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Unique Common Fixed-Point Theorem Of Compatible Mappings Of Type (K) Satisfying Integral Type Inequality On Intuitionistic Fuzzy Metric Space

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Abstract
In this paper we prove a common fixed point theorem for compatible mappings of type (K) that satisfy an integral-type inequality within the framework of intuitionistic fuzzy metric spaces. Additionally, a common fixed point result is derived for self-mappings in such spaces under the same integral-type inequality conditions. In this paper we will generalize the result of Tenguria A., Rajput A. and Mandwariya V. [17].
***Keywords**: Intuitionistic Fuzzy Metric Space, Fixed Point, Compatible Maps of Type (K).*

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I. Introduction

Zadeh [21] introduced the concept of fuzzy set A in X is a function with domain X and value in [0, 1]. Deng [3], Erceg [4], Fang [5], George and Veeramani [7], Kaleva and Seikkala [10], Kramosil and Michalek [11] have introduced the concept of fuzzy metric spaces in different ways. Atanassove [1] introduced and studied the concept of fuzzy metric spaces with the help of continuous t- norms and continuous t-conorms as generalization of fuzzy metric space due to George and Veeramani [7]. Several authors [12, 13, 14, 16] proved some fixed point theorem in intuitionistic fuzzy metric space. Further Coker [2] introduced the intuitionistic fuzzy topological spaces. Turkoglu et al. [18] proved Jungck's common fixed point theorem [9] in the setting of intuitionistic fuzzy metric spaces for commuting mappings. In this paper we prove a common fixed point theorem for compatible mappings of type (k) satisfying integral type inequality.

II. Preliminaries

Definition 2.1. [17]. A binary operation * : [0, 1]× [0, 1] → [0, 1] is continuous t – norm if * is satisfies the following conditions:
(i)* is commutative and associative;
(ii)* is continuous
(iii)$a * 1 = a$, for all $a \in [0,1]$;
(iv)$a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2.2. [17]. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t – conorm if $\diamond$ is satisfies the following conditions:
(i)$\diamond$ is commutative and associative;
(ii) $\diamond$ is continuous;
(iii) $a \diamond 0 = a$ for all $a \in [0,1]$;
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2.3. [17]. A, 5 – tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is continuous t – norm, $\diamond$ is continuous t – conorm and M, N are arbitrary fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$
(i) M (x, y, t) + N(x, y, t) ≤ 1
(ii) M (x, y, 0) = 0
(iii) M (x, y, t) = 1 if and only if x = y;
(iv) M (x, y, t) = M (y, x, t) ≠ 0 for t ≠ 0;
(v) M (x, y, t)*M (y, z, s) ≤ M (x, z, t + s);
(vi) M (x, y, ·): [0,∞) → [0, 1] is continuous.

(vii) $\lim_{t\to\infty} M(x, y, t) = 1$
(viii) N(x, y, 0) = 1
(ix) N(x, y, t) = 0 if and only if x = y;
(x) N(x, y, t) = N(y, x, t) ≠ 0 for t ≠ 0;
(xi) N(x, y, t) ⋄ N(y, z, s) ≥ N(x, z, t + s);
(xii) M(x, y, ·): [0,∞) → [0, 1] is continuous.
(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Definition 2.4. [17]. Let (X, M, N, *, ⋄) be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ is said to be
Convergent to a point $x \in X$ if
$\lim_{x\to\infty} M(x_n, x, t) = 1$ and $\lim_{x\to\infty} N(x_n, x, t) = 0$ for all t > 0.

Cauchy sequence if
$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$ for all t > 0, p > 0.

Definition 2.5. [17]. A sequence $\{x_n\}$ in an intuitionistic fuzzy metric space (X, M, N, *, ⋄) is called complete if and only if every Cauchy sequence in X is convergent.

Definition 2.6. [17]. Let S and T be self mappings of an intuitionistic fuzzy metric space $(X, M, N, * \diamond)$. Then a pair (S, T) is said to be compatible if
$\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1$ and $\lim_{n\to\infty} N(STx_n, TSx_n, t) = 0$
For all t > 0, whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$, for some $u \in X$.

Definition 2.7. [17]. Let S and T be self mapping of an intuitionistic fuzzy metric space (X, M, N, *, ⋄). Then a pair (S, T) is said to be compatible of type (A) if
$\lim_{n\to\infty} M(STx_n, TTx_n, t) = 1$ and $\lim_{n\to\infty} M(TSx_n, SSx_n, t) = 1$
And
$\lim_{n\to\infty} N(STx_n, TTx_n, t) = 0$ and $\lim_{n\to\infty} N(STx_n, TTx_n, t) = 0$
For all t > 0, whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$, for some $u \in X$.

Definition 2.8. [17]. Let S and T be self mapping of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. Then a pair (S, T) is said to be compatible of type (P) if
$\lim_{n\to\infty} M(SSx_n, TTx_n, t) = 1$ and $\lim_{n\to\infty} N(SSx_n, TTx_n, t) = 0$

Definition 2.9. [17]. Let S and T be self mapping of an intuitionistic fuzzy metric space (X, M, N, *, ⋄) are called reciprocally continuous on X if $\lim_{n\to\infty} STx_n = Sx$ and $\lim_{n\to\infty} TSx_n = Tx$, whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$, for some $u \in X$.

Definition 2.10. [17]. Let S and T be self mapping of an intuitionistic fuzzy metric space(X, M, N, *, ⋄). Then a pair (S, T) is said to be compatible of type (K) iff $\lim_{n\to\infty} M(SSx_n, TTx_n, t) = M(Tx, Sx, t)$.
$\lim_{n\to\infty} SSx_n = Tx$ and $\lim_{n\to\infty} TTx_n = Sx$. whenever $\{x_n\}$ is sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$, for some $u \in X$.

Lemma: 2.1. [17]. Let (X, M, *, ⋄) be an intuitionistic fuzzy metric space. Then for all x, y in X, M(x, y, .) is non decreasing.

Lemma: 2.2. [17]. Let (X, M, N, *, ⋄) be an intuitionistic fuzzy metric space. If there exist $q \epsilon (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ and $N(x, y, qt) \leq N(x, y, t)$ for all x, y and t > 0 then.x = y.

Lemma: 2.3. [17]. The only t - norm * satisfying $r * r \geq r$ for all r ϵ [0,1] is the minimum t- norm, that is $a * b = \min\{a, b\}$ for all a, b ϵ[0,1].

III. Main Theorem

Theorem. Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and A, B, S, M, T, and D be self - mappings of X satisfying the following condition:
(i) A(X) ⊆ SM(X) and B(X) ⊆ TD(X)

(ii) $\int_0^{M(Ax,By,kt)} \psi(t)dt \geq \int_0^{M(Ax,TDx,t)*M(By,SMy,t)*M(Ax,SMy,\alpha t)*M(TDx,SMy,t)*M(Bx,SMx,(2-\alpha)t)} \psi(t)dt$

$$\int_0^{N(Ax,By,kt)} \psi(t)dt \leq \int_0^{N(Ax,TDx,t)\diamond N(By,SMy,t)\diamond N(Ax,SMy,\alpha t)\diamond N(TDx,SMy,t)\diamond N(Bx,SMx,(2-\alpha)t)} \psi(t)dt$$

Where, $\alpha: [0,1] \to [0,1]$ is continuous function such that r (t) > t for some 0 < t <1 for all x, y ϵ X, k ϵ (0, 1), $\alpha \,\epsilon\, (0,2)$ and

(iii) S and T are continuous
If (A, TD) and (B, SM) are compatible of type (k), then A, B, S, M, T and D have a unique common fixed point.

Proof. A(X) ⊆ SM(X) and B(X) ⊆ TD(X), So for any $x_0 \,\epsilon\, X$, there exists $x_1 \epsilon X$ such that $Ax_0 = SMx_1$ and for this x_1 there exist $x_2 \epsilon X$ such that $Bx_1 = TDx_2$. Inductively we define a sequence $\{y_n\}$ in X such that.
$y_{2n-1} = Ax_{2n-2} = SMx_{2n-1}$ and $y_{2n} = Bx_{2n-1} = TDx_{2n}$ for all n = 0, 1, 2….
Taking $x = x_{2n}$ and $y = y_{2n+1}$ in (ii), we get

$$\int_0^{M(y_{2n+1},y_{2n+2},kt)} \psi(t)dt = \int_0^{M(Ax_{2n},Bx_{2n+1},kt)} \psi(t)dt$$

$$\geq \int_0^{M(Ax_{2n},TDx_{2n},t)*M(Bx_{2n+1},SMx_{2n},t)*M(Ax_{2n},SMx_{2n+1},\alpha t)*M(TDx_{2n},SMx_{2n+1},t)*M(Bx_{2n},SMx_{2n}(2-\alpha)t)} \psi(t)dt$$

Now, we put $\alpha = 1 - q$ with $q \epsilon (0,1)$, we get

$$\int_0^{M(y_{2n+1},y_{2n+2},kt)} \psi(t)dt \geq \int_0^{M(Ax_{2n},TDx_{2n},t)*M(Bx_{2n},SMx_{2n+1},t)*M(Ax_{2n},SMx_{2n+1},(1-q)t)*M(TDx_{2n},SMx_{2n+1},t)*M(Bx_{2n},SMx_{2n},2-(1-q)t)} \psi(t)dt$$

$$\geq \int_0^{M(y_{2n+1},y_{2n},t)*M(y_{2n+2},y_{2n+1},t)*M(y_{2n+1},y_{2n+1},(1-q)t)*M(y_{2n},y_{2n+1},t)*M(y_{2n+1},y_{2n},(1+q)t)} \psi(t)dt$$

$$\geq \int_0^{M(y_{2n+1},y_{2n},t)*M(y_{2n+2},y_{2n+1},t)*M(y_{2n},y_{2n+1},t)*M(y_{2n+1},y_{2n},(1+q)t)} \psi(t)dt$$

$$\geq \int_0^{M(y_{2n+1},y_{2n},t)*M(y_{2n+2},y_{2n+1},t)*M(y_{2n+1},y_{2n},t)*M(y_{2n+1},y_{2n},qt)} \psi(t)dt$$

$$\geq \int_0^{M(y_{2n+1},y_{2n},t)*M(y_{2n+2},y_{2n+1},t)*M(y_{2n+1},y_{2n},qt)} \psi(t)dt$$

$$\geq \int_0^{M(y_{2n+1},y_{2n},t)*M(y_{2n+2},y_{2n+1},t)} \psi(t)dt$$

And

$$\int_0^{N(y_{2n+1},y_{2n+2},kt)} \psi(t)dt = \int_0^{N(Ax_{2n},Bx_{2n+1},kt)} \psi(t)dt$$

$$\leq \int_0^{N(Ax_{2n},TDx_{2n},t)\diamond N(Bx_{2n+1},SMx_{2n},t)\diamond N(Ax_{2n},SMx_{2n+1},\alpha t)\diamond N(TDx_{2n},SMx_{2n+1},t)\diamond N(Bx_{2n},SMx_{2n}(2-\alpha)t)} \psi(t)dt$$

Now, we put $\alpha = 1 - q$ with $q \,\epsilon\, (0,1)$, we get

$$\int_0^{N(y_{2n+1},y_{2n+2},kt)} \psi(t)dt \leq \int_0^{N(Ax_{2n},TDx_{2n},t)\diamond N(Bx_{2n},SMx_{2n+1},t)\diamond N(Ax_{2n},SMx_{2n+1},(1-q)t)\diamond N\, N(TDx_{2n},SMx_{2n+1},t)\diamond N(Bx_{2n},SMx_{2n},2-(1-q)t)} \psi(t)dt$$

$$\leq \int_0^{N(y_{2n+1},y_{2n},t)N\diamond(y_{2n+2},y_{2n+1},t)\diamond N(y_{2n+1},y_{2n+1},(1-q)t)\diamond N(y_{2n},y_{2n+1},t)\diamond N(y_{2n+1},y_{2n},(1+q)t)} \psi(t)dt$$

$$\leq \int_0^{N(y_{2n+1},y_{2n},t)\diamond N(y_{2n+2},y_{2n+1},t)\diamond N(y_{2n},y_{2n+1},t)\diamond N(y_{2n+1},y_{2n},(1+q)t)} \psi(t)dt$$

$$\leq \int_0^{N(y_{2n+1},y_{2n},t)\diamond N(y_{2n+2},y_{2n+1},t)\diamond N(y_{2n+1},y_{2n},t)\diamond N(y_{2n+1},y_{2n},qt)} \psi(t)dt$$

$$\leq \int_0^{N(y_{2n+1},y_{2n},t)\diamond N(y_{2n+2},y_{2n+1},t)\diamond N(y_{2n+1},y_{2n},qt)} \psi(t)dt$$

$$\leq \int_0^{N(y_{2n+1},y_{2n},t)\diamond N(y_{2n+2},y_{2n+1},t)} \psi(t)dt$$

From lemma 2.1 and 2.3 we have

$\int_0^{M(y_{2n+1},y_{2n+2},kt)} \psi(t)dt \geq \int_0^{M(y_{2n},y_{2n+1},t)} \psi(dt)$ and

$\int_0^{N(y_{2n+1},y_{2n+2},kt)} \psi(t)dt \leq \int_0^{N(y_{2n},y_{2n+1},t)} \psi(dt)$ (1)

Similarly, we have

$\int_0^{M(y_{2n+2},y_{2n+3},kt)} \psi(t)dt \geq \int_0^{M(y_{2n+1},y_{2n+2},t)} \psi(dt)$ and

$\int_0^{N(y_{2n+2},y_{2n+3},kt)} \psi(t)dt \geq \int_0^{N(y_{2n+1},y_{2n+2},t)} \psi(dt)$ (2)

From (1) and (2), we have

$\int_0^{M(y_{n+1},y_{n+2},kt)} \psi d(t) \geq \int_0^{M(y_n,y_{n+1},t)} \psi d(t)$ and

$\int_0^{N(y_{n+1},y_{n+2},kt)} \psi d(t) \leq \int_0^{N(y_n,y_{n+1},t)} \psi d(t)$ (3)

From (3), we have

$$\int_0^{M(y_{n+1},y_{n+2},kt)} \psi(t)dt \geq \int_0^{M\left(y_n,y_{n+1},\frac{t}{k}\right)} \psi(t)dt \geq \int_0^{M\left(y_{n-1},y_{n+1},\frac{t}{k^2}\right)} \psi(t)dt \geq \cdots \ldots.. \geq \int_0^{M\left(y_1,y_2,\frac{t}{k^n}\right)} \psi(t)dt$$
$$\to 1 \; as \; n \to \infty$$

And

$$\int_0^{N(y_n,y_{n+1},kt)} \psi(t)dt \leq \int_0^{N(y_n,y_{n+1},t)} \psi(t)dt \leq \int_0^{N\left(y_{n-1},y_{n+1},\frac{t}{k^2}\right)} \psi(t)dt \leq \cdots \ldots \ldots. \leq \int_0^{N\left(y_1,y_2,\frac{t}{k^n}\right)} \psi(t)dt$$
$$\to 1 \; as \; n \to \infty$$

So, $\int_0^{M(y_n,y_{n+1},t)} \psi(t)\,dt \to$ as $n \to \infty$ for any t > 0. For each $\epsilon > 0$ and each t > 0 we can choose $n_0 \epsilon N$ such that $\int_0^{M(y_n,y_{n+1},t)} \psi(t)dt > 1-\epsilon$ for all $n > n_0$. For n, m ϵ N, we suppose $m \geq n$. Then we have $\int_0^{M(y_n,y_m,t)} \psi(t)dt \geq \int_0^{M\left(y_n,y_{n+1},\frac{t}{m-n}\right)} \psi(t)dt * \int_0^{M\left(y_{n+1},y_{n+2},\frac{t}{m-n}\right)} \psi(t)dt * \ldots \ldots.* \int_0^{M(y_{n+1},y_{n+2},\frac{t}{m-n}} \psi(t)dt \geq$ $1-\epsilon * 1-\epsilon * \ldots \ldots. m-n$ times. This implies

$\int_0^{M(y_n,y_m,t)} \psi(t)dt \leq 1-\epsilon$ and hence $\{y_n\}$ is a Cauchy sequence in X.

Since $(X, M, N, *, \diamond)$ is complete, $\{y_n\}$ converges to some point $z \in X$, and so that $\{Ax_{2n-2}\}$, $\{TDx_{2n}\}$, $\{Bx_{2n-1}\}$, $\{SMx_{2n-1}\}$ also converges to z. since (A, TD) and (B, SM) are compatible of type (k), we have

$AAx_{2n-2} \to TDz, TD(TDx_{2n}) \to Az, BBx_{2n-1} \to SMz, SM(SMx_{2n-1}) \to Bz$ (4)

From (ii), we get by putting $x = Ax_{2n-2}$ and $y = Bx_{2n-1}$

$$\int_0^{M(A(Ax_{2n-2}),B(Bx_{2n-1}),kt)} \psi(t)dt$$
$$\geq \int_0^{M(A(Ax_{2n-2}),TD(Ax_{2n-2}),t)*M(B(Bx_{2n-1}),SM(Bx_{2n-1}),t)*M(A(Ax_{2n-2}),SM(Bx_{2n-1}),\alpha t)*M(TD(Ax_{2n-2}),SM(Bx_{2n-1}),t)*M(B(Ax_{2n-2}),SM(Ax_{2n-2}),(2-\alpha)t)} \psi(t)dt$$

Now, using limit $n \to \infty$ and using (4)

We have,

$$\int_0^{M(TDz,SMz,kt)} \psi(t)dt \geq \int_0^{M(TDz,TDz,t)*M(SMz,SMz,t)*M(TDz,SMz,(1-q)t)*M(TDz,SMz,t)*M(Bz,SMz,(2-(1-q))t)} \psi(t)dt$$

$$\geq \int_0^{M(TDz,TDz,t)*M(SMz,SMz,t)*M(TDz,SMz,t)*M(TDz,SMz,t)*M(Bz,SMz,t)*M(Bz,SMz,qt)} \psi(t)dt$$

$$\geq \int_0^{M(TDz,TDz,t)*M(SMz,SMz,t)*M(TDz,SMz,t)*M(Bz,SMz,t)*M(Bz,SMz,qt)} \psi(t)dt$$

$$\geq \int_0^{1*1*M(TDz,SMz,t)*M(Bz,SMz,t)*M(Bz,SMz,qt)} \psi(t)dt$$

$$\geq \int_0^{M(TDz,SMz,t)*M(Bz,SMz,t)} \psi(t)dt$$

$$\geq \int_0^{M(TDz,SMz,t)*1} \psi(t)dt$$

$$\geq \int_0^{M(TDz,SMz,t)} \psi(t)dt$$

And

$$\int_0^{N(A(Ax_{2n-2}),B(Bx_{2n-1}),kt)} \psi(t)dt$$

$$\leq \int_0^{\substack{N\diamond(A(Ax_{2n-2}),TD(Ax_{2n-2}),t)\diamond N(B(Bx_{2n-1}),SM(Bx_{2n-1}),t)\diamond N(A(Ax_{2n-2}),SM(Bx_{2n-1}),\alpha t)\\ \diamond N(TD(Ax_{2n-2}),SM(Bx_{2n-1}),t)\diamond N(B(Ax_{2n-2}),SM(Ax_{2n-2}),(2-\alpha)t)}} \psi(t)dt$$

Now, using limit $n \to \infty$ and using (4)

We have,

$$\int_0^{N(TDz,SMz,kt)} \psi(t)dt \leq \int_0^{\substack{N(TDz,TDz,t)\diamond N(SMz,SMz,t)\diamond N(TDz,SMz,(1-q)t)\diamond\\ N(TDz,SMz,t)\diamond N(Bz,SMz,(2-(1-q))t)}} \psi(t)dt$$

$$\leq \int_0^{\substack{N(TDz,TDz,t)\diamond N(SMz,SMz,t)\diamond N(TDz,SMz,t)\\ \diamond N(TDz,SMz,t)\diamond N(Bz,SMz,t)\diamond N(Bz,SMz,qt)}} \psi(t)dt$$

$$\leq \int_0^{\substack{N(TDz,TDz,t)\diamond N(SMz,SMz,t)\\ \diamond N(TDz,SMz,t)\diamond N(Bz,SMz,t)\diamond N(Bz,SMz,qt)}} \psi(t)dt$$

$$\leq \int_0^{\substack{1\diamond 1\\ \diamond(TDz,SMz,t)\diamond N(Bz,SMz,t)\diamond N(Bz,SMz,qt)}} \psi(t)dt$$

$$\leq \int_0^{N(TDz,SMz,t)\diamond N(Bz,SMz,t)} \psi(t)dt$$

$$\leq \int_0^{N(TDz,SMz,t)\diamond 1} \psi(t)dt$$

$$\leq \int_0^{N(TDz,SMz,t)} \psi(t)dt$$

SMz = TDz

Now, by putting $x = z$, $y = Bx_{2n-1}$

$$\int_0^{M(Az,B(Bx_{2n-1}),kt)} \psi(t)dt \geq \int_0^{\substack{M(Az,TD(z),t)*M(B(Bx_{2n-1}),SM(Bx_{2n-1}),t)*M(Az,SM(Bx_{2n-1}),\alpha t)\\ *M(TDz,SM(Bx_{2n-1}),t)*M(Bz,SMz,(2-\alpha)t)}} \psi(t)dt$$

Taking limit as $n \to \infty$ and using (4)

$$\int_0^{M(Az,SMz,kt)} \psi(t)dt \geq \int_0^{\substack{M(Az,TDz,t)*M(SMz,SMz,t)*M(Az,SMz,(1-q)t)*\\ M(TDz,SMz,t)*M(Bz,SMz,2-(1-q)t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{M(Az,SMz,t)*M(SMz,SMz,t)*M(Az,SMz,(1-q)t)*\\ M(SMz,SMz,t)*M(Bz,SMz,(1+q)t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{1*1*\\ M(Az,SMz,(1-q)t)*1*M(Bz,SMz,t)*M(Bz,SMz,qt)}} \psi(t)dt$$

$$\geq \int_0^{M(Az,SMz,t)*M(Bz,SMz,t)} \psi(t)dt$$

$$\geq \int_0^{M(Az,SMz,t)*1} \psi(t)dt$$

$$\geq \int_0^{M(Az,SMz,t)} \psi(t)dt$$

Az = SMz.

And

$$\int_0^{N(Az,B(Bx_{2n-1}),kt)}\psi(t)dt \le \int_0^{\substack{N(Az,TD(z),t)\diamond N(B(Bx_{2n-1}),SM(Bx_{2n-1}),t)\diamond N(Az,SM(Bx_{2n-1}),\alpha t)\\ \diamond N(TDz,SM(Bx_{2n-1}),t)\diamond N(Bz,SMz,(2-\alpha)t)}}\psi(t)dt$$

Again taking limit as $n \to \infty$ and using (4)

$$\int_0^{N(Az,SMz,kt)}\psi(t)dt \le \int_0^{\substack{N(Az,TDz,t)\diamond(SMz,SMz,t)\diamond N(Az,SMz,(1-q)t)\diamond\\ N(TDz,SMz,t)\diamond N(Bz,SMz,2-(1-q)t)}}\psi(t)dt$$

$$\le \int_0^{\substack{N(Az,SMz,t)\diamond N(SMz,SMz,t)\diamond N(Az,SMz,(1-q)t)\diamond\\ N(SMz,SMz,t)\diamond N(Bz,SMz,(1+q)t)}}\psi(t)dt$$

$$\le \int_0^{\substack{0\diamond 0\diamond\\ N(Az,SMz,(1-q)t)\diamond 0\diamond N(Bz,SMz,t)\diamond N(Bz,SMz,qt)}}\psi(t)dt$$

$$\le \int_0^{N(Az,SMz,t)\diamond N(Bz,SMz,t)}\psi(t)dt$$

$$\le \int_0^{N(Az,SMz,t)\diamond 0}\psi(t)dt$$

$$\le \int_0^{N(Az,SMz,t)}\psi(t)dt$$

Az = SMz

Hence Az = SMz.

From (ii), We get

$$\int_0^{M(Az,Bz,kt)}\psi(t)dt \ge \int_0^{\substack{M(Az,TDz,t)*M(Bz,SMz,t)*M(Az,SMz,\alpha t)\\ *M(TDz,SMz,t)*M(Bz,SMz,(2-\alpha)t)}}\psi(t)dt$$

$$\ge \int_0^{\substack{M(TDz,TDz,t)*M(Bz,SMz,t)*M(SMz,SMz,(1-q)t)*M(SMz,SMz,t)\\ *M(Bz,SMz,2-(1-q)t)}}\psi(t)dt$$

$$\ge \int_0^{\substack{1*M(Bz,SMz,t)*M(SMz,SMz,qt)\\ *M(Bz,SMz,t)*M(Bz,SMz,qt)}}\psi(t)dt$$

$$\ge \int_0^{M(Bz,SMz,t)}\psi(t)dt$$

$$\ge \int_0^{M(Bz,Az,t)}\psi(t)dt$$

And

$$\int_0^{N(Az,Bz,kt)}\psi(t)dt \le \int_0^{\substack{N(Az,TDz,t)\diamond N(Bz,SMz,t)\diamond N(Az,SMz,\alpha t)\\ \diamond N(TDz,SMz,t)\diamond N(Bz,SMz,(2-\alpha)t)}}\psi(t)dt$$

$$\le \int_0^{\substack{N(TDz,TDz,t)\diamond N(Bz,SMz,t)\diamond N(SMz,SMz,(1-q)t)\diamond N(SMz,SMz,t)\\ \diamond N(Bz,SMz,2-(1-q)t)}}\psi(t)dt$$

$$\le \int_0^{\substack{0\diamond N(Bz,SMz,t)\diamond N(SMz,SMz,qt)\\ \diamond N(Bz,SMz,t)\diamond N(Bz,SMz,qt)}}\psi(t)dt$$

$$\le \int_0^{N(Bz,SMz,t)}\psi(t)dt$$

$$\le \int_0^{N(Bz,Az,t)}\psi(t)dt$$

Az = Bz

Therefore, Bz = Az = SMz = TDz.

Now we show that Bz = z. From (ii) we get

$$\int_0^{M(Ax_{2n},Bz,kt)}\psi(t)dt \ge \int_0^{\substack{M(Ax_{2n},TDx_{2n},t)*M(Bz,SMz,t)*M(Ax_{2n},SMz,\alpha t)\\ *M(TDx_{2n},SMz,t)*M(Bx_{2n},SMx_{2n},(2-\alpha)t)}}\psi(t)dt$$

$$\int_0^{M(Ax_{2n},Bz,kt)}\psi(t)dt \ge \int_0^{\substack{M(Ax_{2n},TDx_{2n},t)*M(Bz,SMz,t)*M(Ax_{2n},SMz,t)\\ *M(TDx_{2n},SMz,t)*M(Bx_{2n},SMx_{2n},t)*M(Bx_{2n},SMx_{2n},qt)}}\psi(t)dt$$

Taking limit as $n \to \infty$, we have

$$\int_0^{M(z,Bz,kt)} \geq \int_0^{\substack{M(z,z,t)*M(Bz,SMz,t)*M(z,SMZ,t)* \\ M(TDz,SMZ,t)*M(z,z,t)*M(z,z,t)}} \psi(t)dt$$

$$\geq \int_0^{\substack{1*M(Bz,SMz,t)*M(z,SMz,t)* \\ M(SMz,SMz,t)*1*1}} \psi(t)dt$$

$$\geq \int_0^{M(Bz,SMz,t)*M(z,SMz,t)} \psi(t)dt$$

$$\geq \int_0^{M(Bz,Bz,t)*M(Z,Bz,t)} \psi(t)dt$$

$$\geq \int_0^{1*M(z,Bz,t)} \psi(t)dt$$

$$\geq \int_0^{M(z,Bz,t)} \psi(t)dt$$

And

$$\int_0^{N(z,Bz,kt)} \psi(t)dt \leq \int_0^{\substack{N(z,z,t)\diamond N(Bz,SMz,t)\diamond N(z,SMZ,t)\diamond \\ N(TDz,SMZ,t)\diamond N(z,z,t)\diamond N(z,z,t)}} \psi(t)dt$$

$$\leq \int_0^{\substack{0\diamond N(Bz,SMz,t)\diamond N(z,SMz,t)\diamond \\ N(SMz,SMz,t)\diamond 0\diamond 0}} \psi(t)dt$$

$$\leq \int_0^{N(Bz,SMz,t)\diamond N(z,SMz,t)} \psi(t)dt$$

$$\leq \int_0^{N(Bz,Bz,t)\diamond N(Z,Bz,t)} \psi(t)dt$$

$$\leq \int_0^{0\diamond N(z,Bz,t)} \psi(t)dt$$

$$\leq \int_0^{N(z,Bz,t)} \psi(t)dt$$

And hence Bz = z.
Therefore Bz = Az = SMz = TDz = z.
Now, by (ii) put x = Mz, y = z

$$\int_0^{M(A(Mz),Bz,kt)} \psi(t)dt \geq \int_0^{\substack{M(A(Mz),TD(Mz),t)*M(Bz,SMz,t)*M(A(Mz),SMz,t) \\ *M(TD(Mz),SMz,t)*M(B(Mz),SM(Mz),t)*M(B(Mz),SM(Mz),qt)}} \psi(t)dt$$

$$\int_0^{M(Mz,z,kt)} \psi(t)dt \geq \int_0^{\substack{M(Mz,Mz,t)*M(z,z,t)*M(Mz,z,t) \\ *M(Mz,z,t)*M(Mz,Mz,t)*M(Mz,Mz,qt)}} \psi(t)dt$$

$$\geq \int_0^{M(Mz,z,t)*M(Mz,z,t)} \psi(t)dt$$

$$\geq \int_0^{M(Mz,z,t)} \psi(t)dt$$

And

$$\int_0^{N(A(Mz),Bz,kt)} \psi(t)dt \leq \int_0^{\substack{N(A(Mz),TD(Mz),t)\diamond N(Bz,SMz,t)\diamond N(A(Mz),SMz,t)\diamond \\ N(TD(Mz),SMz,t)\diamond N(B(Mz),SM(Mz),t)\diamond N(B(Mz),SM(Mz),qt)}} \psi(t)dt$$

$$\int_0^{N(Mz,z,kt)} \psi(t)dt \leq \int_0^{\substack{N(Mz,Mz,t)\diamond N(z,z,t)\diamond N(Mz,z,t)\diamond \\ N(Mz,z,t)\diamond N(Mz,Mz,t)\diamond N(Mz,Mz,qt)}} \psi(t)dt$$

$$\leq \int_0^{N(Mz,z,t)\diamond N(Mz,z,t)} \psi(t)dt$$

$$\leq \int_0^{N(Mz,z,t)} \psi(t)dt$$

Hence Mz = z. since SMz = z which implies Sz = z.
Again by (ii) put x = z, y = Dz

$$\int_0^{M(Az,B(Dz),kt)} \psi(t)dt \geq \int_0^{\substack{M(Az,TDz,t)*M(B(Dz),SM(Dz),t)*M(Az,SM(Dz),t)\\ *M(TDz,SM(Dz),t)*M(Bz,SMz,t)*M(Bz,SMz,qt)}} \psi(t)dt$$

$$\int_0^{M(z,Dz,kt)} \psi(t)dt \geq \int_0^{\substack{M(z,z,t)*M(Dz,Dz,t)*M(z,Dz,t)\\ *M(z,Dz,t)*M(z,z,t)*M(z,z,qt)}} \psi(t)dt$$

$$\geq \int_0^{M(z,Dz,t)*M(z,Dz,t)} \psi(t)dt$$

$$\geq \int_0^{M(z,Dz,t)} \psi(t)dt$$

and

$$\int_0^{N(Az,B(Dz),kt)} \psi(t)dt \leq \int_0^{\substack{N(Az,TDz,t)\diamond N(B(Dz),SM(Dz),t)\diamond N(Az,SM(Dz),t)\\ \diamond N(TDz,SM(Dz),t)\diamond N(Bz,SMz,t)\diamond N(Bz,SMz,qt)}} \psi(t)dt$$

$$\int_0^{M(z,Dz,kt)} \psi(t)dt \leq \int_0^{\substack{N(z,z,t)\diamond N(Dz,Dz,t)\diamond M(z,Dz,t)\\ \diamond N(z,Dz,t)\diamond N(z,z,t)\diamond N(z,z,qt)}} \psi(t)dt$$

$$\geq \int_0^{N(z,Dz,t)\diamond N(z,Dz,t)} \psi(t)dt$$

$$\geq \int_0^{N(z,Dz,t)} \psi(t)dt$$

Hence Dz = z.
Since TDz = z which implies Tz = z.
And hence Az = Bz = Sz = Mz = Tz = Dz = z and z is common fixed point of A, B, S, M, T and D.
Uniqueness: To prove uniqueness of fixed point, let w be an another fixed point of A, B, S, M, T and D. Then Aw = Bw = Sw = Mw = Tw = Dw = w, then from (ii) we get

$$\int_0^{M(z,w,t)} \psi(t)dt = \int_0^{M(Az,Bw,kt)} \psi(t)dt \geq \int_0^{\substack{M(Az,TDz,t)*M(Bw,SMw,t)*M(Az,SMw,\alpha t)\\ *M(TDz,SMw,t)*M(Bz,SMz,(2-\alpha)t)}} \psi(t)dt$$

$$\int_0^{M(Az,Bw,kt)} \psi(t)dt \geq \int_0^{M(z,w,t)} \psi(t)dt$$

and

$$\int_0^{N(z,w,t)} \psi(t)dt = \int_0^{N(Az,Bw,kt)} \psi(t)dt \leq \int_0^{\substack{N(Az,TDz,t)\diamond N(Bw,SMw,t)\diamond N(Az,SMw,\alpha t)\diamond\\ N(TDz,SMw,t)\diamond N(Bz,SMz,(2-\alpha)t)}} \psi(t)dt$$

$$\int_0^{N(Az,Bw,kt)} \psi(t)dt \leq \int_0^{N(z,w,t)} \psi(t)dt$$

From lemma 2.2, we get z = w.
Hence z is a unique common fixed point of A, B, S, M, T and D.

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