. The inductive hypothesis is: if Y∈TT and the length of the proof of Y is less than L then Y'∈TGET. Suppose X∈TT and that the length of the proof of X is L, so X is an axiom or obtained from previous steps using Mp. In the first case, proceed as in the base step. In the second case, Y and Y⊃Z are taken in previous steps of the proof of X, i.e., the lengths of the proofs of Y and Y⊃X are less than L, by the inductive hypothesis it turns out that Y'∈TGET and (Y'(X'))∈TGET, applying R3 infers ((X'))∈TGET, using R5 we conclude X'∈TGET. By the principle of mathematical induction, it is proved that the theorems of GT are graphic theorems.
Part b. If X implies Y, then X⊃Y∈TT, by the part a, (X'(Y'))∈TGET, i.e., λ >>(X'(Y')), if X' is assumed, by R3 follows ((Y')), applying R5 results in Y', so X'>>Y'.
Proposition 12.4 *For X,Y,Z∈GT. If X⇒Y then Y" is validly inferred from X", in the case of the most elementary versions of the rules. Generalization to arbitrary odd or even regions will be presented later.*
Proof. R1. ({_}). If ∼ −λ it is invalid then there is a model with the actual world M such that, M(∼ −λ)=0, by V∼ follows M(−λ)=1, by V− it follows that (∃P∈S)(M<P y P(λ)=0). Which contradicts Vλ. Therefore, R1" is valid.
R2. XZ|⇒ X. Consider an arbitrary model with the actual world M. Suppose that M(X"∩Y")=1, by V∩ we derive M(X')=1. Therefore, R2" is a valid rule in GT.
R3. Y(X)⇔Y(XY). Consider an arbitrary model with the actual world M. Suppose that M(Y"∩∼X")=1, by V∩ follow M(Y")=1, M(∼X")=1, according to V∼ derive M(X")=0, using V∩ we affirm M(X"∩Y")=0, by V∩ we infer M(∼(X"∩Y"))=1, Applying V∼ we get M(Y"∩ ∼(X"∩Y"))=1. It has been proved that M(Y"∩∼X")=1 implies M(Y"∩∼(X"∩Y"))=1. To prove the reciprocal, suppose that (Y"∩∼(X"∩Y"))=1, by V∩ follow M(Y")=1 and M(∼(X"∩Y"))=1, resulting M(X"∩Y")=0, i.e. M(Y")=0 or M(X")=0, but M(Y")=1, so M(X")=0, i.e. M(∼X")=1, and by V∩ we deduce M(Y"∩∼X")=1. It has been proved that M(Y"∩∼(X"∩Y"))=1 implies M(Y"∩∼X")=1. Therefore, R3" is a valid rule in GT.
R4. (X)|⇒ {X}. Consider an arbitrary model with the actual world M. Suppose that M(∼X")=1, i.e. M(X")=0, and since M<M, we can affirm (∃N)(M>N) N(X")=0, by V− we derive M(−X")=1. It has been proved that M(∼X")=1 implies M(−X")=1. Therefore, R4" is a valid rule in GT.
R5. ((X))|⇒ X. Suppose that M(∼∼X")=1 is equivalent to M(∼X")=0, again by the same rule we conclude M(X")=1. It has been proved that M(∼∼X") =1 implies M(X")=1. Therefore, R5" is a valid rule in GT.
Proposition 12.5 *For X,Y,Z,W∈FT. If X⊃Y∈VT are valid in GT:*
a) (X∩Z)⊃(Y∩Z). b) −(Y∩Z)⊃ −(X∩Z). c) −(W∩ −(X∩Z))⊃ −(W∩ −(Y∩Z)).
Proof. Suppose X⊃Y∈VT, so you have the initial result, for every model, (S, Ma, <, V), and for every M∈S, M(X⊃Y)=1. Part a. Is CPC result.
Part b. Be M be the actual world of any model. Suppose that M(−(Y∩Z))=1, then by V−, there is P∈S, M<P and P(Y∩Z)=0. Suppose M(−(X∩Z))=0, times V− it follows that, for every N∈S, M<N implies N(X∩Z)=1, and as M<P then P(X∩Z)=1, times ∩ we have P(X)=1 and P(Z)=1, as P(Y∩Z)=0, by V∩, P(Y)=0 is derived, but as P(X)=1, and by the initial result, P(X⊃Y)=1, then by V⊃, we get P(Y)=1, which is not the case, so M(−(X∩Z))=1. It has been proven that, M(−(Y∩Z))=1 implies M(−(X∩Z))=1, which by V⊃ means that

M(−(Y∩Z)⊃ −(X∩Z))=1. Therefore, −(Y∩Z)⊃ −(X∩Z)∈VT.

Part c. Be M be the actual world of any model. Suppose that M(−(W∩ −(X∩Z)))=1, then by V−, there is P∈S, M<P and P(W∩ −(X∩Z))=0.

Assumption 2, M(−(W∩−(Y∩Z)))=0, times V− it follows that, for every N∈S, M<N implies N(W∩ −(Y∩Z))=1, and as M<P then P(W∩ −(Y∩Z))=1, by V∩ we have P(W)=1 and P(−(Y∩Z))=1, by V− it follows that, there is Q∈S, P<Q and Q(Y∩Z)=0, such as P(W)=1 and P(W∩−(X∩Z))=0, follows P(−(X∩Z))=0, by V− it follows that, for each N∈S, P<N implies N(X∩Z)=1, as P<Q, in particular, Q(X∩Z)=1, follows V∩ followed by Q(X)=1 and Q(Z)=1, for the initial result, we have Q(X⊃Y)=1, follows V⊃ Q(Y)=1, and having Q(Z)=1, is derived Q(Y∩Z)=1, which is not the case, so M(−(W∩ −(Y∩Z)))=1. It has been proven that, M(−(W∩ −(X∩Z)))=1 implies M(−(W∩ −(Y∩Z)))=1, which by V⊃ means that M(−(W∩ −(X∩Z))⊃ −(W∩ −(Y∩Z)))=1. Therefore, −(W∩ −(X∩Z))⊃ −(W∩ −(Y∩Z))∈VT.

Proposition 12.6 *For X,Y,Z,W∈FT. If X⊃Y∈TT are GT theorems:*

a) (X∩Z)⊃(Y∩Z). b) −(Y∩Z)⊃ −(X∩Z). c) −(W∩ −(X∩Z))⊃ −(W∩ −(Y∩Z)).

Proof. A direct consequence of propositions 10.10 and 12.5.

Proposition 12.7 *For X,Y,Z,W,V∈FT. If X⟩⟩Y then, a) X_p⟩⟩Y. b) Y_i⟩⟩X.*

Proof. Induction in the number, *n*, of negations surrounding X.

Base step. n=0. X∩Z⟩⟩Y∩Z, is satisfied by proposition 11.7.

n=1. There are 2 possibilities, –{X∩Z} and ~ {X∩Z}. For proposition 12.6, we have –{Y∩Z} ⊃–{X∩Z}, for CPC we have ~ {Y∩Z} ⊃~ {X∩Z}, so the proposition is satisfied when n=1.

n=2. There are 4 possibilities, –{W∩–{X∩Z}}, –{W∩~ {X∩Z}}, ~ {W∩–{X∩Z}} y ~ {W∩~ {X∩Z}}. By proposition 12.6, we have that, –{W∩–{X∩Z}} ⊃–{W∩–{Y∩Z}} y –{W∩~ {X∩Z}} ⊃–{W∩~ {Y∩Z}}, the other 2 cases ~ {W∩–{X∩Z}} ⊃~ {W∩–{Y∩Z}} y ~ {W∩~ {X∩Z}} ⊃~ {W∩~ {Y∩Z}} are taken for CPC, so the proposition is satisfied when n=2.

Inductive step. Part a. As an inductive hypothesis we have that, if X is surrounded by *2n* negations, then X> >Y. By proposition 12.6 we have that, –{W∩–{X∩Z}} ⊃–{W∩–{Y∩Z}} y –{W∩~ {X∩Z}} ⊃–{W∩~ {Y∩Z}}, and by CPC we have ~ {W∩–{X∩Z}} ⊃~ {W∩–{Y∩Z}} y ~ {W∩~ {X∩Z}} ⊃ ~ {W∩~ {Y∩Z}}, in the region surrounded by *2n* negations, and these are the only cases for which two other negations can be added to X. Therefore, if X is surrounded by *2n+2* slices, i.e. by *2(n+1)* slices, then X>>Y.

Part b. As an inductive hypothesis we have that, if X is surrounded by *2n+1* negations, then Y>>X. By proposition 12.6 we have that, –{W∩–{X∩Z}} ⊃–{W∩–{Y∩Z}} y –{W∩~ {X∩Z}} ⊃–{W∩~ {Y∩Z}}, and by CPC we have ~ {W∩–{X∩Z}} ⊃~ {W∩–{Y∩Z}} y ~ {W∩ ~ {X∩Z}} ⊃~ {W∩~ {Y∩Z}}, in the region surrounded by *2n+1* negations, and these are the only cases for which two other negations can be added to X. Therefore, if X is surrounded by *2n+1+2* negations, i.e. by *2(n+1)+1* negations, then Y>>X. By the principle of mathematical induction, the proposition has been proved.

Proposition 12.8 *For X,Y,Z∈GET. a) Primitive GET rules are valid rules in GT semantics. b) If X∈GET then X"∈VG.*

Proof. Part a. Direct consequence of propositions 12.4 and 12.7.

Part b. If X∈TT then λ >>X, then there are R1, ..., Rn∈RTRA, and there are X_1, ..., X_{n-1} ∈GET, such that $\lambda R_1 X_1 R_2 X_2 ... X_{n-1} R_n X$. The proof is performed by induction over the length L of the demonstration. Base step. L=1. It means that only one of the primitive rules was applied, then X"∈VG.

Inductive step. Inductive hypothesis: The proposition is valid if L < n+1 with n > 0. Be L=n+1, so $\lambda R_1 X_1 R_2 X_2 ... X_{n-1} R_n X_n R_{n+1} X$, i.e. $\lambda R_1 X_1 R_2 X_2 ... X_{n-1} R_n X_n$ and $X_n R_{n+1} X$, both demonstrations with a length shorter than n+1. Applying the inductive hypothesis, it turns out that X_n"∈VG and from X_n" is validly inferred X", hence X"∈VG. By the principle of mathematical induction, the truth of the proposition is concluded.

Proposition 12.9 *For X,Y∈GET. a) If X∈TGET then X"∈TT.* b) If X>>Y then X" implies Y".

Proof. Part a. By proposition 10.10 we have that, X"∈VT if and only if X"∈TT, and by proposition 12.8 we have that, if X∈TGET then X"∈VG. Therefore, if X∈TGET then X"∈TT.

Part b. Consequence of part a and proposition 10.10.

Definition 12.10 *Be T1=[_]':FG→GG and T2=[_]":GG→FG, be the translation functions presented in definition 12.1. They are defined: the composite function T1oT2:GG→GG such that (T1oT2)[X]=T1[T2[X]], the composite function, T2oT1:FG→FG such that (T2oT1)[X]=T2[T1[X]], the identity function in FG, Id_{FG}:FG→FG such that (Id_{FG})[X]=X, the identity function in G Id_{GG}:GG→GG such that (Id_{GG})[X]=X.*

Proposition 12.11 *a) T1oT2 = Id_{GG}. b) T2oT1= Id_{FT}. c) T1 is the inverse function of T2. d) T2 is the inverse function of T1.* Proof similar to proposition 6.12.

Proposition 12.12 *For G,H∈GET, and X,Y∈FT. a) G∈TGET if and only if G"∈TT. b) X'∈TT if and only if X∈TGET. c) G>>H if and only if G"⟩⟩H". d) X'>>Y' if and only if X⟩⟩Y.*

Proof. Part a. By proposition 12.9 we have that, if G∈TGET then G"∈TT, furthermore, by proposition 12.3 we have that, if G”∈TT then (G”)’∈TGET, but by proposition 12.11 we know that, (G”)’=G, resulting that, if G”∈TT then G∈TGET, and since we have the reciprocal, we conclude that, G∈TGET if and only if G”∈TT.

Part b. By proposition 12.3 we have that, if X∈TT then X’∈TGET, furthermore, by proposition 12.9 we have that, if X’∈TGET then (X’)”∈TT, by proposition 12.11 (X’)”=X, resulting that, if X’∈TGET then X∈TT, and since we have the reciprocal, we conclude that, X’∈TGET if and only if X∈TT.

Part c. By proposition 12.3 we have, if G”⟩⟩H” then [G”]’>>[H”]’, but [G”]’=G and [H”]’=H, so if G”⟩⟩H” then G>>H, in addition by proposition 12.9 we have the reciprocal. Therefore, G>>H if and only if G”⟩⟩H”.

Part d. By proposition 12.9 we have, if X’⟩⟩Y’ then [X’]”>>[Y”]’, by proposition 12.11 we have [X’]”=X and [Y’]”=Y, so if X’⟩⟩Y’ then X>>Y, by proposition 12.3 we have the reciprocal. Therefore, X>>Y if and only if X’⟩⟩Y’.

XIII. Final Remark GT And GET

Remark 13.1 *a) GET is paraconsistent. b) GT is paraconsistent.*

Proof similar to proposition 7.2.

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