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Analysis Of A Queueing System With Interruptions, Self-Healing Mechanisms, And Service Protection In Random Environment

Author

Abstract
This paper explores the various factors that can cause service interruptions in queueing models and their consequences on system performance. We consider a single server queueing model with interruptions. These interruptions arise from a finite number of environmental factors. For a subset of these factors, the interrup- tion is mild, allowing service to continue at a reduced rate. There is a possibility for self-correction of these mild interruptions, after which the server resumes service at the normal rate. The duration of uninterrupted service with a mild interrup- tion is measured by an interruption clock. When this clock triggers, the server is taken out for repair, and the interrupted customer's service resumes after the re- pair is complete. Interruptions caused by the remaining environmental factors are severe, immediately requiring the server to be taken out for repair. In these severe cases, considering the interruption's severity, protected service is provided to the interrupted customer for the remaining phases of service. We analyze the stabil- ity of the system, calculate the steady-state probability vector using the matrix analytic method, and numerically substantiate important performance measures. keywords: Interruption, Environmental factor, Self-correction, Protected service, Interruption clock, Ignored interruption

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I. Introduction

Interruptions are a common Interruptions are an inherent part of various real-world systems, often influencing their operational efficiency. In queueing systems, service disruptions can have a profound impact on performance, altering queue dynamics and overall system effectiveness. These interruptions, characterized as temporary halts in the service process, have been widely studied in queueing theory to better understand their implications. Foundational work by White and Christie (1958) [16] established the basis for research in this area, leading to extensive studies on queueing models incor- porating service interruptions. For a comprehensive overview of such models, we refer readers to the work of A. Krishnamoorthy, Pramod P.K., and S.R. Chakravarthy (2012) [10]. In certain scenarios, to mitigate customer dissatisfaction, systems may attempt to maintain service continuity during interruptions whenever feasible. In practice, servers often continue operating during disruptions, albeit at reduced efficiency.

Prior research typically assumed servers ceased service entirely during lengthy, un- predictable breakdowns. The concept of postponing interruptions until the service fin- ishes is discussed by Gaver Jr D.P (1962) [5] and by Hans (J.P.C.) Blanc (2012) [7]. Post- ponement of interruption refers to delaying the interruption until the current service finishes. Kalidass and Kasturi (2012) [8], and Deepa, Kalidass and Vijayalakshmi (2021) [1] considered a breakdown policy called working break down where customers receive service at a reduced rate when the system experiences partial failure. In [15] the authors examined M/M/1 queues with working breakdowns and delayed repair, in which the system is repaired immediately (with probability p) or continues to provide service for customers at a lower rate (with probability 1-p) when a breakdown occurs. For more related researches on queueing models with working breakdowns, interested readers are referred to Liu and Song(2014)[13], Liou (2015)[3], Chen, Yen, and Wang (2016)[2], and Yang and Cho (2019)[17]. In these works related to queue with interruption, more or less the interrupted service is resumed or restarted after completion of interruption.

External shocks, often unpredictable and ranging from mild to destructive, can dis- rupt service processes. Identifying and promptly addressing the root causes of these shocks is crucial for maintaining service process stability. The paper by A. Krish- namoorthy, Jaya and lakshmi(2015) [11] presents a unique case of an M/M/1 queue where interruptions arise from a finite number of environmental factors and remain unidentified for a short duration. In [12] the interruption causing environmental fac- tors and duration of interruption are the deciding factors of whether to repeat, resume the service after interruption, or replace the server. In these cases, there is an equal probability of either server damage or self-correction of the interruption. Automatic systems increasingly incorporate self-correction, as exemplified by the recent work by Hafaiedh and Slimane(2022)[6] on designing autonomous systems that can identify and fix problems themselves. These autonomic computing systems, inspired by the human body's self-regulating nervous system, strive for a high degree of self-management. In [14], Jaya(2019) explores the concept of partially ignored interruptions with the possi- bility of self-correction. Additionally, protected service, discussed in [4] and [9] provides a mechanism where service continues after the fixation of interruption, albeit with some modifications. Protected services offer a safety net that minimizes the impact of inter- ruptions on your operations and customer satisfaction.

Consider the example of a router transmitting a data packet. The transmission can be interrupted due to various environmental factors like, Low signal strength: Service continues at a reduced rate (retransmission with error correction), Channel congestion: Service continues at a reduced rate (waiting for a less congested slot), Hardware failure: The router is immediately taken offline for repair. The interrupted packet transmission might be restarted with priority (protected service) after repair. Software crash: Similar to hardware failure, immediate repair with potential service resumption after fixing the software issue. here are some specific advantages to con- sidering a queueing model where a server experiencing a partial breakdown can still provide service at a reduced capacity:

- Reduced Customer Wait Times: Compared to a complete server breakdown, customers might experience some service even during the partial breakdown. This can help keep the queue moving and potentially reduce overall waiting times.
- Maintaining System Functionality: Even with degraded performance, the server can continue to handle some requests. This is crucial for systems where even limited service is better than complete downtime, such as emergency hotlines or critical infrastructure. During a partial breakdown, even with slower service, the queue keeps moving compared to a complete shutdown.
- improved System Efficiency: Self-correction minimizes downtime caused by in- terruptions. This allows the server to resume full or near-full capacity service quicker, leading to a faster reduction in queue length and waiting times.
- Reduced Customer Dissatisfaction: Shorter wait times due to faster recovery from interruptions translate to a better customer experience. This is crucial for maintaining customer satisfaction, especially in situations where even minor delays can be frustrating.

Every queuing model aims to achieve a balance between hassle-free service, minimal cost, and high customer satisfaction. In this model, we consider a single-server queue- ing system with Poisson arrivals and Erlang-distributed service times. Interruptions can occur during service, and we will explore how the system handles them. Each interruption duration follows an exponential distribution. There are n environmental factors that can trigger these interruptions. However, if the interruption originates from one of the first m factors, it's considered mild. In such cases, service continues at a reduced rate, and the interruption is essentially ignored. We introduce an "inter- ruption clock" that starts ticking upon any interruption. This clock's duration is also exponentially distributed. During this time, there's a chance for the interruption to self-correct, allowing service to resume at the normal rate from the halted phase. If self-correction occurs, the service rate returns to normal, the interruption clock stops and service continues for the customer. When the interruption clock reaches its limit, the server undergoes repair. The interrupted customer's service resumes after the repair is complete. If a customer's service is interrupted but finishes before the clock triggers repair, the next customer in line begins service on the (potentially still operational) interrupted server. Interruptions caused by the remaining $n-m$ factors are considered severe. The server immediately goes for repair, bypassing the clock. Protected ser- vice—an uninterrupted service—is provided for the remaining duration of the affected customer's service due to the critical nature of these interruptions.

The rest of the paper is organized as follows. Section 2 provides a detailed description of the queueing model. The mathematical formulation of the queueing model is pre- sented in Section 3. Section 4 analyzes the behaviour of the service process within the model. Key performance measures for the model are discussed in Section 5. Section 6 presents numerical examples to illustrate the model's behaviour.

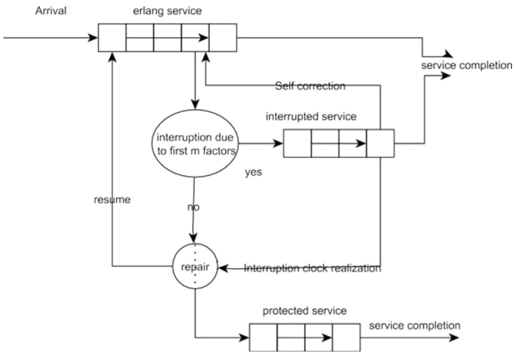

Fig 1: Model description

II. Model Description

We consider a single-server infinite waiting space queueing system. Customers arrive according to a Poisson process with arrival rate λ. The customers are served in FIFO service order. Service times follow an Erlang distribution with shape parameter K and scale parameter μ. During service, interruptions can occur due to n environmental factors.These factors are ranked from 1 to n based on the severity of the interruption they cause. Interruption stream follow a Poisson process with parameter β. Factor i, $(1 \leq i \leq n)$ triggers an interruption with probability p_i. If the interruptions originate from one of the first m factors ($m < n$), it is considered mild. Then the actual service phase does not change and the service continues at a reduced rate $\mu_i (i = 1, 2, \ldots, m)$. An interruption clock start ticking at the onset of interruption. This clock is exponentially distributed with parameter δ_i, $(i = 1, 2, \ldots, m)$. During this interrupted service period, there is a chance for self-correction, which is exponentially distributed with parameter γ_i, $i = 1, 2, \ldots, m$. If self-correction occurs, the service rate returns to μ. If the customer finishes their service before a pre-defined repair window opens (triggered by a random clock), the system moves on to the next customer(assuming it is still operational). Once the repair window opens, the server is automatically flagged for repair. Interruptions caused by the remaining $n - m$ factors are considered severe.The server immediately undergoes repair.The repair time depends on the specific environ-mental factor that caused the interruption. Each factor has its own associated repair time distribution, denoted by the parameter η_i, $i = 1, 2, \ldots, n$. Once the repair is com- plete, the interrupted service is resumed from the halted phase. Service protection, ensuring no further interruptions to the remaining service, is provided to the customer whose service was disrupted by the i^{th} factor, where $i = m + 1, \ldots, n$, starting from the moment service resumes following repair.

III. Mathematical Description

The behavior of the queueing system described above can be analyzed using a Markov chain. Let $\mathbf{X} = \{X(t), t \geq 0\} = \{(N(t), S(t), I_1(t), I_2(t)), t \geq 0\}$ where $N(t)$ is the number of customers in the system, $S(t)$ is the status of the server, $I_1(t)$ is the envi- ronmental factor causing interruption and $I_2(t)$ is the phase of service:

$$S(t) = \begin{cases} 0, & \text{servier is idle ;} \\ 1, & \text{service without any interruption ;} \\ 2, & \text{if interrupted service going on;} \\ 3, & \text{if server under repair;} \\ 4, & \text{if protected service is going on.} \end{cases}$$

The state space of the process is
$\{(0,0) \cup (r,1,i) \cup (r,2,j,i) \cup (r,3,l,i) \cup (r,4,i); r = 1, \ldots, \infty; i = 1, \ldots, K; j = 1, \ldots, m; l = 1, \ldots, n\}$.

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The transitions in the Markov Chain and the correspopnding rates are discribed below:

$(0,0) \xrightarrow{\lambda} (1,1,1)$, $(0,0) \xrightarrow{-\lambda} (0,0)$, $(1,1,K) \xrightarrow{\mu} (0,0)$,

μj μ

$(1,2,j,K) \longrightarrow (0,2), j = 1,\dots,m,\ (1,4,K) \longrightarrow (0,0)$

The below given transitions are for $r = 2,\dots,\infty$;

$(r,1,i) \xrightarrow{\mu} (r,1,i+1)$, for $i = 1,\dots,K-1$

$(r,1,K) \xrightarrow{\mu} (r-1,1,1)$,

$(r,1,i) \xrightarrow{\beta p_j} (r,2,j,i)$ for $i = 1,\dots,K, = 1,\dots,m$

$(r,1,i) \xrightarrow{\beta p_j} (r,3,j,i)$ for $i = 1,\dots,K, j = m+1,\dots,n$

$(r,2,j,i) \xrightarrow{\mu_j} (r,2,j,i+1)$ for $i = 1,\dots,K-1, j = 1,\dots,m$

$(r,2,j,K) \xrightarrow{\mu_j} (r-1,2,j,1)$ for $j = 1,\dots,m$

$(r,2,j,i) \xrightarrow{\delta_j} (r,1,i)$, $(r,2,j,i) \xrightarrow{\gamma_j} (r,3,j,i)$ for $i = 1,\dots,K, j = 1,\dots,m$

$(r,3,j,i) \xrightarrow{\eta_j} (r,1,i)$ for $i = 1,\dots,K, j = 1,\dots,m$

$(r,3,j,i) \xrightarrow{\eta_j} (r,4,i)$ for $i = 1,\dots,K, j = m+1,\dots,n$

$(r,4,i) \xrightarrow{\mu} (r,4,i+1)$ for $i = 1,\dots,K-1$

$(r,4,K) \xrightarrow{\mu} (r-1,1,1)$

$(r,l,i) \xrightarrow{\lambda} (r+1,l,i)$ for $l = 1,4.,\ i = 1,\dots,K,$

$(r,l,j,i) \xrightarrow{\lambda} (r+1,l,j,i)$ for $l = 2,3.,\ i = 1,\dots,K, j = 1,\dots,m$

$(r,1,i) \xrightarrow{-\lambda-\mu-\beta} (r,1,i)$, for $i = 1,\dots,K$

$(r,2,j,i) \xrightarrow{-\lambda-\mu_j-\gamma_j-\delta_j} (r,2,j,i)$ for $i = 1,\dots,K, j = 1,\dots,m$

$(r,3,j,i) \xrightarrow{-\lambda-\eta_j} (r,3,j,i)$ for $i = 1,\dots,K, j = 1,\dots,n$

$(r,4,i) \xrightarrow{-\lambda-\mu} (r,4,i)$, for $i = 1,\dots,K$

The infinitesimal generator matrix of the process is given by

$$Q = \begin{bmatrix} B_0 & B_1 & & & \\ B_2 & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & A_2 & A_1 & A_0 \\ & & & \ddots & \ddots & \ddots \end{bmatrix}$$

where $B_0 = [-\lambda]$, $B_1 = \begin{bmatrix} \lambda & 0 \end{bmatrix}$ is an $1 \times (2+m+n)K$ matrix

B_2 is a column matrix of order $(2+m+n)K \times 1$. A_0, A_1 and A_2 are square matrices of order $(2+m+n)K$

IV. Analysis Of Service Process

The service time follows PH distribution with representation $(\boldsymbol{\alpha}, S)$ where $\boldsymbol{\alpha} = (1,0,\dots,0)_{1\times(m+n+2)K}$ and
$$S = \begin{bmatrix} C'_0 & C_1 & C_2 & 0 \\ C_3 & C'_4 & C_5 & 0 \\ C_6 & 0 & C'_7 & C_8 \\ 0 & 0 & 0 & C'_9 \end{bmatrix}$$

$C'_0 = C_0 + \lambda I$, $C'_4 = C_4 + \lambda I$, $C'_7 = C_7 + \lambda I$ and $C'_9 = C_9 + \lambda I$, where C_0 is a matrix of order K,

$$C_0(i,j) = \begin{cases} -\lambda - \beta - \mu, & \text{for } i = j; \\ \mu, & \text{for } j = i+1; i = 1,\dots,K-1 \\ 0, & \text{otherwise.} \end{cases}$$

$C_1 = (\beta p' \otimes I_K)_{K\times Km}$, $C_2 = \begin{bmatrix} 0 & \beta p'' \otimes I_K \end{bmatrix}_{K\times Km}$ where $p = (p', p'')$ with $p' = (p_1,\dots,p_m)$ and $p'' = (p_{m+1},\dots,p_n)$.

Let $\boldsymbol{\gamma} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{pmatrix}$ then $C_3 = \boldsymbol{\gamma} \otimes I_K$ and C_4 is a matrix of order mK,

$$C_4(i,j) = \begin{cases} \vartheta_r, & \text{for } i = j; i = (r-1)K + l; r = 1, \ldots, m; l = 1, \ldots, K; \\ \mu_r, & \text{for } i + 1 = j, i = (r-1)K + l; r = 1, \ldots, m; l = 1, \ldots, K-1; \\ 0, & \text{otherwise.} \end{cases}$$

where $\vartheta_r = -\lambda - \mu_r - \gamma_r - \delta_r$.

C_5 is a matrix of order $mK \times nK$,

$$C_5(i,j) = \begin{cases} \delta_t, & \text{for } i = j; i = (t-1)K + l, t = 1, \ldots, m; l = 1, \ldots, K \\ 0, & \text{otherwise;} \end{cases}$$

Let $\boldsymbol{\eta} = \begin{pmatrix} \boldsymbol{\eta}' \\ \boldsymbol{\eta}'' \end{pmatrix}$ with $\boldsymbol{\eta}' = (\eta_1, \eta_2, \ldots, \eta_m)^T$ and $\boldsymbol{\eta}'' = (\eta_{m+1}, \ldots, \eta_n)^T$

then $C_6 = \begin{pmatrix} \boldsymbol{\eta}' \otimes I_K \\ 0 \end{pmatrix}_{(nK \times K)}$.

C_7 is a matrix of order nK,

$$C_7 = \begin{cases} -\lambda - \eta_r, & \text{for } i = j; i = (r-1)K + l, r = 1, \ldots, n; l = 1, \ldots, K \\ 0, & \text{otherwise.} \end{cases}$$

$C_8 = \begin{pmatrix} 0 \\ \boldsymbol{\eta}'' \otimes I_K \end{pmatrix}_{(nK \times K)}$. C_9 is a matrix of order K,

$$C_9 = \begin{cases} -\lambda - \mu, & \text{for } i = j; i = (r-1)K + l, l = 1, \ldots, K; \\ \mu, & \text{for } j = i + 1; i = (r-1)K + l, l = 1, \ldots, K-1; \\ 0, & \text{otherwise.} \end{cases}$$

The absorbing state is represented by $S^0 = B_2$ which is a column matrix.

- The response time of the service process, $E(S) = -\boldsymbol{\alpha} S^{-1} e$.
- Hence the expected service rate $\mu_s = \frac{1}{E(S)}$.
- **Theorem**:*The queueing system is stable when $\lambda < \mu_s$.*

V. Stationary Distribution

The stationary distribution, under the condition of stability, $\lambda < \mu_s$ of the model, has Matrix Geometric solution. Let $\chi=(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots)$ be the steady state probability vector of the Markov chain $\{Z(t), t \geq 0\}$. Each $\mathbf{x}_i$, $i > 0$ are vectors with $(2 + m + n)K$ elements. We assume that $\mathbf{x}_2 = \mathbf{x}_1.R$, and $\mathbf{x}_i = \mathbf{x}_1.R^{i-1}$, $i \geq 2$, where R is the minimal non- negative solution to the matrix quadratic equation

$R^2A_2 + RA_1 + A_0=0$.

From $\chi Q=0$ we get

$\mathbf{x}_0B_0 + \mathbf{x}_1B_2 = 0$.

$\mathbf{x}_0B_1 + \mathbf{x}_1(A_1 + RA_2) = 0$.

Solving the above two equations we get $\mathbf{x}_0$ and $\mathbf{x}_1$ subject to the normalizing condition $\mathbf{x}_0e + \mathbf{x}_1(I - R)^{-1}e=1$. The stationary distribution, under the condition of stability, $\lambda < \mu_s$ of the model, has Matrix Geometric solution. Let $\chi=(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots)$ be the steady state probability vector of the Markov chain $\{Z(t), t \geq 0\}$. Each $\mathbf{x}_i$, $i > 0$ are vectors with $(2 + m + n)K$ elements. We assume that $\mathbf{x}_2 = \mathbf{x}_1.R$, and $\mathbf{x}_i = \mathbf{x}_1.R^{i-1}$, $i \geq 2$, where R is the minimal non- negative solution to the matrix quadratic equation $R^2A_2 + RA_1 + A_0=0$.

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Expected number of interruptions during the service of any cus- tomer

Let $N'(t)$ be the number of interruptions due to first m environmental factors during the service of a particular customer at time t. $S(t)$ be the status of the server at time t.

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$$S(t) = \begin{cases} 1, & \text{when service is going on ;} \\ 2, & \text{if interrupted service going on;} \\ 3, & \text{if server under repair} \end{cases}$$

$I_1(t)$ is the environmental factor causing interruption and $I_2(t)$ is the phase of service. Then $\{(N'(t), S(t), I_1(t), I_2(t)), t \geq 0\}$ is a Markov chain with state space $\{(r, 1, i) \cup (r, 2, j, i) \cup (r, 3, j, i); r = 1, 2, \ldots; i = 1, \ldots, K; j = 1, \ldots, m\} \cup \nabla$ where ∇ represents the absorbing state. The infinitesimal generator matrix of the process is given by

$$\hat{Q} = \begin{bmatrix} 0 & 0 & 0 & & & \\ U'_2 & U'_1 & U'_0 & & & \\ U_2 & 0 & U_1 & U_0 & & \\ U_2 & 0 & 0 & U_1 & U_0 & \\ \vdots & & & \ddots & \ddots & \ddots \end{bmatrix}$$

where U'_2 is a column matrix of order $K \times 1$. $U'_2 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \mu \end{bmatrix}$,

$$U'_1 = \begin{cases} -\beta \sum_{r=1} p_r - \mu, & \text{for } i = j; i, j = 1, \ldots, K \\ \mu, & \text{for } j = i + 1; i = 1, \ldots, K - 1 \\ 0, & \text{otherwise.} \end{cases}$$

$U'_0 = (\beta p' \otimes I_K)_{K \times Km}$.

U_2 is a column matrix of order $(1 + 2m)K \times 1$ and

$$U_2(i, 1) = \begin{cases} \mu j, & \text{for } i = Kr; r = 1, 2, \ldots, m; \\ \mu, & \text{for } i = K(2m + 1); \\ 0, & \text{otherwise.} \end{cases}$$

$$U_1 = \begin{bmatrix} D_0 & D_1 & D_2 \\ 0 & D_3 & D_4 \\ 0 & 0 & D_5 \end{bmatrix}_{(1+2m)K \times (1+2m)K}$$

$$D_0(i, j) = \begin{cases} \vartheta_r, & \text{for } i = j; i = rK + l; r = 0, \ldots, m - 1; l = 1, \ldots, K; \\ \mu_r, & \text{for } i + 1 = j, i = rK + l; r = 0, \ldots, m - 1; \\ & l = 1, \ldots, K - 1; \\ 0, & \text{otherwise.} \end{cases}$$

where $\vartheta_r = -\mu_r - \gamma_r - \delta_r$.

$$D_1(i, j) = \begin{cases} \gamma_t, & \text{for } i = j; i = (t - 1)K + l, t = 1, \ldots, m; l = 1, \ldots, K \\ 0, & \text{otherwise.} \end{cases}$$

$D_2 = \delta \otimes I_K$ and

$$D_3(i, j) = \begin{cases} -\eta_r, & \text{for } i = j; i = (r - 1)K + l, r = 1, \ldots, m; l = 1, \ldots, K \\ 0, & \text{otherwise.} \end{cases}$$

$$D_4(i, j) = \begin{cases} \eta_r, & \text{for } i = K(1 + m + (r - 1)) + j; r = 1, \ldots, m; \& j = 1, \ldots, K; \\ 0, & \text{otherwise.} \end{cases}$$

$$D_5(i, j) = \begin{cases} -\beta - \mu, & \text{for } i = j; \\ \mu, & \text{for } j = i + 1; \\ 0, & \text{otherwise.} \end{cases}$$

$$U_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ D_6 & 0 & 0 \end{bmatrix}_{(1+2m)K \times (1+2m)K}. \quad D_6 = (\beta p' \otimes I_K)_{K \times Km}$$

Let Z_k be the probability that there are exactly k interruptions during the service of a customer due to first m environmental factors.

Then $Z_k = \begin{cases} \boldsymbol{\alpha}(-U'_1)^{-1}U'_2 & \text{for } k = 0 \\ \boldsymbol{\alpha}[(-U_1)^{-1}U_0][(-U_1)^{-1}U_0]^{k-1}(-U_1)^{-1}U_2, & \text{for } k = 1, 2, 3, \ldots \end{cases}$

- The expected number of interruptions due to first m environmental factors during single service $E(I) = \sum_{k=0}^{\infty} kZ_k$.

VI. Performance Measures

The next step involves analyzing the steady-state probability vector to uncover crucial performance measures for the system. The important measures are as follows.

Expected waiting time

We consider the customer who joined as the m^{th} customer in the queue. During the time of arrival of m^{th} customer one customer in the system may be in service or the server may be under repair and other customers are waiting in the queue. So the waiting time of the tagged customer is the time until absorption of the Markov chain $W = \{(M(t), S(t), I_1(t), I_2(t)), t \geq 0\}$ where $M(t)$ is the rank of the tagged customer, $S(t)$, $I_1(t)$ and $I_2(t)$ are as defined in earlier sections. The waiting time of the tagged customer follows phase type distribution with representation $(\boldsymbol{\omega}, T)$ where

$$T = \begin{bmatrix} S & S^0\boldsymbol{\alpha} & & & & \\ & S & S^0\boldsymbol{\alpha} & & & \\ & & S & S^0\boldsymbol{\alpha} & & \\ & & & S & S^0\boldsymbol{\alpha} & \\ & & & \ddots & \ddots & \ddots \\ & & & & \ddots & \ddots \end{bmatrix}$$ $\boldsymbol{\omega}$ is the initial probability vector.

Depending on the state of the server at the time of joining, the expected waiting time of the tagged customer, $E_W^m = -S^{-1}(I - (S^0\boldsymbol{\alpha}S^{-1})^{(r-1)})(I - S^0\boldsymbol{\alpha}S^{-1})^{-1}e$.

- The expected waiting time of any customer who waits in the queue,

$$E(W) = \sum_{m=1}^{\infty} \mathbf{x}_m E_W^m$$

Other important performance measures

- Probability that the system is idle, $P(I) = \mathbf{x}_0$.
- Probability that the system is working without interruption,

$$P(WI) = \sum_{i=1}^{\infty} (\mathbf{x}_{i0}\boldsymbol{e} + \mathbf{x}_{i3}\boldsymbol{e})$$

.

- Probability that the system is under repair $P(R) = \sum_{i=1}^{\infty} \mathbf{x}_{i2}\boldsymbol{e}$.
- Probability that the system is under protection $P(p) = \sum_{i=1}^{\infty} \mathbf{x}_{i3}\boldsymbol{e}$.

- Expected number of customers in the system, $E(C) = \sum_{i=1} i\mathbf{x}_i\boldsymbol{e}.$
- Effective interruption rate, $E_{int} = \beta \sum_{i=1} \mathbf{x}_{i1}\boldsymbol{e}.$
- Effective rate of self correction, $E_{selfcorr} = \sum_{i=1}\sum_{j=1} \delta_j\mathbf{x}_{i1j}\boldsymbol{e}.$
- Effective rate of protection, $E_{protection} = \sum_{i=1}^{\infty}\sum_{j=m+1} \eta_j\mathbf{x}_{i2j}\boldsymbol{e}.$

VII. Numerical Illustrations

In this section assuming arbitrary values for the parameters, subject to stability, we obtained the numerical values for important performance measures. Let $n = 4$, $m = 2$, $\mu = 7$, $\mu_1 = 5$, $\mu_2 = 4$; $\eta_1 = 4, \eta_2 = 3, \eta_3 = 2, \eta_4 = 1, \beta = .5; \gamma_1 = 1, \gamma_2 = 2; \delta_1 = 1, \delta_2 = .5; p_1 = p_2 = p_3 = p_4 = 0.25$. The conclusion drawn are purely based on the values of input parameters.

Effect of λ on various performance measures

Table 1: Effect of λ on various performance measures

λ	$E(C)$	$P(I)$
1	0.9230	0.5012
1.5	1.9157	0.3134
2	3.5376	0.1910
2.5	6.1867	0.1166
3	10.5083	0.0716
3.5	17.1259	0.0443
4	24.7322	0.0276

As the arrival rate λ increases the expected number of customers in the system $E(C)$ increases, but probability for idleness of the server $P(I)$ decreases which are on expected lines (refer Table 1).

Effect of μ on various performance measures

Assuming $\lambda = 2$ and varying μ we get the following values for different performance measures.

Table 2: Effect of μ on various performance measures

μ	$E(S)$	$E(C)$	$P(I)$	$P(R)$	E_{int}	$E_{selfcorr}$
3	2.2897	29.2040	0.0175	0.1422	0.3322	0.0983
4	1.7508	15.5352	0.0471	0.1437	0.3364	0.0978
5	1.4180	8.0176	0.0891	0.1398	0.3281	0.0938
6	1.1917	5.0105	0.1384	0.1338	0.3147	0.0885
7	1.0279	3.5376	0.1910	0.1267	0.2986	0.0827
8	0.9037	2.6963	0.2437	0.1192	0.2814	0.0769
9	0.8063	2.16207	0.2946	0.1117	0.2641	0.0713

As the initial service rate μ increases the expected service time $E(s)$, the expected number of customers in the system $E(C)$, probability for repair $P(R)$, expected rate of interruption E_{int} and expected rate of self correction $E_{selfcorr}$ decrease but probability for idleness of the server $P(I)$ increase which are on expected lines. μ increases means number of service completion in unit time increases. So rate of self correction, rate of interruption and probability for repair in unit time reduces (see Table 2).

Effect of β on various performance measures

Table 3: Effect of β on various performance measures

β	$E(S)$	$E(C)$	$P(I)$	$P(R)$	E_{int}	$E_{selfcorr}$
.5	1.4180	8.0176	0.0891	0.1398	0.3281	0.0938
1	1.5863	11.0407	0.0688	0.2230	0.5230	0.1502
2	1.8247	16.4010	0.0476	0.31577	0.7394	0.2140
3	1.9818	20.3997	0.0373	0.3648	0.8539	0.2484
4	2.0906	23.1333	0.0314	0.3946	0.9231	0.2695
5	2.1691	24.9636	0.0277	0.4141	0.9685	0.2834

From Table 3 we note that as the interruption rate β increases effective service time $E(S)$, the expected number of customers in the system $E(S)$, probability for repair $P\ (R)$, expected rate of interruption E_{int} and expected rate of self correction $E_{selfcorr}$ increases but probability for idleness of the server $P\ (I)$ decrease which are on expected lines.

Effect of γ on various performance measures

Assuming $\gamma_1 = \gamma_2 = \gamma$ and varying over its value we get the following table for different performance measures. As the interruption clock realization rate γ increases effective service time $E(S)$, the expected number of customers in the system $E(C)$, probability for repair $P\ (R)$, expected rate of interruption E_{int} increase but probability for idleness

Table 4: Effect of γ on various performance measures

γ	$E(S)$	$E(C)$	$P(I)$	$P(R)$	E_{int}	$E_{selfcorr}$	$E_{protection}$
0.5	1.4069	7.7491	0.0915	0.1303	0.3254	0.1184	0.1627
1	1.4136	7.9371	0.0898	0.1367	0.3283	0.1030	0.1642
1.5	1.4192	8.0705	0.0887	0.1413	0.3302	0.0919	0.165
2	1.4240	8.1718	0.0878	0.1448	0.3315	0.0832	0.1657
2.5	1.4282	8.2523	0.0871	0.1475	0.3324	0.0763	0.1662
3	1.4318	8.3184	0.0865	0.1498	0.3331	0.0705	0.1665
3.5	1.4349	8.3739	0.0860	0.1517	0.33367	0.0656	0.1668

of the server $P\ (I)$ and expected rate of self correction $E_{selfcorr}$ decrease which are on expected lines. Rate of protection decrease with increase in γ. As the realization rate of interruption clock increase the server immediately goes for repair reducing the chance for self correction (see Table 4).

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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