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Applications Of Imprecise Probabilities In Operations Research

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Abstract
The theory has been generally accepted and established in recent years but a thorough introduction is required to make the concept accessible to a wider audience. Probabilities provide a detailed introduction to incorrect likelihoods, including hypotheses and implementations that illustrate the current state of the art. Important reading for academic researchers, institutes of science and other organisations and professionals working in fields such as risk management and engineering. Imprecise probability generalizes probability theory to require partial probability specifications and applies where information is sparse, implicitly or in dispute, where it is difficult to define a uniquely unique probability distribution. The theory seeks to more accurately reflect the information available. There is a long history of the concept of using unrefined probabilities. The first systematic diagnosis dates from the mid-19th century at least. The theory has been that strongly in the late 1990s, and the term imprecise probability was influenced by detailed foundations.

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I. Introduction

With its principle of buying and selling game costs, Walley expands the conventional subjective probability theory while the approach of Weichsel Berger generalizes Kolmogorov's axioms without forcing an interpretation. Inaccurate probability assignments to non-empty closed convex sets of probability distributions are typically considered conditions of consistency. The theory also offers a formal basis for models used in robust statistics and non-parametric figures as a welcome by-product. Also included are ideas focused on the combination of Choquet and so-called two-monotone, totally monotonous capabilities that became very common under the name of the Dempster-Shafer trust in artificial intelligence. In addition, the notion of game theory is closely related to Shafer and Vovk.

Perhaps the simplest generalization is to substitute a single probability definition for an interval. Lower and high odds, suggested by $\underline{P}(A)$ and $\overline{P}(A)$ or more generally, lower and upper expectations (previsions), [4] [5] [6] [7] aim to fill this gap:

- the special case with $\underline{P}(A) = \overline{P}(A)$ for all events A provides precise probability, whilst
- $\underline{P}(A) = 0$ and $\overline{P}(A) = 1$ represents no constraint at all on the specification of $P(A)$, with

a flexible continuum in between.

Some approaches, condensed under the name non-added substance probabilities,[8] straightforwardly utilize one of these set capacities, accepting the other one to be normally characterized to such an extent that $\underline{P}(A^c) = 1 - \overline{P}(A)$, with A^c the complement of *A*.Other related concepts understand the corresponding intervals $[\underline{P}(A), \overline{P}(A)]$ for all events as the basic entity.

Mathematical Models

Therefore, although the term imprecise probability appears to have been developed in the 90s and encompasses many extensions of probability theory, even though the word unfortunate misname allows for a more reliable quantification of uncertainty than reliable probability:

- Previsions
- Random set theory
- Dempster-Shafer evidence theory
- Lower and upper probabilities, or interval probabilities
- Belief functions

- Possibility and necessity measures
- Lower and upper previsions
- Comparative probability orderings
- Partial preference orderings
- Sets of desirable gambles
- p-boxes
- Robust Bayes methods

Interpretation of Imprecise Probabilities

Walley proposed a synthesis of various theories of imprecise probability [7], but this is by no means the first attempt to formalize imprecise probabilities. Walley's language of imprecise probabilities, when it comes to probability definitions, is based on the analytical version of the Bayesian interpretation. Walley describes higher and lower chances as special cases of upper and lower chances and the Bruno de Finetti gambling system. In simple terms, the lower forecast of a decision-making officer is the highest price the decision-maker thinks he or she will buy a game, and the top forecast is the lowest price the decision-maker is sure he or she can buy the opposite of the bet. The reward at which the decision maker is prepared to take the game on either hand, when the top and the bottom predictions are equal, they reflect together the appropriate reward for the decision-maker. A fair price contributes to correct probabilities.

The discrepancy between the upper and lower predictions of a decision maker or the gap between accuracy and inaccuracy is the main disparity between theories regarding probabilities. These gaps naturally occur in betting markets which, due to asymmetric knowledge, are financially illiquid. Henry Kyburg also repeatedly makes this gap because of his probabilities intervals, although he and Isaac Levi also provide other reasons for intervals or distributions which reflect believing states.

Issues with Imprecise Probabilities

One difficulty with unknown probabilities is that the use of an interval is often unilateral, rather than a broader or narrower degree of caution or audacity. It could be a level of trust, level of fluidity, or acceptance threshold. This is not so much a concern at lower and upper intervals deriving from a variety of probability distributions, e.g. a set of priors followed by conditioning on each part of the set. However, it may contribute to the question as to why some distributions are included and others are not included in the priors.

Another point question is why, rather than a single number, one could be right about two numbers, one lower bound and one upper bound. This question can be purely theoretical, provided that the power of a model with intervals is intrinsically higher than a model with points of view. It raises concerns about unsuitable claims for accuracy at endpoints as well as point values.

More realistic is what kind of decision theory will take advantage of unreliable probabilities. Yager's thesis persists for fuzzy measures. Levi's works are instructive for convex distribution sets. The method asks if it is better to take a decision than to simply follow the average or a Hurwicz law to regulate the boldness of the interval. In the literature there are other methods.

Robust Solution Concept

Consider a decision problem under uncertainty where the decision is based on the maximization of a scalar (real valued) outcome. The simplest representation of uncertainty depends on a finite set $\Omega\ (|\Omega| = m)$ of predefined scenarios. The finaloutcome is uncertain and only its realizations under various scenarios $\omega \in \Omega$ are known. Exactly, for each scenario *w* the corresponding outcome realization is given as a function of the decision variables $y_\omega = f_\omega(\mathbf{x})$ where x denotes a vector of decision variables to be selected from the feasible set $Q \subset R^n$ of constraints under consideration. Let us define the set of attainable outcomes $A = \{\mathbf{y} = (y)_{\omega\in\Omega} : y_\omega = f_\omega(\mathbf{x})\ \forall\ \omega \in \Omega,\quad \mathbf{x} \in Q\}$. We are interested in larger outcomes under each scenario. Hence, the decision under uncertainty can be considered a multiple criteria optimization problem

$$\max\ \{\ (y_\omega)_{\omega\in\Omega}\ :\ \mathbf{y} \in A\ \}. \qquad (1)$$

From the point of view of dynamic under vulnerability, the model (1) just indicates that we are keen on augmentation of results under all situations $\omega \in \Omega$. So as to make the different target model operational for the choice help process, one needs to expect some arrangement idea balanced to the leader's inclinations

Inside the choice issues under hazard it is accepted that the specific estimations of the fundamental situation probabilities $p_\omega\ (\omega \in \Omega)$ are given or can be evaluated. This is a reason for the stochastic programming approaches where the arrangement idea relies upon the augmentation of the normal worth (the mean result)

$$\mu(\mathbf{y}) = \sum_{\omega\in\Omega} y_\omega p_\omega \qquad (2)$$

or some risk function. In particular, the risk functions $\mu_{\delta^k}(\mathbf{y}) = \mu(\mathbf{y}) - \delta^k(\mathbf{y})$ based on the downside semi deviations

$$\delta^k(\mathbf{y}) = \left[\sum_{\omega\in\Omega} \max\{\mu(\mathbf{y}) - y_\omega p_\omega, 0\}^k\right]^{1/k} \qquad (3)$$

are consistent with the second degree stochastic dominance and thereby coherent. Among them, the Mean Absolute Deviation (δ^1) related risk function can be expressed as the mean of downside distribution $\mu_{\delta^1}(\mathbf{y}) = \sum_{\omega\in\Omega} \min\{\mu(\mathbf{y}), y_\omega\} p_\omega$.

Many scholars have recently implemented the second-order quintile risk indicators in various ways. Generally, the (worst) tail mean is defined as the mean within the given (quantile) tolerance of the worst outcome. The tail mean sum is generally called tail VaR, average VaR, or Conditional VaR (where VaR reads after Value-at-risk), in the sense of risk literature, and in particular with respect to financial application. In addition, the most popular name for CVaR is now. Nonetheless, since we consider the metric without a formally defined probabilistic space with regard to distributions, we will refer it to as the tail mean. The threshold mean maximization is consistent with stochastic dominance in the second degree and complies with coherent risk calculation criteria.

For any probabilities p_ω and tolerance level β the corresponding tail mean can be mathematically formalized as follows. Having defined the right-continuous cumulative distribution function (cdf): $F_{\mathbf{y}}(\eta) = \mathrm{Prob}[y_w \le \eta]$, we introduce the quantile function $F_{\mathbf{y}}^{(-1)}$ as the left continuous inverse of the cumulative distribution function Fy:

$$F_{\mathbf{y}}^{(-1)}(\beta) = \inf\ \{\eta : F_{\mathbf{y}}(\eta) \ge \beta\} \quad \text{for } 0 < \beta \le 1.$$

By integrating $F_{\mathbf{y}}^{(-1)}$ one gets the (worst) tail mean

$$\mu_\beta(\mathbf{y}) = \frac{1}{\beta}\int_0^\beta F_{\mathbf{y}}^{(-1)}(\alpha)d\alpha \quad \text{for } 0 < \beta \le 1. \qquad (4)$$

The utter Lorenz curve's point value. Finally, the tail means are related directly to the dual risk theory of preference.

Maximization of the tail β-mean

$$\max_{\mathbf{y}\in A} \mu_\beta(\mathbf{y}) \qquad (5)$$

defines the tail β-mean solution concept. When parameter β approaches 0, the tail β-mean tends to the smallest outcome

$$M(\mathbf{y}) = \min\{y_\omega : \omega \in \Omega\} = \lim_{\beta\to 0_+} \mu_\beta(\mathbf{y}).$$

On the other hand, for $\beta = 1$ the corresponding tail mean becomes the standard mean $(\mu_1(\mathbf{y}) = \mu(\mathbf{y}))$.

Note that, due to the finite number of scenarios, the tail ˇ-mean is well defined by the following optimization

$$\mu_\beta(\mathbf{y}) = \min_{u_\omega}\left\{\frac{1}{\beta}\sum_{\omega\in\Omega} y_\omega u_\omega : \sum_{\omega\in\Omega} u_\omega = \beta,\ 0 \le u_\omega \le p_\omega\ \forall\ \omega \in \Omega\right\}. \qquad (6)$$

Problem (6) is a Linear Program for a given result vector y while it gets nonlinear for y being a vector of variables as in the tail β-mean problem (5). Notably, this trouble can be overwhelmed by a comparable LP detailing of the ˇ-imply that permits one to actualize the β-mean problem (5) with helper linear disparities. In particular, the accompanying hypothesis reviews LP model for persistent appropriations which stays substantial for a general dispersion. In spite of the fact that we present another verification which can be additionally summed up for a group of hearty arrangement ideas we consider.

Theorem 1. For any outcome vector *y* with the corresponding probabilities $p_\omega,$ and for any real value $0 < \beta \le 1,$ the tail *β-mean* outcome is given by the followinglinear program:

$$\mu_\beta(\mathbf{y}) = \max_{t, d_\omega} \left\{ t - \frac{1}{\beta} \sum_{\omega \in \Omega} p_\omega d_\omega : y_\omega \ge t - d_\omega,\ d_\omega \ge 0\ \forall\ \omega \in \Omega \right\}. \quad (7)$$

Proof: The theorem can be proven by taking advantage of the LP dual to (6). Introducing dual variable t corresponding to the equation $\sum_{\omega \in \Omega} u_\omega = \beta$ and variables d_ω corresponding to upper bounds on u! one gets the LP dual (7). Due to the duality theory, for any given vector y the tail β-mean $\mu_\beta(\mathbf{y})$ can be found as the optimal value of the LP problem (7).

The odds of the situation are always unclear or unreliable. Uncertainty can then be represented by limits (intervals) with independently varying potential probability values. We rely on this interpretation in order to describe a solid solution definition. Typically, we take into account uncertain hypercube probabilities:

$$\mathbf{u} \in U = \left\{ (u_1, u_2, \ldots, u_m) : \sum_{\omega \in \Omega} u_\omega = 1,\ \Delta_\omega^l \le u_\omega \le \Delta_\omega^u\ \forall\ \omega \in \Omega \right\} \quad (8)$$

where obviously

$$\sum_{\omega \in \Omega} \Delta_\omega^l \le 1 \le \sum_{\omega \in \Omega} \Delta_\omega^u.$$

With the emphasis on the mean result as the primary measure of device performance, we get the robust mean solution principle

$$\max_{\mathbf{y}} \min_{\mathbf{u}} \left\{ \sum_{\omega \in \Omega} u_\omega y_\omega : \mathbf{u} \in U,\ \mathbf{y} \in A \right\}. \quad (9)$$

However, provided that all limitations of achieved set A remain unchanged while the probabilities are disturbed, the robust medium solution can be reprogrammed as

$$\max_{\mathbf{y} \in A} \min_{\mathbf{u} \in U} \sum_{\omega \in \Omega} u_\omega y_\omega = \max_{\mathbf{y} \in A} \left\{ \min_{\mathbf{u} \in U} \sum_{\omega \in \Omega} u_\omega y_\omega \right\} = \max_{\mathbf{y} \in A} \mu^U(\mathbf{y}) \quad (10)$$

where

$$\begin{aligned} \mu^U(\mathbf{y}) &= \min_{\mathbf{u} \in U} \sum_{\omega \in \Omega} u_\omega y_\omega \\ &= \min_{u_\omega} \left\{ \sum_{\omega \in \Omega} y_\omega u_\omega : \sum_{\omega \in \Omega} u_\omega = 1,\ \Delta_\omega^l \le u_\omega \le \Delta_\omega^u\ \forall\ \omega \in \Omega \right\} \end{aligned} \quad (11)$$

represent the worst case mean outcomes for given outcome vector $\mathbf{y} \in A$ with respect to the probabilities set *U*.

For different risk functions used instead of the mean, similar solid solution principles can be developed. The corresponding robust tail is used to optimize the CVaR β-mean solution can be expressed as

$$\max_{\mathbf{y} \in A} \mu_\beta^U(\mathbf{y}) \quad (12)$$

Where,

$$\mu_\beta^U(\mathbf{y}) = \min_{\mathbf{u} \in U} \min_{u'_\omega} \left\{ \frac{1}{\beta} \sum_{\omega \in \Omega} y_\omega u'_\omega : \sum_{\omega \in \Omega} u'_\omega = \beta,\ 0 \le u'_\omega \le u_\omega\ \forall\ \omega \in \Omega \right\}. \quad (13)$$

represents the worst case tail ˇ-mean outcome for given outcome vector $\mathbf{y} \in A$ with respect to the probabilities set *U*.

Portfolio Optimization

The problem we are considering with portfolio optimization follows the original formulation of Markowitz and relies on single investment model duration. An investor allocates the capital to different securities at the beginning of the transaction, assigning each security a non-negative (capital share) weight. Let $J = \{1, 2, \ldots, n\}$ denote a set of securities considered for an investment. For each security $j \in J$, its rate of return is represented by a random variable Rj with a given mean $\mu_j = \mathbb{E}\{R_j\}$. Further, let $\mathbf{x} = (x_j)_{j=1,\ldots,n}$ Note the vector of decision variables xj that represents portfolio weights. xj The weights must comply with a range of restrictions for a portfolio. The best way to evaluate a workable set *Q* is to render weights one and not negative (short sales are not permitted), i.e.

$$Q = \left\{ \mathbf{x} : \sum_{j \in J} x_j = 1, \quad x_j \geq 0 \quad \forall j \in J \right\}. \tag{14}$$

Hereafter, we examine the set *Q* with restrictions (14) in detail. Nonetheless, the results presented can be easily generalized to a general realistic LP as a system of linear equations and inequalities, thus permitting short sales, maximum limits on single shares or portfolio structure limits facing a real-life investor to be included.

$$y_\omega = \sum_{j \in J} r_j^\omega x_j. \tag{15}$$

Following Theorem and taking into account (15), for any arbitrary intervals $[\Delta_\omega^l, \Delta_\omega^u]$ (for all $\omega \in \Omega$) of probabilities, the corresponding robust portfolio optimization problem (10) can be given by the following LP problem:

$$\begin{aligned} \max_{\mathbf{x}, \mathbf{y}, t, d_\omega^u, d_\omega^l} \quad & t - \sum_{\omega \in \Omega} \Delta_\omega^u d_\omega^u + \sum_{\omega \in \Omega} \Delta_\omega^l d_\omega^l : \\ \text{s.t.} \quad & \sum_{j \in J} x_j = 1, \quad x_j \geq 0 \qquad \text{for } j \in J \\ & d_\omega^u - d_\omega^l - t + \sum_{j \in J}^{n} r_j^\omega x_j \geq 0, \; d_\omega^u, d_\omega^l \geq 0 \text{ for } \omega \in \Omega \end{aligned} \tag{16}$$

Where *t* is an unbounded variable.

As a particular case of relaxed lower bounds on scenario probabilities $(\Delta_\omega^l = 0 \; \forall \omega \in \Omega)$, , following Corollary 2 one gets the classical CVaR portfolio optimization model

$$\begin{aligned} \max_{\mathbf{x}, \mathbf{y}, t, d_\omega} \quad & t - \frac{1}{\beta} \sum_{\omega \in \Omega} p_\omega d_\omega \\ \text{s.t.} \quad & \sum_{j \in J} x_j = 1, \quad x_j \geq 0 \qquad \text{for } j \in J \\ & d_\omega - t + \sum_{j \in J} r_j^\omega x_j \geq 0, \; d_\omega \geq 0 \text{ for } \omega \in \Omega \end{aligned} \tag{17}$$

with probabilities $p_\omega = \Delta_\omega^u / \sum_{\omega \in \Omega} \Delta_\omega^u$ and the tolerance level $\beta = 1 / \sum_{\omega \in \Omega} \Delta_\omega^u$.

Except from the corresponding portfolio constraints (14), model (17) contains m nonnegative variables d_ω plus single variable t and m corresponding linear inequalities. Hence, its dimensionality is proportional to the number of scenarios m. Exactly, the LP model contains $m + n + 1$ variables and $m + 1$ constraints. It does not cause any computational difficulties for a few hundreds scenarios as in many computational analysis based on historical data, However, in the case of more sophisticated simulation models employed for scenario generation one that get many thousand scenarios. This increasing lead to the LP model (27) with huge number of variables and constraints thus decreasing the computational efficiency of the model.

The dual model (13) helps one to formulate the corresponding robust portfolio optimization problem (10), for any arbitrary intervals of probabilities (8), as the following LP issue:

$$
\begin{aligned}
&\min_{\mathbf{u},q} \; q \\
&\text{s.t.} \;\; q - \sum_{\omega \in \Omega} r_j^{\omega} u_{\omega} \geq 0 \text{ for } j \in J \\
&\qquad \sum_{\omega \in \Omega} u_{\omega} = 1 \\
&\qquad \Delta_{\omega}^{l} \leq u_{\omega} \leq \Delta_{\omega}^{u} \quad \text{for } \omega \in \Omega.
\end{aligned}
$$

(18)

The dual model takes the following form for the particular case of the CVaR (17) model, which describes the case of relaxed lower limits.

$$
\begin{aligned}
&\min_{\mathbf{u},q} \; q \\
&\text{s.t.} \;\; q - \sum_{\omega \in \Omega} r_j^{\omega} u_{\omega} \geq 0 \quad \text{for } j = 1, \ldots, n \\
&\qquad \sum_{\omega \in \Omega} u_{\omega} = 1 \\
&\qquad 0 \leq u_{\omega} \leq \frac{p_{\omega}}{\beta} \quad \text{for } \omega \in \Omega.
\end{aligned}
$$

(19)

The dual LP model contains m variables u_{ω}, but only $n+1$ limitations (n disparities and one condition) barring the straightforward limits on u! not influencing the problem intricacy. In reality, the quantity of requirements in (19) is corresponding to the portfolio size n, in this manner it is free from the quantity of situations. Precisely, there are $m+1$ variables and $n+1$ This ensures a high computational productivity of the double model in any event, for exceptionally huge number of situations. Note that conceivable extra portfolio structure prerequisites are normally demonstrated with rather modest number of linear requirements in this manner producing modest number of extra variables in the double model. Absolutely, the ideal portfolio shares xj are not straightforwardly spoken to inside the arrangement vector of problem (29) yet they are effectively accessible as the double variables (shadow costs) for imbalances Additionally, the double model (19) might be viewed as a unique case inside the general hypothesis of double portrayals of reasonable proportions of hazard, following from conjugate duality.

II. Conclusion

We have studied the robust middle solution principle where uncertainties differ independently from one scenario likelihood to another by limit (intervals). In general, such an approach results in complex operations Analysis models with vector (probability) coefficients. However, we have shown that operations work with robust mid-solution concepts can express itself with linear help disparities similar to the tail –mean solution principle based on the average maximization in terms of the worst outcomes. In addition, the Operations Research solution for upper probability limits is the tail-mean for a suitable value. The operations research solution can be found for upper and lower limits by optimizing the properly combined mean and tail mean. Thus, a relatively stable mean solution can be represented with optimization problems that are almost close to the tail medium at arbitrary probability intervals, which can easily be enforced with auxiliary linear inequality. When considering the tail medium as the basic optimization criterion (CVaR optimization), the correct robust solution definition may be expressed as the standard tail mean with a properly specified tolerance level and rescaled probabilities for any arbitrary perturbation collection.

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