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Mathematical Model for Evaluating the Impact of Education on Drug Addiction

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Abstract:
Mathematical modelling provides a powerful framework for analysing and predicting dynamics of addiction to a drug. This study presents a mathematical model to evaluate the impact of education on drug addiction. The analysis highlights how increased awareness and preventive education can reduce addiction rates and promote rehabilitation. The model employs a compartmental approach, consisting of six non-linear differential equations, to analyze these dynamics. The basic reproduction number is derived to determine the threshold for the persistence or elimination of addiction. Equilibrium points are identified and it analyses the drug-free equilibrium that stands for the absence of addiction in the population and the endemic equilibrium, which indicates the presence of addiction. The local stability of the equilibrium points is conducted to find out whether the system remains stable or not. Empirical data was collected from treatment and rehabilitation centers through surveys and medical records, focusing on education levels, awareness, relapse rates and program effectiveness. Numerical simulations confirm the theoretical results, highlighting the need for targeted interventions to reduce addiction prevalence and formulating public health strategies.
Keywords: *Mathematical Model, Reproduction Number, Next Generation Matrix, Global Stability, Lyapunov Function*

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I. Introduction

Mathematical modelling is the process of using mathematics to represent real-world systems or phenomena. It involves translating a problem from the physical, biological, economic or social world into mathematical terms, which can then be analysed or solved using various mathematical techniques. The aim of mathematical modelling is to gain insight into the system being studied, predict future behaviour or inform decision-making. Mathematical modelling is used to understand, analyse and predict complex systems across various fields. It helps in disease spread prediction, climate modelling, optimizing industrial processes and financial risk assessment. The process of converting a real word problem into the language of mathematics is described in figure 1.

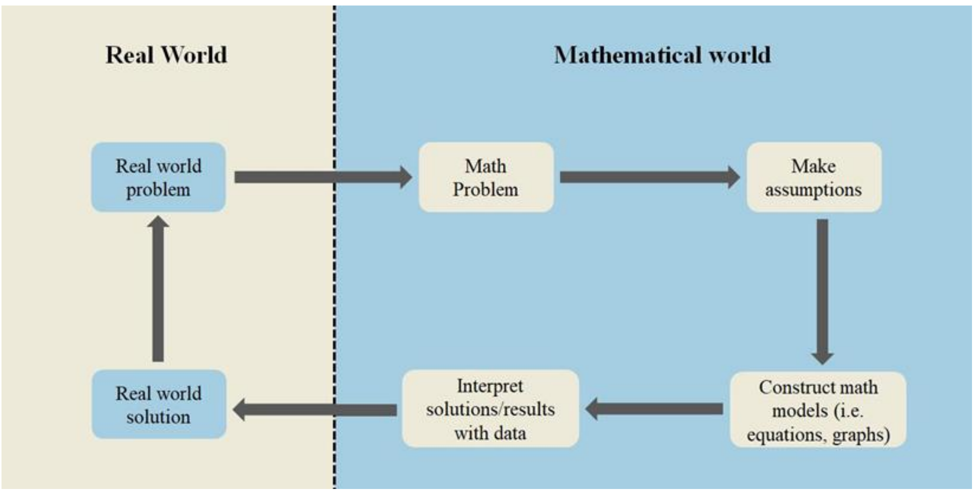

Figure 1: Scientific Process to connect real world problem with Mathematics

Mathematical Model approach is particularly valuable in the field of epidemiology, where it is used to study the spread and control of diseases within populations. By applying mathematical models to epidemics, we can simulate disease transmission, estimate key epidemiological parameters and evaluate the potential impact of various public health interventions. There are two primary types of epidemic models: deterministic and stochastic.

A stochastic model includes randomness, using probability distributions to reflect uncertainty. So even with the same starting conditions, outcomes can differ across simulations. It is ideal for unpredictable systems like stock markets or weather forecasting. A deterministic model has no randomness - outcomes are fully determined by initial conditions and fixed parameters. It often divides populations into compartments with predictable transitions between them.

Mathematical modelling is playing an important role in spread and control of many addictions including drug addiction. Drug addiction is a chronic and relapsing condition characterized by compulsive substance use despite harmful consequences, disrupting the brain's reward system and leading to physical and psychological dependence. The development of addiction is influenced by genetic, environmental and social factors, with peer pressure, mental health issues and trauma acting as key triggers. Over time, prolonged substance uses results in severe health complications, including organ damage, cognitive impairments and an increased risk of infectious diseases, which complicate the individual's struggle with recovery.

In India, drug addiction is a significant public health concern, affecting millions across various demographics. According to the 2019 National Survey, approximately 16 crore people aged 10 - 75 are current alcohol users, 3.1 crore use cannabis and 2.06% of the population uses opioids. Recent reports from December 2024 show over 10.47 crore people are affected by substance abuse, including 15 million children and adolescents. The growing prevalence of inhalant use, particularly among children and adolescents, exacerbates this issue, with a higher usage rate among youth (1.17%) compared to adults (0.58%).

The intersection of drug addiction and youth crime is particularly concerning, with increase the regions witnessing a rise in juvenile delinquents struggling with addiction. This highlights the urgent need for targeted interventions, especially for youth who are at greater risk of long-term dependency.

To address this crisis, a multifaceted approach is required, combining early intervention, public awareness campaigns and comprehensive rehabilitation programs. Policy changes and community engagement are essential for creating a supportive environment for recovery. Targeting at-risk groups, such as children, adolescents and young adults, can significantly reduce the harmful impacts of addiction on both individuals and society. Collaborative efforts involving healthcare professionals, policymakers and community leaders are key to breaking the cycle of addiction, reducing its prevalence and improving public health outcomes across the country.

II. Literature Review

Babaei, A., et al. (2020)[3] developed a mathematical model to explore the interaction between drug addiction and HIV/AIDS transmission in Iranian prisons. Initially, they analysed the stability of both addiction and HIV/AIDS models independently, excluding medical treatment. The study was later extended by incorporating parameters for drug addiction treatment to assess its impact on HIV/AIDS spread. Findings highlight that rehabilitative treatments can significantly reduce the disease's transmission, as demonstrated through comparative reproduction numbers under different treatment scenarios.

Binuyo, A. O. (2021)[4] developed a mathematical model to analyse drug substance addiction among students in Nigerian tertiary institutions. The model focuses on the dynamics of mood-altering substance use and addiction, with the basic reproduction number derived via the next generation method. The drug-free equilibrium is locally asymptotically stable when this number is less than one. The study found that increased student recruitment and relapse rates raise the addiction number, while higher interaction rates between users and non-users lead to more drug use. Policy intervention is recommended to curb drug sales and use among students.

Bansal, K., et al. (2022)[5] proposed a fractional-order mathematical model to analyse the public health impact of illegal drug use. The model is built on epidemiological principles, with drug transmission driven by social interactions between susceptible individuals and drug users. A threshold value is derived to assess model stability. Using a Lyapunov function, stability of the addiction equilibrium is examined. The model is further extended to incorporate time delay, leading to conditions for Hopf bifurcation. Numerical simulations validate the theoretical findings and highlight the model's dynamic behaviour.

Andrawus, J., et al. (2024)[1] proposed a deterministic nonlinear model to study drug abuse and addiction, incorporating interventions like awareness and rehabilitation. The model's mathematical analysis confirmed solution positivity, boundedness, and the existence of two equilibria: the drug-free equilibrium (DFE) and the drug-endemic equilibrium point (DEEP). The DFE is globally asymptotically stable if $R_0 < 1$, while the DEEP is globally stable when $R_0 > 1$ and $\delta_1 = \delta_2 = 0$, as shown via a Go-Volterra-type Lyapunov function. Simulations emphasized that increasing awareness and rehabilitation rates significantly helps in curbing drug addiction.

Zanib, S. A., et al. (2024)[15] addressed drug addiction as a global public health issue and introduced a novel mathematical model with compartments: S_D, E_D, H_D, L_D, R_D, and C_D, to study its dynamics. The model distinguishes between heavily and lightly addicted individuals, incorporating rehabilitation as a control strategy. Using the RK4 method in Maple, the study emphasizes early identification and intervention to prevent escalation of addiction severity. Simulation results highlight that interaction rates (γ) and rehabilitation rates δ_1 and δ_2 are key factors in reducing the drug-addicted population.

Ullah, A., et al. (2024)[14] presented a modified NERA model to address marijuana use, introducing a new class for addicted individuals under treatment - termed the hospitalized class. The model employs first-order nonlinear ODEs to represent marijuana consumption dynamics and classifies the population into non-users, experimental users, recreational users, addicts, and hospitalized individuals. Validation through invariant region analysis and the basic reproduction number R_0 helped assess initial transmission potential. Sensitivity analysis identified key factors influencing marijuana spread, and prevention-based control strategies were tested using RK4 simulations in MATLAB, confirming their effectiveness.

III. Mathematical Model

Here, we formulate a model based on the SIRS (Susceptible (S), Infected (I), Recovered (R)) model of drugs addiction. The entire population is divided into six compartments based on addiction status, which are referred to as state variables. The compartments are denoted as, Susceptible population (S), Exposed population (E), Lightly addicted educated population (M_E), Lightly addicted uneducated population (M_{UE}), Heavily addicted population (H) and Recovered population (R).

- Let us assume homogeneous population mixing, i.e., each individual can contact any other individual.
- The transitions between the different subpopulations are determined as follows:
  a. Newly recruited individuals enter the susceptible subpopulation $S(t)$. Susceptible individuals become exposed to drug $E(t)$ use through interaction with drug-exposed individuals at a rate α.
  b. An individual exposed to drugs $E(t)$ may be either lightly addicted and educated $M_E(t)$ or lightly addicted and uneducated $M_{UE}(t)$.
  c. Exposed individuals $E(t)$ do not get heavily addicted $H(t)$ directly.
  d. An educated/uneducated person with a lightly drug addicted may be either heavily addicted $H(t)$ or recovered $R(t)$ through self-realization.
  e. Recovered person $R(t)$ becomes susceptible $S(t)$ again due to loss of immunity.
  f. The death rate in compartments (S), (E) and (R) is same and denoted by μ. The death rate in addicted classes $\left(M_E(t)\right)$, $(M_{UE}(t))$ and (H) is same and denoted by δ.

The notations and parametric values used in this model are given in the following Table 1.

Table 1: Variables and its descriptions

Variables/ Parameters	Description
$S(t)$	People who do not use drug yet but might in the future
$E(t)$	Number of individuals who have been exposed to drugs but have not progressed to addiction
$M_E(t)$	Number of educated individuals with lightly Drug addicted
$M_{UE}(t)$	Number of uneducated individuals with lightly Drug addicted
$H(t)$	Number of people who are heavily drug addicted
$R(t)$	the number of individuals who have successfully quit drug addiction
$N(t)$	Total population
B	Recruitment rate
α	The contact rate of transmission from susceptible individuals to the exposed class
β	The rate of the exposed and educated individuals who becomes lightly drug addicted
π	The rate of the exposed and uneducated individuals who becomes lightly drug addicted
θ	The rate at which lightly drug addicted and educated individuals becomes highly drug addicted
ε	The rate at which lightly drug addicted and uneducated individuals becomes highly drug addicted
λ	The rate at which educated individuals recover from lightly drug addicted
γ	The rate at which individuals recover from highly drug addicted
σ	The rate at which uneducated individuals recover from lightly drug addicts
ω	The rate at which recovered individuals relapse and become susceptible (S) to drug addiction again
δ	The death rate due to drug addiction
μ	Natural death rate.

The mathematical model, formulated and depicted in Figure 2, utilizes the notations specified in Table 1 to represent the dynamics of drug addiction.

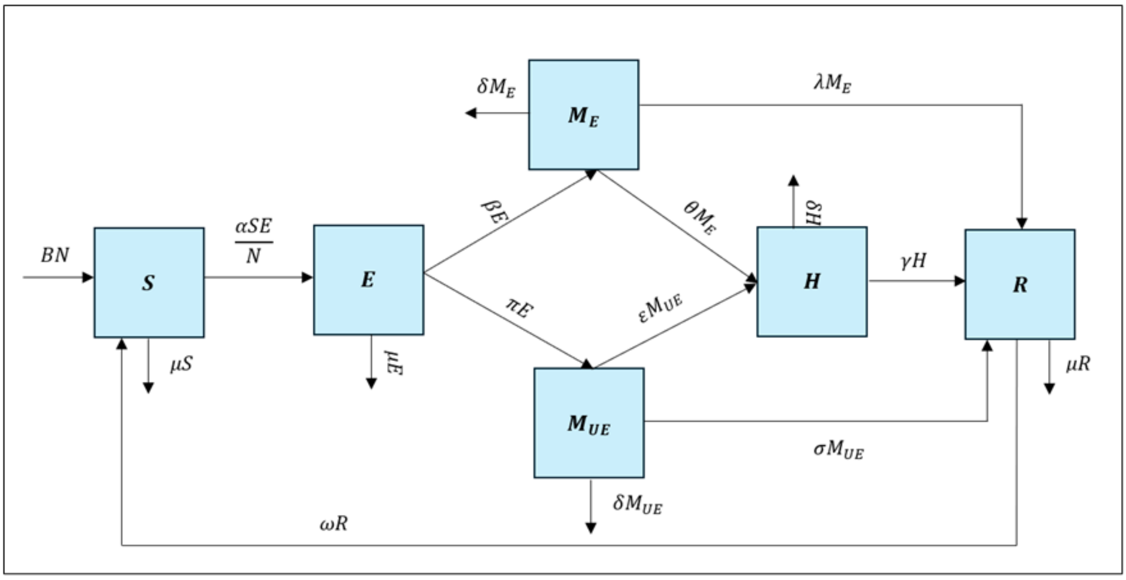

Figure 2: Mathematical Model

By taking the above assumptions into account and the transitions of how people move among classes, we can construct a system of differential equations representing the model of the evolution of drug addiction as follows:

$$\frac{dS(t)}{dt} = B - \frac{\alpha S(t)E(t)}{N(t)} + \omega R(t) - \mu S(t)$$

$$\frac{dE(t)}{dt} = \frac{\alpha S(t)E(t)}{N(t)} - \beta E(t) - \pi E(t) - \mu E(t)$$

$$\frac{dM_E(t)}{dt} = \beta E(t) - \lambda M_E(t) - \theta M_E(t) - \delta M_E(t)$$

$$\frac{dM_{UE}(t)}{dt} = \pi E(t) - \varepsilon M_{UE}(t) - \sigma M_{UE}(t) - \delta M_{UE}(t)$$

$$\frac{dH(t)}{dt} = \theta M_E(t) + \varepsilon M_{UE}(t) - \gamma H(t) - \delta H(t)$$

$$\frac{dR(t)}{dt} = \lambda M_E(t) + \gamma H(t) + \sigma M_{UE}(t) - \mu R(t) - \omega R(t) \qquad \ldots\ldots\ldots.1$$

with

$$S(t) + E(t) + M_E(t) + M_{UE}(t) + H(t) + R(t) = N(t)$$

IV. Basic Properties

Invariant Region

It is necessary to prove that all solutions of system (1) with positive initial data will remain positive for all times $t > 0$. This will be established by the following lemma.

Lemma 1: All feasible solution $S(t), E(t), M_E(t), M_{UE}(t), H(t), R(t)$ of system equation (1) are bounded by the region

$$A = \left\{(S, E, M_E, M_{UE}, H, R) \in \mathbb{R}^6 : S + E + M_E + M_{UE} + H + R \leq \frac{B}{\mu}\right\}$$

Proof. From the system equation (1)

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dE(t)}{dt} + \frac{dM_E(t)}{dt} + \frac{dM_{UE}(t)}{dt} + \frac{dH(t)}{dt} + \frac{dR(t)}{dt}$$

$$\frac{dN(t)}{dt} = B - \mu\big(S(t) + E(t) + M_E(t) + M_{UE}(t) + H(t) + R(t)\big)$$

implies that

$$\frac{dN(t)}{dt} \le B - \mu N(t)$$

It follows that

$$N(t) \le \frac{B}{\mu} + N(0)e^{-\mu t}$$

Where $N(0)$ is the initial value of total number of people, thus $\lim_{t\to\infty} \sup N(t) \le \frac{B}{\mu}$

Then $$S(t) + E(t) + M_E(t) + M_{UE}(t) + H(t) + R(t) \le \frac{B}{\mu}$$

Hence, for the analysis of model (1), we get the region which is given by the set:

$$A = \left\{(S, E, M_E, M_{UE}, H, R) \in \mathbb{R}^6 : S + E + M_E + M_{UE} + H + R \le \frac{B}{\mu}\right\}$$

which is a positively invariant set for (1), so we only need to consider the dynamics of system on the set A, the non-negative sets of solutions.

Positivity of the solutions of the model

Lemma 2: If $S(0) \ge 0, E(0) \ge 0, M_E(0) \ge 0, M_{UE}(0) \ge 0, H(0) \ge 0$ and $R(0) \ge 0$ then the solution of system (1) $S(t), E(t), M_E(t), M_{UE}(t), H(t)$ and $R(t)$ are positive for all $t > 0$.

Proof. From the system equation (1)

$$\frac{dS(t)}{dt} = B - \frac{\alpha S(t)E(t)}{N(t)} + \omega R(t) - \mu S(t)$$

To seek positivity, we can write

$$\frac{dS(t)}{dt} \ge B - \mu S(t)$$

$$\Rightarrow \frac{dS(t)}{dt} + \mu S(t) \ge B$$

The integrating factor of above equation is given by

$$I.F. = e^{\int \mu dt} = e^{\mu t}$$

Multiplying $e^{\mu t}$ on both side of the equation, we get

$$\frac{d}{dt}\left(e^{\mu t}S(t)\right) \ge Be^{\mu t}$$

Now, by integrating above equation, we have

$$S(t) \ge \frac{B}{\mu} + ce^{-\mu t}$$

where c is an integrating constant.

Considering the initial value at $t = 0, S(t) \ge S(0)$

$$S(0) \ge \frac{B}{\mu} + c \Rightarrow S(0) - \frac{B}{\mu} \ge c$$

Substituting the value of c into the above equation, we obtain

$$S(t) \ge \frac{B}{\mu} + \left(S(0) - \frac{B}{\mu}\right)e^{-\mu t}$$

So, at $t = 0$ and $t \longrightarrow \infty, S(t) \ge 0$. By repeating the above procedure, we can prove the positivity of all other state variables.

Consequently, it is clear that $\forall\ t \ge 0$.

$$S(t) \ge 0, E(t) \ge 0, M_E(t) \ge 0, M_{UE}(t) \ge 0, H(t) \ge 0, R(t) \ge 0$$

V. Equilibria And Their Stability Analysis

Equilibrium points and reproduction number (R_0)

Now to find equilibrium points of the mathematical model, put right hand side equals to zero in equations given in system (1). In this paper python software is used. This analysis helps in understanding the long-term behaviour of drugs dynamics under different conditions and interventions.

The drug - free equilibrium $E^0\left(\frac{B}{\mu}, 0,0,0,0,0\right)$ is achieved when there are no active drug users in the population ($E = M_E = M_{UE} = H = R = 0$). The drug present equilibrium $E^*(S^*, E^*, {M_E}^*, {M_{UE}}^*, H^*, R^*)$ is achieved when drug users exist. Where:

$$S^* = \frac{N(\beta + \mu + \pi)}{\alpha}$$

$$E^{*} = \frac{(\delta+\gamma)(\mu+\omega)(\delta+\varepsilon+\sigma)(\delta+\lambda+\theta)(\alpha B - N\beta\mu - N\mu^2 - N\mu\pi)}{\begin{gathered}\alpha(\beta\delta^3\mu + \beta\delta^3\omega + \beta\delta^2\varepsilon\mu + \beta\delta^2\varepsilon\omega + \beta\delta^2\gamma\mu + \beta\delta^2\gamma\omega + \beta\delta^2\lambda\mu + \beta\delta^2\mu\sigma + \beta\delta^2\mu\theta + \beta\delta^2\omega\sigma \\ +\beta\delta^2\omega\theta + \beta\delta\varepsilon\gamma\mu + \beta\delta\varepsilon\gamma\omega + \beta\delta\varepsilon\lambda\mu + \beta\delta\varepsilon\mu\theta + \beta\delta\varepsilon\omega\theta + \beta\delta\gamma\lambda\mu + \beta\delta\gamma\mu\sigma + \beta\delta\gamma\mu\theta + \beta\delta\gamma\omega\sigma \\ +\beta\delta\lambda\mu\sigma + \beta\delta\mu\sigma\theta + \beta\delta\omega\sigma\theta + \beta\varepsilon\gamma\lambda\mu + \beta\varepsilon\gamma\mu\theta + \beta\gamma\lambda\mu\sigma + \beta\gamma\mu\sigma\theta + \delta^3\mu 2 + \delta^3\mu\omega + \delta^3\mu\pi \\ +\delta^3\omega\pi + \delta^2\varepsilon\mu^2 + \delta^2\varepsilon\mu\omega + \delta^2\varepsilon\mu\pi + \delta^2\varepsilon\omega\pi + \delta^2\gamma\mu^2 + \delta^2\gamma\mu\omega + \delta^2\gamma\mu\pi + \delta^2\gamma\omega\pi + \delta^2\lambda\mu^2 \\ +\delta^2\lambda\mu\omega + \delta^2\lambda\mu\pi + \delta^2\lambda\omega\pi + \delta^2\mu^2\sigma + \delta^2\mu^2\theta + \delta^2\mu\omega\sigma + \delta^2\mu\omega\theta + \delta^2\mu\pi\sigma + \delta^2\mu\pi\theta + \delta^2\omega\pi\theta \\ +\delta\varepsilon\gamma\mu^2 + \delta\varepsilon\gamma\mu\omega + \delta\varepsilon\gamma\mu\pi + \delta\varepsilon\lambda\mu^2 + \delta\varepsilon\lambda\mu\omega + \delta\varepsilon\lambda\mu\pi + \delta\varepsilon\lambda\omega\pi + \delta\varepsilon\mu^2\theta + \delta\varepsilon\mu\omega\theta + \delta\varepsilon\mu\pi\theta \\ +\delta\varepsilon\omega\pi\theta + \delta\gamma\lambda\mu^2 + \delta\gamma\lambda\mu\omega + \delta\gamma\lambda\mu\pi + \delta\gamma\lambda\omega\pi + \delta\gamma\mu^2\sigma + \delta\gamma\mu^2\theta + \delta\gamma\mu\omega\sigma + \delta\gamma\mu\omega\theta + \delta\gamma\mu\pi\sigma \\ +\delta\gamma\mu\pi\theta + \delta\gamma\omega\pi\theta + \delta\lambda\mu^2\sigma + \delta\lambda\mu\omega\sigma + \delta\lambda\mu\pi\sigma + \delta\mu^2\sigma\theta + \delta\mu\omega\sigma\theta + \delta\mu\pi\sigma\theta + \varepsilon\gamma\lambda\mu^2 + \varepsilon\gamma\lambda\mu\omega \\ +\varepsilon\gamma\lambda\mu\pi + \varepsilon\gamma\mu^2\theta + \varepsilon\gamma\mu\omega\theta + \varepsilon\gamma\mu\pi\theta + \gamma\lambda\mu^2\sigma + \gamma\lambda\mu\omega\sigma + \gamma\lambda\mu\pi\sigma + \gamma\mu^2\sigma\theta + \gamma\mu\omega\sigma\theta + \gamma\mu\pi\sigma\theta)\end{gathered}}$$

$$M_{E}^{*} = \frac{\beta(\delta+\gamma)(\mu+\omega)(\delta+\varepsilon+\sigma)(\alpha B - N\beta\mu - N\mu^2 - N\mu\pi)}{\begin{gathered}\alpha(\beta\delta^3\mu + \beta\delta^3\omega + \beta\delta^2\varepsilon\mu + \beta\delta^2\varepsilon\omega + \beta\delta^2\gamma\mu + \beta\delta^2\gamma\omega + \beta\delta^2\lambda\mu + \beta\delta^2\mu\sigma + \beta\delta^2\mu\theta + \beta\delta^2\omega\sigma \\ +\beta\delta^2\omega\theta + \beta\delta\varepsilon\gamma\mu + \beta\delta\varepsilon\gamma\omega + \beta\delta\varepsilon\lambda\mu + \beta\delta\varepsilon\mu\theta + \beta\delta\varepsilon\omega\theta + \beta\delta\gamma\lambda\mu + \beta\delta\gamma\mu\sigma + \beta\delta\gamma\mu\theta + \\ \beta\delta\gamma\omega\sigma + \beta\delta\lambda\mu\sigma + \beta\delta\mu\sigma\theta + \beta\delta\omega\sigma\theta + \beta\varepsilon\gamma\lambda\mu + \beta\varepsilon\gamma\mu\theta + \beta\gamma\lambda\mu\sigma + \beta\gamma\mu\sigma\theta + \delta^3\mu 2 + \delta^3\mu\omega + \\ \delta^3\mu\pi + \delta^3\omega\pi + \delta^2\varepsilon\mu^2 + \delta^2\varepsilon\mu\omega + \delta^2\varepsilon\mu\pi + \delta^2\varepsilon\omega\pi + \delta^2\gamma\mu^2 + \delta^2\gamma\mu\omega + \delta^2\gamma\mu\pi + \delta^2\gamma\omega\pi + \\ \delta^2\lambda\mu^2 + \delta^2\lambda\mu\omega + \delta^2\lambda\mu\pi + \delta^2\lambda\omega\pi + \delta^2\mu^2\sigma + \delta^2\mu^2\theta + \delta^2\mu\omega\sigma + \delta^2\mu\omega\theta + \delta^2\mu\pi\sigma + \delta^2\mu\pi\theta \\ +\delta^2\omega\pi\theta + \delta\varepsilon\gamma\mu^2 + \delta\varepsilon\gamma\mu\omega + \delta\varepsilon\gamma\mu\pi + \delta\varepsilon\lambda\mu^2 + \delta\varepsilon\lambda\mu\omega + \delta\varepsilon\lambda\mu\pi + \delta\varepsilon\lambda\omega\pi + \delta\varepsilon\mu^2\theta + \delta\varepsilon\mu\omega\theta \\ +\delta\varepsilon\mu\pi\theta + \delta\varepsilon\omega\pi\theta + \delta\gamma\lambda\mu^2 + \delta\gamma\lambda\mu\omega + \delta\gamma\lambda\mu\pi + \delta\gamma\lambda\omega\pi + \delta\gamma\mu^2\sigma + \delta\gamma\mu^2\theta + \delta\gamma\mu\omega\sigma + \\ \delta\gamma\mu\omega\theta + \delta\gamma\mu\pi\sigma + \delta\gamma\mu\pi\theta + \delta\gamma\omega\pi\theta + \delta\lambda\mu^2\sigma + \delta\lambda\mu\omega\sigma + \delta\lambda\mu\pi\sigma + \delta\mu^2\sigma\theta + \delta\mu\omega\sigma\theta + \delta\mu\pi\sigma\theta \\ +\varepsilon\gamma\lambda\mu^2 + \varepsilon\gamma\lambda\mu\omega + \varepsilon\gamma\lambda\mu\pi + \varepsilon\gamma\mu^2\theta + \varepsilon\gamma\mu\omega\theta + \varepsilon\gamma\mu\pi\theta + \gamma\lambda\mu^2\sigma + \gamma\lambda\mu\omega\sigma + \gamma\lambda\mu\pi\sigma + \gamma\mu^2\sigma\theta \\ +\gamma\mu\omega\sigma\theta + \gamma\mu\pi\sigma\theta)\end{gathered}}$$

$$M_{UE}^{*} = \frac{\pi(\delta+\gamma)(\mu+\omega)(\delta+\lambda+\theta)(\alpha B - N\beta\mu - N\mu^2 - N\mu\pi)}{\begin{gathered}\alpha(\beta\delta^3\mu + \beta\delta^3\omega + \beta\delta^2\varepsilon\mu + \beta\delta^2\varepsilon\omega + \beta\delta^2\gamma\mu + \beta\delta^2\gamma\omega + \beta\delta^2\lambda\mu + \beta\delta^2\mu\sigma + \beta\delta^2\mu\theta + \\ \beta\delta^2\omega\sigma + \beta\delta^2\omega\theta + \beta\delta\varepsilon\gamma\mu + \beta\delta\varepsilon\gamma\omega + \beta\delta\varepsilon\lambda\mu + \beta\delta\varepsilon\mu\theta + \beta\delta\varepsilon\omega\theta + \beta\delta\gamma\lambda\mu + \beta\delta\gamma\mu\sigma + \\ \beta\delta\gamma\mu\theta + \beta\delta\gamma\omega\sigma + \beta\delta\lambda\mu\sigma + \beta\delta\mu\sigma\theta + \beta\delta\omega\sigma\theta + \beta\varepsilon\gamma\lambda\mu + \beta\varepsilon\gamma\mu\theta + \beta\gamma\lambda\mu\sigma + \beta\gamma\mu\sigma\theta + \\ \delta^3\mu^2 + \delta^3\mu\omega + \delta^3\mu\pi + \delta^3\omega\pi + \delta^2\varepsilon\mu^2 + \delta^2\varepsilon\mu\omega + \delta^2\varepsilon\mu\pi + \delta^2\varepsilon\omega\pi + \delta^2\gamma\mu^2 + \delta^2\gamma\mu\omega + \\ \delta^2\gamma\mu\pi + \delta^2\gamma\omega\pi + \delta^2\lambda\mu^2 + \delta^2\lambda\mu\omega + \delta^2\lambda\mu\pi + \delta^2\lambda\omega\pi + \delta^2\mu^2\sigma + \delta^2\mu^2\theta + \delta^2\mu\omega\sigma + \\ \delta^2\mu\omega\theta + \delta^2\mu\pi\sigma + \delta^2\mu\pi\theta + \delta^2\omega\pi\theta + \delta\varepsilon\gamma\mu^2 + \delta\varepsilon\gamma\mu\omega + \delta\varepsilon\gamma\mu\pi + \delta\varepsilon\lambda\mu^2 + \delta\varepsilon\lambda\mu\omega + \\ \delta\varepsilon\lambda\mu\pi + \delta\varepsilon\lambda\omega\pi + \delta\varepsilon\mu^2\theta + \delta\varepsilon\mu\omega\theta + \delta\varepsilon\mu\pi\theta + \delta\varepsilon\omega\pi\theta + \delta\gamma\lambda\mu^2 + \delta\gamma\lambda\mu\omega + \delta\gamma\lambda\mu\pi + \\ \delta\gamma\lambda\omega\pi + \delta\gamma\mu^2\sigma + \delta\gamma\mu^2\theta + \delta\gamma\mu\omega\sigma + \delta\gamma\mu\omega\theta + \delta\gamma\mu\pi\sigma + \delta\gamma\mu\pi\theta + \delta\gamma\omega\pi\theta + \delta\lambda\mu^2\sigma + \\ \delta\lambda\mu\omega\sigma + \delta\lambda\mu\pi\sigma + \delta\mu^2\sigma\theta + \delta\mu\omega\sigma\theta + \delta\mu\pi\sigma\theta + \varepsilon\gamma\lambda\mu^2 + \varepsilon\gamma\lambda\mu\omega + \varepsilon\gamma\lambda\mu\pi + \varepsilon\gamma\mu^2\theta + \\ \varepsilon\gamma\mu\omega\theta + \varepsilon\gamma\mu\pi\theta + \gamma\lambda\mu^2\sigma + \gamma\lambda\mu\omega\sigma + \gamma\lambda\mu\pi\sigma + \gamma\mu^2\sigma\theta + \gamma\mu\omega\sigma\theta + \gamma\mu\pi\sigma\theta)\end{gathered}}$$

$$H^{*} = \frac{(\mu+\omega)(\alpha B - N\beta\mu - N\mu^2 - N\mu\pi)(\beta\delta\theta + \beta\varepsilon\theta + \beta\sigma\theta + \delta\varepsilon\pi + \varepsilon\lambda\pi + \varepsilon\pi\theta)}{\begin{gathered}\alpha(\beta\delta^3\mu + \beta\delta^3\omega + \beta\delta^2\varepsilon\mu + \beta\delta^2\varepsilon\omega + \beta\delta^2\gamma\mu + \beta\delta^2\gamma\omega + \beta\delta^2\lambda\mu + \beta\delta^2\mu\sigma + \beta\delta^2\mu\theta + \beta\delta^2\omega\sigma \\ +\beta\delta^2\omega\theta + \beta\delta\varepsilon\gamma\mu + \beta\delta\varepsilon\gamma\omega + \beta\delta\varepsilon\lambda\mu + \beta\delta\varepsilon\mu\theta + \beta\delta\varepsilon\omega\theta + \beta\delta\gamma\lambda\mu + \beta\delta\gamma\mu\sigma + \beta\delta\gamma\mu\theta + \beta\delta\gamma\omega\sigma \\ +\beta\delta\lambda\mu\sigma + \beta\delta\mu\sigma\theta + \beta\delta\omega\sigma\theta + \beta\varepsilon\gamma\lambda\mu + \beta\varepsilon\gamma\mu\theta + \beta\gamma\lambda\mu\sigma + \beta\gamma\mu\sigma\theta + \delta^3\mu 2 + \delta^3\mu\omega + \delta^3\mu\pi \\ +\delta^3\omega\pi + \delta^2\varepsilon\mu^2 + \delta^2\varepsilon\mu\omega + \delta^2\varepsilon\mu\pi + \delta^2\varepsilon\omega\pi + \delta^2\gamma\mu^2 + \delta^2\gamma\mu\omega + \delta^2\gamma\mu\pi + \delta^2\gamma\omega\pi + \delta^2\lambda\mu^2 \\ +\delta^2\lambda\mu\omega + \delta^2\lambda\mu\pi + \delta^2\lambda\omega\pi + \delta^2\mu^2\sigma + \delta^2\mu^2\theta + \delta^2\mu\omega\sigma + \delta^2\mu\omega\theta + \delta^2\mu\pi\sigma + \delta^2\mu\pi\theta + \delta^2\omega\pi\theta \\ +\delta\varepsilon\gamma\mu^2 + \delta\varepsilon\gamma\mu\omega + \delta\varepsilon\gamma\mu\pi + \delta\varepsilon\lambda\mu^2 + \delta\varepsilon\lambda\mu\omega + \delta\varepsilon\lambda\mu\pi + \delta\varepsilon\lambda\omega\pi + \delta\varepsilon\mu^2\theta + \delta\varepsilon\mu\omega\theta + \delta\varepsilon\mu\pi\theta \\ +\delta\varepsilon\omega\pi\theta + \delta\gamma\lambda\mu^2 + \delta\gamma\lambda\mu\omega + \delta\gamma\lambda\mu\pi + \delta\gamma\lambda\omega\pi + \delta\gamma\mu^2\sigma + \delta\gamma\mu^2\theta + \delta\gamma\mu\omega\sigma + \delta\gamma\mu\omega\theta + \delta\gamma\mu\pi\sigma \\ +\delta\gamma\mu\pi\theta + \delta\gamma\omega\pi\theta + \delta\lambda\mu^2\sigma + \delta\lambda\mu\omega\sigma + \delta\lambda\mu\pi\sigma + \delta\mu^2\sigma\theta + \delta\mu\omega\sigma\theta + \delta\mu\pi\sigma\theta + \varepsilon\gamma\lambda\mu^2 + \varepsilon\gamma\lambda\mu\omega \\ +\varepsilon\gamma\lambda\mu\pi + \varepsilon\gamma\mu^2\theta + \varepsilon\gamma\mu\omega\theta + \varepsilon\gamma\mu\pi\theta + \gamma\lambda\mu^2\sigma + \gamma\lambda\mu\omega\sigma + \gamma\lambda\mu\pi\sigma + \gamma\mu^2\sigma\theta + \gamma\mu\omega\sigma\theta + \gamma\mu\pi\sigma\theta)\end{gathered}}$$

$$R^{*} = \frac{\begin{gathered}(\alpha B - N\beta\mu - N\mu^2 - N\mu\pi)(\beta\delta^2\lambda + \beta\delta\varepsilon\lambda + \beta\delta\gamma\lambda + \beta\delta\gamma\theta + \beta\delta\lambda\sigma + \beta\varepsilon\gamma\lambda + \beta\varepsilon\gamma\theta + \beta\gamma\lambda\sigma + \\ \beta\gamma\sigma\theta + \delta^2\pi\sigma + \delta\varepsilon\gamma\pi + \delta\gamma\pi\sigma + \delta\lambda\pi\sigma + \delta\pi\sigma\theta + \varepsilon\gamma\lambda\pi + \varepsilon\gamma\pi\theta + \gamma\lambda\pi\sigma + \gamma\pi\sigma\theta)\end{gathered}}{\begin{gathered}\alpha(\beta\delta^3\mu + \beta\delta^3\omega + \beta\delta^2\varepsilon\mu + \beta\delta^2\varepsilon\omega + \beta\delta^2\gamma\mu + \beta\delta^2\gamma\omega + \beta\delta^2\lambda\mu + \beta\delta^2\mu\sigma + \beta\delta^2\mu\theta + \beta\delta^2\omega\sigma \\ +\beta\delta^2\omega\theta + \beta\delta\varepsilon\gamma\mu + \beta\delta\varepsilon\gamma\omega + \beta\delta\varepsilon\lambda\mu + \beta\delta\varepsilon\mu\theta + \beta\delta\varepsilon\omega\theta + \beta\delta\gamma\lambda\mu + \beta\delta\gamma\mu\sigma + \beta\delta\gamma\mu\theta + \beta\delta\gamma\omega\sigma \\ +\beta\delta\lambda\mu\sigma + \beta\delta\mu\sigma\theta + \beta\delta\omega\sigma\theta + \beta\varepsilon\gamma\lambda\mu + \beta\varepsilon\gamma\mu\theta + \beta\gamma\lambda\mu\sigma + \beta\gamma\mu\sigma\theta + \delta^3\mu 2 + \delta^3\mu\omega + \delta^3\mu\pi \\ +\delta^3\omega\pi + \delta^2\varepsilon\mu^2 + \delta^2\varepsilon\mu\omega + \delta^2\varepsilon\mu\pi + \delta^2\varepsilon\omega\pi + \delta^2\gamma\mu^2 + \delta^2\gamma\mu\omega + \delta^2\gamma\mu\pi + \delta^2\gamma\omega\pi + \delta^2\lambda\mu^2 \\ +\delta^2\lambda\mu\omega + \delta^2\lambda\mu\pi + \delta^2\lambda\omega\pi + \delta^2\mu^2\sigma + \delta^2\mu^2\theta + \delta^2\mu\omega\sigma + \delta^2\mu\omega\theta + \delta^2\mu\pi\sigma + \delta^2\mu\pi\theta + \delta^2\omega\pi\theta \\ +\delta\varepsilon\gamma\mu^2 + \delta\varepsilon\gamma\mu\omega + \delta\varepsilon\gamma\mu\pi + \delta\varepsilon\lambda\mu^2 + \delta\varepsilon\lambda\mu\omega + \delta\varepsilon\lambda\mu\pi + \delta\varepsilon\lambda\omega\pi + \delta\varepsilon\mu^2\theta + \delta\varepsilon\mu\omega\theta + \delta\varepsilon\mu\pi\theta \\ +\delta\varepsilon\omega\pi\theta + \delta\gamma\lambda\mu^2 + \delta\gamma\lambda\mu\omega + \delta\gamma\lambda\mu\pi + \delta\gamma\lambda\omega\pi + \delta\gamma\mu^2\sigma + \delta\gamma\mu^2\theta + \delta\gamma\mu\omega\sigma + \delta\gamma\mu\omega\theta + \delta\gamma\mu\pi\sigma \\ +\delta\gamma\mu\pi\theta + \delta\gamma\omega\pi\theta + \delta\lambda\mu^2\sigma + \delta\lambda\mu\omega\sigma + \delta\lambda\mu\pi\sigma + \delta\mu^2\sigma\theta + \delta\mu\omega\sigma\theta + \delta\mu\pi\sigma\theta + \varepsilon\gamma\lambda\mu^2 + \varepsilon\gamma\lambda\mu\omega \\ +\varepsilon\gamma\lambda\mu\pi + \varepsilon\gamma\mu^2\theta + \varepsilon\gamma\mu\omega\theta + \varepsilon\gamma\mu\pi\theta + \gamma\lambda\mu^2\sigma + \gamma\lambda\mu\omega\sigma + \gamma\lambda\mu\pi\sigma + \gamma\mu^2\sigma\theta + \gamma\mu\omega\sigma\theta + \gamma\mu\pi\sigma\theta)\end{gathered}}$$

To determine the drugs generation number R_0 using the next generation matrix method, we first identify the infection classes in our model. The matrices F and V are constructed based on the rates of transition between different drugs states. Let,

$$F = \begin{bmatrix} \frac{\alpha B}{N\mu} & 0 & 0 \\ \beta & 0 & 0 \\ \pi & 0 & 0 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} \beta + \pi + \mu & 0 & 0 \\ 0 & \lambda + \theta + \delta & 0 \\ 0 & 0 & \varepsilon + \sigma + \delta \end{bmatrix}$$

Thus, the next generation matrix is

$$FV^{-1} = \begin{bmatrix} \frac{\alpha B}{N\mu(\beta + \pi + \mu)} & 0 & 0 \\ \frac{\beta}{\beta + \pi + \mu} & 0 & 0 \\ \frac{\pi}{\beta + \pi + \mu} & 0 & 0 \end{bmatrix}$$

The drug generation number R_0 is found by calculating the spectral radius $\rho(FV^{-1})$, which simplifies to:

$$R_0 = \frac{\alpha B}{N\mu(\beta + \pi + \mu)}$$

This R_0 value represents the average number of new drug addicts that a single addict would generate in a fully susceptible population. It is a key metric for understanding the potential spread and persistence of drug addiction within the community, influence of various rates of transition and treatment.

Local Stability Analysis

Local Stability at E^0

Evaluating the Jacobian matrix of system (1) at E^0 gives

$$J(E^0) = \begin{bmatrix} -\mu & -\frac{\alpha B}{N\mu} & 0 & 0 & 0 & \omega \\ 0 & \frac{\alpha B}{N\mu} - \beta - \mu - \pi & 0 & 0 & 0 & 0 \\ 0 & \beta & -\delta - \lambda - \theta & 0 & 0 & 0 \\ 0 & \pi & 0 & -\delta - \varepsilon - \sigma & 0 & 0 \\ 0 & 0 & \theta & \varepsilon & -\delta - \gamma & 0 \\ 0 & 0 & \lambda & \sigma & \gamma & -\mu - \omega \end{bmatrix}$$

The eigen values are given by

$$\lambda_1 = -\mu, \lambda_2 = \frac{B\alpha}{N\mu} - \beta - \mu - \pi, \lambda_3 = -\delta - \lambda - \theta, \lambda_4 = -\delta - \varepsilon - \sigma, \lambda_5 = -\delta - \gamma, \lambda_6 = -\mu - \omega$$

Hence E^0 is locally asymptotically stable if $R_0 < 1$. For $R_0 = 1$, if $\lambda_i < 0$ for $i = 1,3,4,5,6$ and $\lambda_2 = 0$, E^0 is locally stable. If $R_0 > 1$, the characteristic equation has a real positive eigenvalue, and therefore E^0 is unstable.

Local Stability at E^*

Evaluating the Jacobian matrix of system (1) at E^*gives

$$J(P^*) = \begin{bmatrix} -\frac{\alpha E^*}{N} - \mu & -\frac{\alpha S^*}{N} & 0 & 0 & 0 & \omega \\ \frac{\alpha E^*}{N} & -\beta - \mu - \pi + \frac{\alpha S^*}{N} & 0 & 0 & 0 & 0 \\ 0 & \beta & -\delta - \lambda - \theta & 0 & 0 & 0 \\ 0 & \pi & 0 & -\delta - \varepsilon - \sigma & 0 & 0 \\ 0 & 0 & \theta & \varepsilon & -\delta - \gamma & 0 \\ 0 & 0 & \lambda & \sigma & \gamma & -\mu - \omega \end{bmatrix}$$

$$J(E^*) = \begin{bmatrix} b_{11} & b_{12} & 0 & 0 & 0 & b_{16} \\ b_{21} & b_{22} & 0 & 0 & 0 & 0 \\ 0 & b_{32} & b_{33} & 0 & 0 & 0 \\ 0 & b_{42} & 0 & b_{44} & 0 & 0 \\ 0 & 0 & b_{53} & b_{54} & b_{55} & 0 \\ 0 & 0 & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix}$$

where,
$b_{11} = -\frac{\alpha E^*}{N} - \mu, b_{12} = -\frac{\alpha S^*}{N}, b_{16} = \omega, b_{21} = \frac{\alpha E^*}{N}, b_{22} = -\beta - \mu - \pi + \frac{\alpha S^*}{N}, b_{32} = \beta,$
$b_{33} = -\delta - \lambda - \theta, b_{42} = \pi, b_{44} = -\delta - \varepsilon - \sigma, b_{53} = \theta, b_{54} = \varepsilon, b_{55} = -\delta - \gamma, b_{63} = \lambda,$
$b_{64} = \sigma, b_{65} = \gamma, b_{66} = -\mu - \omega$
The characteristic Polynomial of E^* is given by

$$\lambda^6 + B_1\lambda^5 + B_2\lambda^4 + B_3\lambda^3 + B_4\lambda^2 + B_5\lambda + B_6 = 0$$

where,
$B_1 = -(b_{11} + b_{22} + b_{33} + b_{44} + b_{55} + b_{66})$
$B_2 = b_{11}b_{22} + b_{11}b_{33} + b_{11}b_{44} + b_{11}b_{55} + b_{11}b_{66} - b_{12}b_{21} + b_{22}b_{33} + b_{22}b_{44} + b_{22}b_{55} + b_{22}b_{66} + b_{33}b_{44} + b_{33}b_{55} + b_{33}b_{66} + b_{44}b_{55} + b_{44}b_{66} + b_{55}b_{66}$
$B_3 = -b_{11}b_{22}b_{33} - b_{11}b_{22}b_{44} - b_{11}b_{22}b_{55} - b_{11}b_{22}b_{66} - b_{11}b_{33}b_{44} - b_{11}b_{33}b_{55} - b_{11}b_{33}b_{66} - b_{11}b_{44}b_{55} - b_{11}b_{44}b_{66} - b_{11} b_{55}b_{66} + b_{12}b_{21}b_{33} + b_{12}b_{21}b_{44} + b_{12}b_{21}b_{55} + b_{12}b_{21}b_{66} - b_{22}b_{33}b_{44} - b_{22}b_{33}b_{55} - -b_{22}b_{33}b_{66} - b_{22}b_{44}b_{55} - b_{22}b_{44}b_{66} - b_{22}b_{55}b_{66} - b_{33}b_{44}b_{55} - b_{33}b_{44}b_{66} - b_{33}b_{55}b_{66} - b_{44}b_{55}b_{66}$
$B_4 = b_{11}b_{22}b_{33}b_{44} + b_{11}b_{22}b_{33}b_{55} + b_{11}b_{22}b_{33}b_{66} + b_{11}b_{22}b_{44}b_{55} + b_{11}b_{22}b_{44}b_{66} + b_{11}b_{22}b_{55}b_{66} + b_{11}b_{33}b_{44}b_{55} + b_{11}b_{33}b_{44}b_{66} + b_{11}b_{33}b_{55}b_{66} + b_{11}b_{44}b_{55}b_{66} - b_{12}b_{21}b_{33}b_{44} - b_{12}b_{21}b_{33} b_{55} - b_{12}b_{21}b_{33}b_{66} - b_{12}b_{21}b_{44}b_{55} - b_{12}b_{21}b_{44}b_{66} - b_{12}b_{21}b_{55}b_{66} - b_{16}b_{21}b_{32}b_{63} - b_{16}b_{21}b_{42}b_{64} + b_{22}b_{33}b_{44}b_{55} + b_{22}b_{33}b_{44}b_{66} + b_{22}b_{33}b_{55}b_{66} + b_{22}b_{44}b_{55}b_{66} + b_{33}b_{44}b_{55}b_{66}$
$B_5 = -b_{11}b_{22}b_{33}b_{44}b_{55} - b_{11}b_{22}b_{33}b_{44}b_{66} - b_{11}b_{22}b_{33}b_{55}b_{66} - b_{11}b_{22}b_{44}b_{55}b_{66} - b_{11}b_{33}b_{44}b_{55}b_{66} + b_{12}b_{21}b_{33}b_{44}b_{55} + b_{12}b_{21}b_{33} b_{44}b_{66} + b_{12}b_{21}b_{33}b_{55}b_{66} + b_{12}b_{21}b_{44}b_{55}b_{66} - b_{16}b_{21}b_{32}b_{44}b_{63} - b_{16}b_{21}b_{32}b_{53}b_{65} + b_{16}b_{21}b_{32}b_{55}b_{63} + b_{16}b_{21}b_{33}b_{42}b_{64} - b_{16}b_{21}b_{42}b_{54}b_{65} + b_{16}b_{21}b_{42}b_{55}b_{64} - b_{22}b_{33}b_{44}b_{55}b_{66}$
$B_6 = b_{11}b_{22}b_{33}b_{44}b_{55}b_{66} - b_{12}b_{21}b_{33}b_{44}b_{55}b_{66} + b_{16}b_{21}b_{32}b_{44}b_{53}b_{65} - b_{16}b_{21}b_{32}b_{44}b_{55}b_{63} + b_{16}b_{21}b_{33}b_{42}b_{54}b_{65} - b_{16}b_{21}b_{33}b_{42}b_{55}b_{64}$

To determine the local stability of the equilibrium point E^* using the Routh-Hurwitz criteria, we evaluate the polynomial equation associated with the system dynamics. The roots of this polynomial will have negative real parts if the following conditions are met:

$$B_1 > 0, \quad \begin{vmatrix} B_1 & 1 \\ B_3 & B_2 \end{vmatrix} > 0, \quad \begin{vmatrix} B_1 & 1 & 0 \\ B_3 & B_2 & B_1 \\ B_5 & B_4 & B_3 \end{vmatrix} > 0, \quad \begin{vmatrix} B_1 & 1 & 0 & 0 \\ B_3 & B_2 & B_1 & 1 \\ B_5 & B_4 & B_3 & B_2 \\ 0 & B_6 & B_5 & B_4 \end{vmatrix} > 0,$$

$$\begin{vmatrix} B_1 & 1 & 0 & 0 & 0 \\ B_3 & B_2 & B_1 & 1 & 0 \\ B_5 & B_4 & B_3 & B_2 & B_1 \\ 0 & B_6 & B_5 & B_4 & B_3 \\ 0 & 0 & 0 & B_6 & B_5 \end{vmatrix} > 0, \quad \begin{vmatrix} B_1 & 1 & 0 & 0 & 0 & 0 \\ B_3 & B_2 & B_1 & 1 & 0 & 0 \\ B_5 & B_4 & B_3 & B_2 & B_1 & 1 \\ 0 & B_6 & B_5 & B_4 & B_3 & B_2 \\ 0 & 0 & 0 & B_6 & B_5 & B_4 \\ 0 & 0 & 0 & 0 & 0 & B_6 \end{vmatrix} > 0$$

Hence, the equilibrium point E^* is locally asymptotically stable if all the above conditions are satisfied.

Global Stability Analysis

Global Stability at E^0

Consider Lyapunov function as given below
$Y = S + E + M_E + M_{UE} + H + R$

$$\frac{dY}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dM_E}{dt} + \frac{dM_{UE}}{dt} + \frac{dH}{dt} + \frac{dR}{dt}$$
$$= B - \mu S - \mu E - \delta M_E - \delta M_{UE} - \delta H - \mu R$$
$$= B - \mu\left(\frac{B}{\mu}\right) - \mu(E + M_E + M_{UE} + H + R)$$
$$= -\mu(E + M_E + M_{UE} + R + H)$$

We have $\frac{dY}{dt} \leq 0$ with $\frac{dY}{dt} = 0$ only if $E = M_E = M_{UE} = H = R = 0$. This condition indicates that the Lyapunov function Y is non-increasing over time and only remains constant when the system reaches the drug-free equilibrium.

By LaSalle's Invariance Principle, all system trajectories will ultimately converge to the largest invariant set where $\frac{dY}{dt} = 0$, which in this case is the equilibrium point $E^0\left(\frac{B}{\mu}, 0,0,0,0,0\right)$. Hence, every solution of the system converges to E^0 as time progresses, confirming that the drug-free equilibrium is globally asymptotically stable.

Global Stability at E^*

Consider Lyapunov function as given below

$$Y(t) = \frac{1}{2}[(S - S^*) + (E - E^*) + (M_E - M_E{}^*) + (M_{UE} - M_{UE}{}^*) + (H - H^*) + (R - R^*)]^2$$

$$Y'(t) = [(S - S^*) + (E - E^*) + (M_E - M_E{}^*) + (M_{UE} - M_{UE}{}^*) + (H - H^*) + (R - R^*)]$$
$$(S' + E' + M_E{}' + M_{UE}{}' + H' + R')$$

$$Y'(t) = [(S - S^*) + (E - E^*) + (M_E - M_E{}^*) + (M_{UE} - M_{UE}{}^*) + (H - H^*) + (R - R^*)]$$
$$(B - \mu S - \mu E - \delta M_E - \delta M_{UE} - \delta H - \mu R)$$

$$Y'(t) = [(S - S^*) + (E - E^*) + (M_E - M_E{}^*) + (M_{UE} - M_{UE}{}^*) + (H - H^*) + (R - R^*)]$$
$$(\mu S^* + \mu E^* + \mu M_E{}^* + \mu M_{UE}{}^* + \mu H^* + \mu R^* - \mu S - \mu E - \mu M_E - \mu M_{UE} - \mu H - \mu R)$$
$$= -\mu[(S - S^*) + (E - E^*) + (A - A^*) + (W - W^*) + (I - I^*) + (R - R^*)]^2$$
$$\leq 0$$

where $B = \mu S^* + \mu E^* + \mu M_E{}^* + \mu M_{UE}{}^* + \mu H^* + \mu R^*$

Therefore, based on the Lyapunov function $Y(t)$ and its derivative $Y'(t)$, which satisfies $Y'(t) \leq 0$ indicating that $Y(t)$ is non-increasing, we conclude that the unique positive equilibrium point E^* is globally asymptotically stable.

VI. Data Collection

To study the impact of education on drug addiction, a structured questionnaire was developed to collect data from various treatment centers and rehabilitation facilities. The questionnaire focused on key aspects such as the educational background of patients, awareness programs conducted at the centers, relapse rates and the effectiveness of educational interventions. Additional data points included total admissions for drug rehabilitation, number of drug users not classified as addicted, lightly addicted patients categorized by education level (above and below Grade 10), transitions from light to heavy addiction among both educated and uneducated individuals, current treatment status of lightly and heavily addicted patients, total recoveries and the number of addiction-related deaths.

The collected data was then used to estimate the values of different parameters and transition rates in our mathematical model. This helped us understand how drug addiction spreads and how effective various intervention strategies might be.

The value of different parameters were extracted from the collected data and initial conditions were determined for numerical simulations. The initial values represent different groups within the population and are as follows: $S(t) = 600$, $E(t) = 531$, $M_E(t) = 335$, $M_{UE}(t) = 185$, $H(t) = 222$, $R(t) = 381$. Here, total population$\left(N(t)\right)$ is 2254.

Table 2: Parameters and its value

Parameters	Value	Descriptions
B	0.016	Recruitment rate
α	0.88	The contact rate of transmission from susceptible individuals to the exposed class
β	0.63	The rate of the exposed and educated individuals who becomes lightly drug addicted
π	0.35	The rate of the exposed and uneducated individuals who becomes lightly drug addicted
θ	0.46	The rate at which lightly drug addicted and educated individuals becomes highly drug addicted
ε	0.36	The rate at which lightly drug addicted and uneducated individuals becomes highly drug addicted
λ	0.50	The rate at which educated individuals recover from lightly drug addicted
γ	0.41	The rate at which individuals recover from highly drug addicted
σ	0.66	The rate at which uneducated individuals recover from lightly drug addicts
ω	0.62	The rate at which recovered individuals relapse and become susceptible (S) to drug addiction again
δ	0.03	The death rate due to drug addiction
μ	0.009	Natural death rate.

VII. Sensitivity Analysis

Sensitivity indices of R_0 to all the different parameters tell us that how crucial each parameter is to the addiction spread. This helps us choose the right parameters responsible for making the scenario endemic.

Local Sensitivity Analysis

To assess the influence of individual parameters on the basic reproduction number R_0, a local sensitivity analysis was performed. Local sensitivity indices were derived using the normalized forward sensitivity index:

$$(R_0)_p = \frac{\partial R_0}{\partial p} \cdot \frac{p}{R_0}$$

where p denotes a parameter. This index measures the relative change in R_0 with respect to a relative change in parameter p.

We calculate the sensitivity indices for those parameters on which the value of basic reproduction depends. The results are given in Table 3.

Table 3: Local Sensitivity indices of R_0 to the parameters

Parameters	Sign	Value
N	-	1
B	+	1
α	+	1
β	-	0.64
π	-	0.35
μ	-	1.009

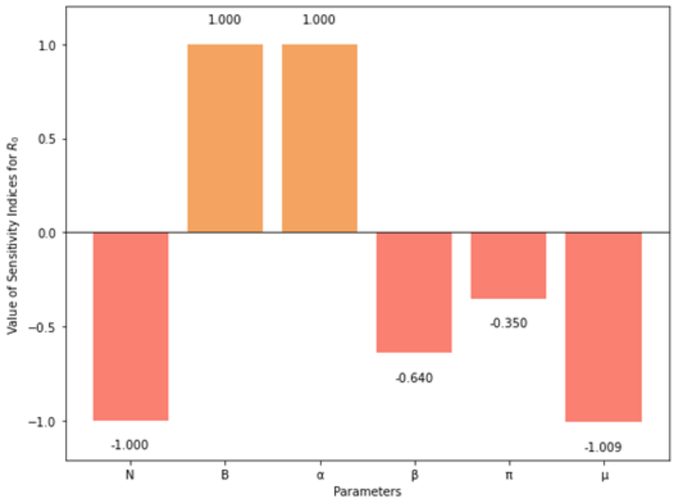

Figure 3: Local Sensitivity analysis for reproduction number R_0

Global Sensitivity Analysis

Global sensitivity analysis approach was adopted, as it accounts for the simultaneous variation of all parameters across their probable ranges, thereby providing a comprehensive understanding of parameter importance.

Since local sensitivity only reflects behaviour near baseline values, we also performed a global sensitivity analysis using Partial Rank Correlation Coefficients (PRCCs). This approach considers the simultaneous variation of all parameters across their probable ranges, capturing nonlinearities and interactions.

For global sensitivity analysis, each parameter was varied within ± 10 % of its nominal (baseline) value. A total of 1,000 parameter sets were generated using Latin Hypercube Sampling (LHS) within biologically probable range. For each set, R_0 was calculated. PRCC values were then estimated between each parameter and R_0, with significance tested using corresponding P - values. PRCC values close to $+1$ (or -1) indicate a strong positive (or negative) monotonic relationship.

Table 4: Global Sensitivity indices of R_0 to the parameters

Parameters	Sign	Value
N	+	0.0091
B	+	0.9610
α	+	0.9340
β	-	0.8522

π	-	0.6523
μ	-	0.9478

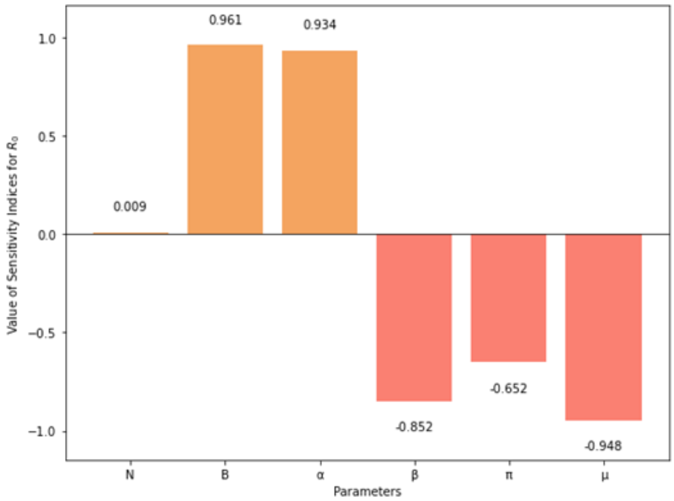

Figure 3: Local Sensitivity analysis for reproduction number R_0

Our analysis shows that the effect of parameters on R_0 depends on whether we look at local or global sensitivity. In the local case, when we study small changes around the baseline values, the progression rate (α) and natural death rate (μ) have the strongest impact on R_0. However, when we consider global sensitivity using Latin Hypercube Sampling with ± 10 % variation in all parameters, the birth rate (B) and natural death rate (μ) emerge as the most influential. This means that while α directly affects R_0 at the baseline, in a wider range of scenarios the demographic factors $(B \text{ and } \mu)$ play a more dominant role in shaping the overall behaviour of the system.

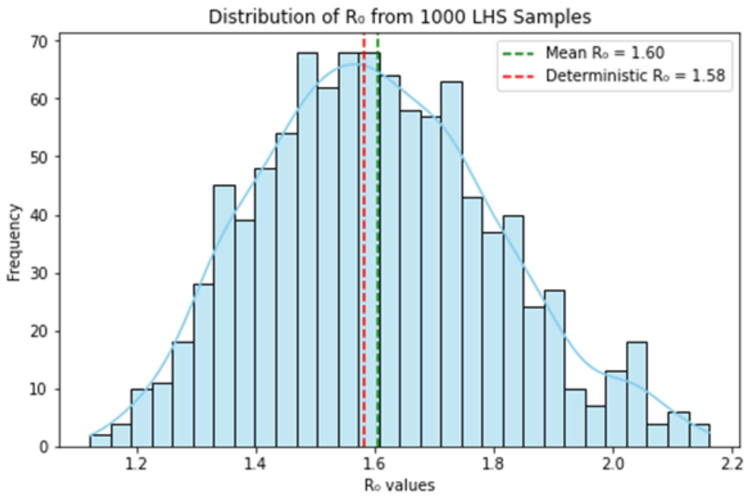

Figure 4: Distribution of R_0 from 100 LHS Samples

Figure 5 shows the spread of R_0 values from 1000 samples with ± 10 % change in parameters. The values form a single peak, close to a bell shape, with an average of 1.58. The fixed (deterministic) value of 1.60 is almost the same as the average, which means the chosen parameters represent the system well. Since most R_0 values are above 1, the results suggest that the addiction is likely to continue spreading even when parameters change within this range.

VIII. Result And Discussion

To predict future trends of the addiction, we conducted simulations using the extracted parameters and initial values of table 2.

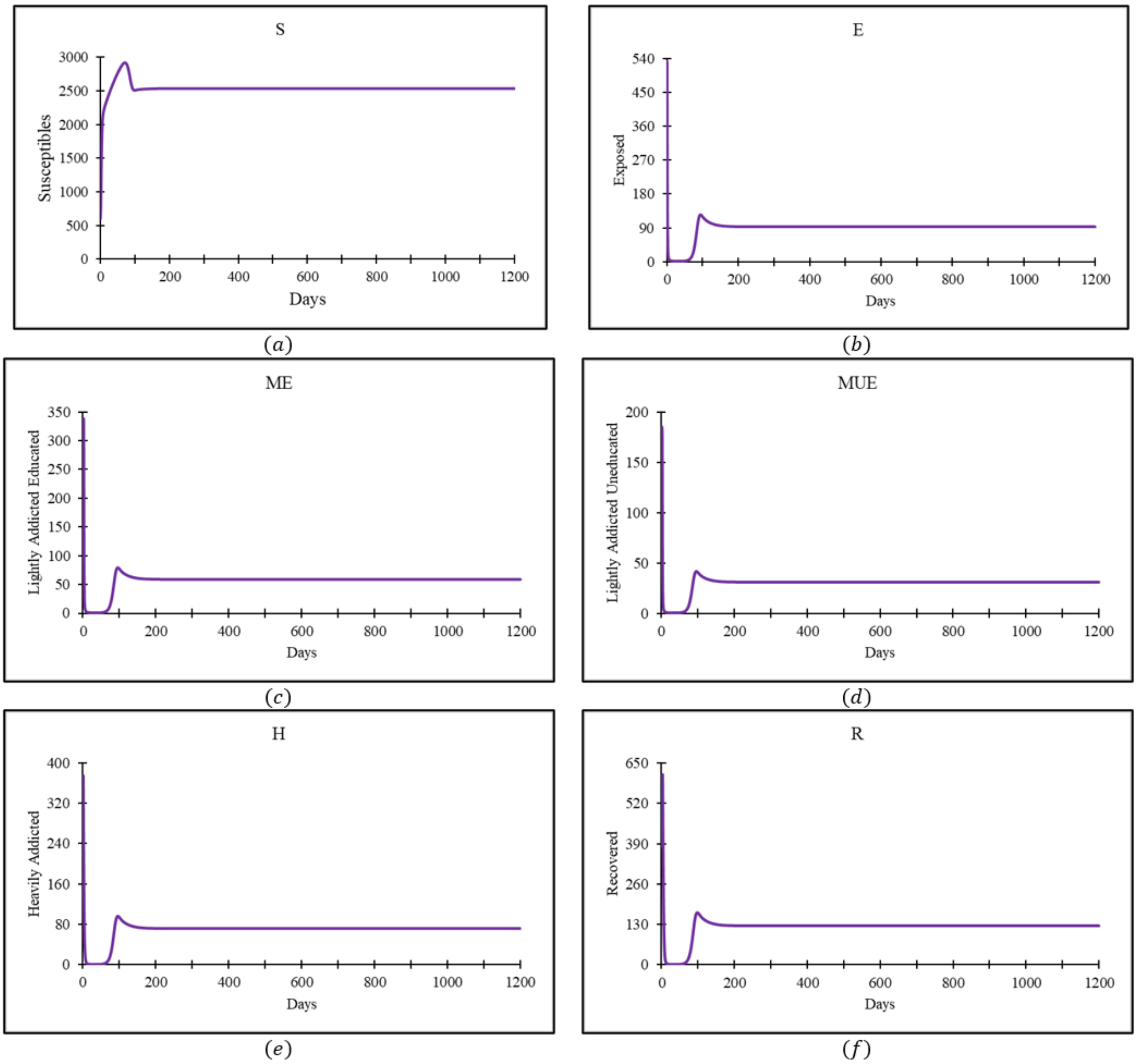

Figure 6: Graph of different compartments (population) vs. days based on the initial and parameter values: (a) Susceptible population vs. Days, (b) Engaged population vs. Days, (c) Lightly Addicted Educated population vs. Days, (d) Lightly addicted uneducated population vs. Days, (e) Heavily Addicted population vs. Days, (f) Recovered population vs. Days. Parameters: [Table 2]

Figure 6(a) shows that the susceptible population initially increases, reaching a peak quickly due to a lack of awareness or control measures. However, after a brief fluctuation, the population stabilizes and remains steady throughout the simulation period. This indicates that once interventions like education or awareness programs take effect, the number of individuals at risk of drug addiction stays constant without further growth.

Figure 6(b) shows the exposed population experiences a sharp rise early on as individuals transition from susceptibility to exposure due to influence or availability of drugs. This is followed by a slight decline and stabilization, indicating that once initial exposure spreads, the rate of new exposures decreases - likely due to the impact of educational awareness or social resistance forming.

Figure 6(c) for lightly addicted educated individuals shows a rapid increase followed by a gradual decline and long-term stabilization. This trend suggests that while a portion of the exposed population becomes lightly addicted, education helps control the addiction severity and progression, resulting in fewer long - term addicted cases among the educated group.

Figure 6(d) Similar to the educated group, the uneducated lightly addicted population initially spikes but at a lower peak, then stabilizes over time. The lower steady state implies that uneducated individuals are more vulnerable but may also face higher transition into heavier addiction or lower rates of seeking help, keeping their numbers comparatively suppressed or redirected to other compartments.

Figure 6(e) The heavily addicted population rises quickly and then settles at a steady level. This shows that a segment of the lightly addicted individuals (especially the uneducated ones) likely progresses to heavy addiction early. However, stabilization afterward reflects the influence of recovery programs or natural limits in the population's progression to heavy addiction.

Figure 6(f) This compartment sees a significant early increase as education and intervention efforts enable addicted individuals to recover. After reaching a peak, the number stabilizes, indicating that a sustainable recovery rate is achieved. The early success of recovery efforts shows that education not only prevents addiction but also actively contributes to rehabilitation.

IX. Conclusion

Drug addiction is a serious social and public health issue that can affect individuals, families and communities. Mathematical modelling helps to understand how addiction spreads and how different actions can reduce its impact. In this study, a mathematical model with real-life data is used to show how people move through different stages - starting from being at risk, getting exposed, becoming lightly or heavily addicted and then becoming aware, getting help and finally recovering.

A critical outcome of the model is the basic reproduction number, $R_0 = 1.58$, which exceeds the threshold of 1. This means that one addicted person can lead to more than one new case, which highlights the importance of early action and education in preventing the spread of addiction. The graphs show that after some initial changes, the numbers of people in each group become steady, meaning the situation becomes stable over time due to education and awareness.

These findings show that education and awareness programs are important and effective in reducing drug addiction. When addiction rates go down, it creates a positive cycle that helps improve and expand prevention efforts. Adding education to public health plans is a low-cost way to stop drug addiction from increasing and to create a healthier, stronger community. In the future, the model can be improved by adding more real-life data, looking at how different ages and backgrounds are affected and including factors like peer pressure and relapse. These changes will help test how well different programs and policies might work before putting them into action.

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