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An Examination Of The Effects Of Manipulatives On University Mathematics Students' Motivation, Retention Rate, And Mathematical Performance

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Abstract:
According to some claims, using manipulatives helps pupils perform better while also enhancing their conceptual knowledge, problem-solving abilities, and attitudes toward mathematics. The purpose of this study was to determine whether or not first-year students at the University of Guyana, Turkeyen Campus, might improve their mathematical performance by using manipulatives as cognitive tools instead of lectures.
Over ten weeks, the mathematical performance of two groups from the MTH1202 – Calculus 1 class was compared, and the lecturers' anecdotal recordings were gathered. Before receiving any instruction in that unit, these two groups—a control group and a treatment group—had to complete a pretest. After that, the treatment group was shown how to solve issues using practical manipulatives, while the control group received instruction via a conventional lecture format. Following the six weeks of teaching for both groups, a posttest was administered. This was followed by a retest which was administered four weeks after the post-test.
Many researchers believe that the absence of usage of manipulatives is a major contributing reason to the low performance of pupils. This has been confirmed by the study's results, which showed an increase in the level of performance, level of retention, and motivation for students who were exposed to manipulatives as compared to those who were not. The results showed a lot of parallels to those reported in the literature.
Key Words: *First-Year Student, Concrete Materials; Effectiveness; Lecture Method; Manipulatives*

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I. Introduction

Universities have long taught mathematics through the lecture format. Felder and Prince (2006) state that in a typical lecture, the broad principles of the subject are covered first, then practice questions build up to a final test. Answers to the following questions are first given relatively little thought: why this issue, what are some real-life uses, and what solutions may be found for its practical application? Furthermore, the only encouragement these students get is the idea that this material will be significant in the future.

To effectively plan to fulfill the requirements and interests of their students, lecturers must have a thorough understanding of how students develop. Psychology knowledge contributes to the effectiveness of the teaching and learning process. Among those whose theories are most well-known are those of Jean Piaget, Jerome Bruner, and Howard Gardner. These three theorists have all influenced schooling in some way. Because of Gardner's idea of multiple intelligences, educators have attempted to include more projects and activities that stimulate different types of intelligences. Using Piaget's theory, educators have positioned their curriculum around the level that corresponds with the cognitive growth of their students. Bruner's concept has primarily been applied to mathematics. As a result, it's important to choose instructional strategies that will push pupils to think and reason more critically. Among these techniques is the use of manipulatives.

Knowing the logic behind and mechanics of mathematical operations is known as conceptual knowledge. Understanding the guidelines and processes involved in mathematical work is known as procedural knowledge (Van de Walle, 2007). A learner will not be able to apply a valid mathematical technique outside of the context in which it was taught if they study it without truly comprehending how it operates. According to research, students are more likely to learn and remember what is taught if they are actively involved in the process rather than just watching (Gallenstein, 2003). Students must experience something to fully comprehend it; they cannot learn it by being taught (Singer & Revenson, 1997).

Students typically only retain information for a short amount of time if they are taught to complete computations mechanically or by memorization. Information associated with preexisting knowledge is retained longer and can be retrieved for use at a later time. Using manipulatives is one technique to create that connection between the lesson's content and its application in real life. A student-centered method of instruction that gives students more responsibility for their education is the use of manipulatives in lectures rather than the traditional lecture-based deductive approach.

It has been found that positive mindsets enhance students' academic ability in mathematics, hence math lecturers should assist students in cultivating positive mindsets (Good, Aronson, & Inzlicht, 2003). It is a challenge for lecturers to engage students in their lectures with this new generation of technologically savvy learners. Utilizing a manipulative will help to direct students' attention and inspire them to view learning as a problem-solving exercise utilizing something tangible.

Another way to encourage multiple intelligences is to include manipulatives in the mathematics curriculum. In addition to the logical-mathematical intellect, instruction also incorporates the altitudinal and kinesthetic intelligence. Students discussing verbally while working in groups the acts occurring or what is found via those actions is another easy way to boost verbal-linguistic and interpersonal intelligence (Martin, 1996). Additionally, students could journal their ideas and discoveries.

Sidhu (1995) lists the following as some objectives of logical thinking development in mathematics education: reasoning ability; analytical thinking, and critical thinking; the ability to make decisions and adopt a heuristic mindset; and the capacity to find solutions and proofs on one's own. It is challenging to accomplish these aims using the lecture format that is typically employed in university mathematics classes. Studies indicate that the utilization of manipulatives can aid in the formation of coherent, well-founded conceptual understandings of mathematical concepts. When using manipulatives to solve an issue, students are better able to recall their actions and articulate their thoughts (Kay|Bovalino, 2019).

According to Heddens (2005), incorporating manipulative materials into mathematics instruction can help students develop their ability to: relate mathematical symbolism to real-world situations; collaborate with others to solve issues, talk about mathematical notions and ideas, and express their mathematical reasoning; give presentations in front of large groups of people; recognize that there are numerous approaches to problem-solving; recognize that it is possible to symbolize mathematical issues in a variety of ways; and recognize that they are capable of solving mathematical problems on their own, independent of teachers' instructions.

Instead of focusing solely on increasing calculation efficiency, manipulatives are meant to improve students' understanding of mathematics (Larbi & Mavis, 2016). This is in contrast to the absorption theory, which was employed in the past and saw students as passive learners who retain information through drill, practice, memorization, and reinforcement (Cain-Caston, 1996). By employing this approach, pupils were not motivated to think creatively to improve their problem-solving skills. Not only should mathematics be taught to students, but they should also actively engage in mathematical thought. We might be able to teach the pupil a problem-solving comprehension that has previously been absent by using manipulatives.

In math classes, students who use manipulatives typically perform better than those who don't. All grade levels, skill levels, and topics show an increase in performance (Cain-Caston, 1996). Additionally, Cain-Caston (1996) discovered that manipulatives were used more frequently. the study participants' understanding of mathematics and their mathematical progress. Dorward (2002) discovered that when instructors employ tangible resources and are informed about their application, students' attitudes toward mathematics improve. Additionally, sustained use of these resources improves mathematical performance.

Clements and McMillen (1996) also discovered that the use of manipulatives improved test results for retention and problem-solving. They also observed that when instructors employ real materials and are conversant with their application, students' attitudes toward mathematics improve. According to Moch (2002), children who had previously shown little interest in mathematics were now willing and excited to participate and pick up new concepts. The children looked forward to future opportunities to explore additional concepts and loved the chance to explore and ponder via exercises utilizing manipulatives.

According to McClung (1998), the use of manipulatives in the classroom increases student and lecturer attention and engagement. In their analysis, Carbonneau & Marley (2015) found 55 papers that contrasted teaching with concrete manipulatives versus education using abstract mathematical symbols. The results showed that the physical manipulatives had small to moderate effect sizes in favor of their use and moderate to large impacts on retention. The findings of Sowell (1989), who carried out the first meta-analysis on the efficacy of physical manipulatives 25 years earlier, were corroborated and expanded upon by these studies. It has been discovered that prolonged usage of manipulatives in mathematics improves students' verbalization of mathematical thinking, discussion of notions and ideas in mathematics, connection of applications of mathematical symbolism to practical issues, collaborative learning, thinking outside the box to solve problems in numerous ways, articulating issues and solutions with a range of mathematical symbols, delivering presentations, accepting accountability for their education, and developing self-assurance in their capacity to solve mathematical issues alone, without consulting lecturers.

Manipulative investigation, especially self-directed exploration, has a favorable effect on students' attitudes toward learning and fosters an interesting learning environment in the classroom. (Moch, 2001). Making learning enjoyable was one of the advantages that multiple researchers discovered when utilizing manipulatives

(Moch, 2001). Using manipulatives increases student engagement in the course and gives them a means of expressing what they are thinking (Naiser, Wright, & Capraro, 2003). Additionally, manipulatives offer educators a useful way to learn about their pupils' thoughts (Naiser, Wright & Capraro, 2003). With the use of manipulatives, students can establish patterns and create the crucial connections that best suit their learning styles (DeGeorge & Santoro 2004; Suth & Moyer-Packenham, 2007; Bellonio, 2012).
The following inquiries will be investigated in this study:
1. Does the performance of students who use manipulatives in lectures change significantly from that of those who do not?
2. Is there a connection, if any, between the usage or lack thereof of manipulatives and students' conceptual retention?
3. Does teaching pupils with manipulatives encourage them to do math?

II. Material and Methods

Quantitative data was gathered using the Non-Equivalent Control Group approach, a quasi-experimental study approach. The experimental group and the control group, which did not include assigning people to groups at random, were two comparable groups in this design.

Figure 1: The Design of the Non-Equivalent Control Group.

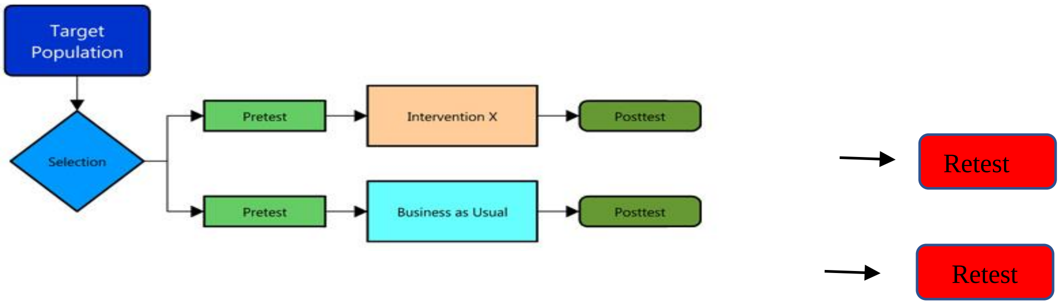

Figure 1 depicts a pre-test, post-test, and re-test which are all part of the Non-Equivalent Control Group Design. Following their selection, both groups will receive the pre-test. The chosen therapy will only be administered to the experimental group following the pre-test. The test group will experience using manipulatives in the classroom, while the control group will listen to lectures as normal. Both groups will receive a post-test after the course of treatment. There will be another test (re-test) given four weeks later. The success of the treatment (using manipulatives) will next be assessed by comparing the test group's findings with those of the control group.

Students, both male and female, registered for MTH1202 – Calculus 1 at the University of Guyana were the study's target group. 46 students were chosen at random from groups that were divided up according to instruction content. There were at least 20 pupils in each session, both male and female, chosen at random. So, the researchers used two tutorials— one serving as a control group and the other as an experimental group —that were comparable in terms of the number of male and female students.

Since cluster sampling chooses groups rather than individuals, it was the sampling technique used. Two groups: the experimental and control were randomly selected from the two instructional groups whose names were written on pieces of paper and put in a bag. The study's dependent variable is the student's test-taking performance, while the independent variable is the mathematics taught using manipulatives. The pupils' gender, age, socioeconomic status, and IQ are examples of extraneous variables, whereas motivation, exhaustion, mathematical anxiety, enthusiasm, maturity, aptitude, and illness are examples of intervening variables.

The independent and dependent variables were measured using pre-, post-, and retest tests. For both the control group and the experimental group, the same three tests were given. Before the course of therapy began, a pre-test was given to the students to determine their preparation, areas of weakness, and strengths in solving mathematical problems. One week following the conclusion of the treatment, the post-test was given to the students to gauge how much their performance may improve (dependent variable). Four weeks following the post-test, a retest was given to ascertain the retention level of both groups by looking over, evaluating, and contrasting the data. Since they are uncontrollable, the intervening and extraneous variables were not measured.

The study employed paper-and-pen tests and anecdotal records as data collection instruments. These tools were utilized to collect quantitative data to assess pupils' performance. There was consistency in the way these tests were given and graded. The pretest and posttest used comparable skills but distinct problem sets. Both exams were created to ensure their validity and applicability through the predefined standard administration and scoring of the questions, test circumstances, scoring methods, and interpretations. There were twenty (20) multiple-choice questions on each test. The University of Guyana (UG) course outline of Calculus 1's subtopics served as the basis for all of the pre-and post-test questions.

To collect qualitative data, observational notes from the lecturer were utilized in addition to the tests. To ascertain whether or not students in the investigational group are motivated to finish mathematics when they are introduced to manipulatives, the lecturer's observational notes were examined. Throughout the treatment, the students' activities, responses, and comments were recorded. This was accomplished by promptly noting significant occurrences and events as they happened.

Data analysis

Students' test results from the pre-, post-, and retest were used as quantitative data, and their results were statistically examined. The test scripts were thoroughly marked, and then each student's scores were entered into Microsoft Excel spreadsheets. Calculations of mean and standard deviation were made using the SPSS software. Moreover, correlation and T-tests were conducted with SPSS. Furthermore, observational notes from the lecturer were collected as qualitative data. This was then used to address the following questions.

Do students' performances in lectures who use manipulatives change noticeably from those of those who do not?

To answer this question, the researcher made use of the test results—both pre-and post-test. The variation in scores between the pre-and post-tests was used to calculate the difference score (gain 1). The real progress each student achieved was displayed in their Gain 1 score. A t-test was used to determine whether there exists a significant difference between the performances of the students who were exposed to manipulatives and those who were not. This test was carried out at the 5% significant level.

Does the use of manipulatives or lack thereof have any bearing on how well students retain concepts?

The results of pre-and retest were used to determine if the use of manipulatives has any bearing on student retention. Gain 2 scores (the difference between the pre-test and retest) were used. The gain 2 scores were represented on a line graph. After calculating the Gain 2 scores, the researchers used the Pearson Moment Correlation to ascertain whether there exists a relationship between student's performance on the retest and pretest.

Does using manipulatives encourage pupils to do math?

To address this question, the researcher would have obtained qualitative data. The researcher was able to ascertain whether manipulatives encourage students to complete mathematics by using the data gathered from the lecturer's observational notes. Recordings of a lecturer's observations about various facets of students' learning are called observational notes or remarks (Moore & McCabe, 2005).

For this study, the lecturer's observational notes were organized into three headings.: date, student name, and remarks or observations. When it was feasible, or as soon as the lesson was over, a description of significant incidences and events that were witnessed by students in the experimental group throughout the therapy was immediately documented.

III. Results and Discussion

Table 1: Pre-test and post-test results

Group	N	Pre-test		Post-test	
		Mean	Standard Deviation	Mean	Standard Deviation
Control	23	8.39	3.21	10.48	3.17
Experimental	23	9.09	3.15	12.96	3.06

Table 1 displays the summary of the results for both the control and experimental group

Examining Table 1, it was discovered that the experimental group's scores were nearly identical to the control group for the pretest. The experimental group's mean score was 8.39, whereas the control group's was 9.09, as shown in the table. This means that the student's performance was below 50% in both groups. The pre-test means for the experimental and control groups also showed that students' comprehension of the material was essentially the same. This was highlighted by the fact that the experimental group's standard deviation was 3.15 whereas the control group's was 3.21. As a result, the pupils' results matched the group mean fairly closely. This provided the perfect setting for carrying out the research.

Furthermore, a study of Table 1 reveals that the experimental group scored better on the post-test than the control group, as indicated by the experimental group's mean scores of 12.96 compared to the control group's mean of 10.48. The experimental group's standard deviation was 3.06, while the control group's was 3.17. The standard deviations stayed consistent.

Figure 2: Comparison between Gain I score

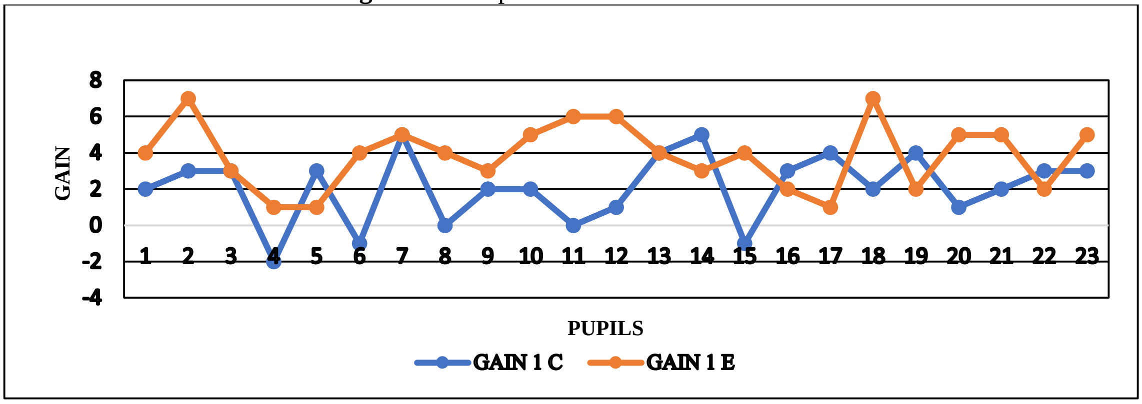

Figure 2 shows Gain I scores for the experimental and control groups

As seen in Figure 2 above, the experimental group's students performed superior to the control group. As indicated by the orange line versus the blue line, it is clear that the experimental group's pupils did better. Still, not enough data is available to conclude that student performance in the experimental group and the control group differs noticeably. The t-test was employed to determine whether there was a significant difference between the two means. The significant difference in the student's performance between the two groups was statistically analyzed using the t-test. Table 2 presents a synopsis of this analysis.

Table 2: t-test results

The t-test for equality of means in independent samples								
Group	No.	df	Mean	Mean Diff.	SD	p-value	95% Confidence Interval	
							Lower	Upper
Experimental	23		3.87		1.817			
Control	23	44	2.09	1.78	1.905	0.0023	0.67370	2.88630

Table 2 shows the t-test results to determine the significant difference

The independent samples t-test, which demonstrated this with 95% confidence, revealed from Table 2 that students exposed to manipulatives had gained scores that lay between a lower limit of 0.67370 below the mean and an upper limit of 2.88630 above the mean. Any *p-value* less than 0.05 is statistically significant because the t-test was conducted with a 5% significance threshold. As a result, the means differ from one another.

The t-test yielded a *p-value* of 0.0023, which is less than 0.05 and indicates that the research's conclusions are significant, as Table 2 illustrates. As a result, at the 5% significant level, there is sufficient evidence to conclude that student performance in the experimental group differs substantially from that of the control group (*p-value* = 0.0023).

The second research question aimed to determine the degree to which using manipulatives will aid students in remembering material more effectively than traditional lecture style. Four weeks after the experimental group's treatment was finished, a retest was given concurrently to the control group and the experimental group to aid in this analysis. Table 3 presents the findings.

Table 3: The retest scores' mean and standard deviation

Group	**N**	**Re-test**	
		Mean	**Standard Deviation**
Control	23	13.02	2.11
Experimental	23	16.89	2.45

Table 3 shows the retest mean and standard deviation for both groups

The experimental group's mean score was greater than the control group, as Table 3 demonstrates. According to this, it would indicate that the performance of students in the group that were exposed to manipulatives was better than those who were not. Gain II scores were calculated by computing the difference between the retest and the pretest scores. The researchers then compared the Gain I score with the Gain II scores. The relationship between is made evident in Figures 3 and 4.

Figure 3: The control group's Gain I and Gain II scores

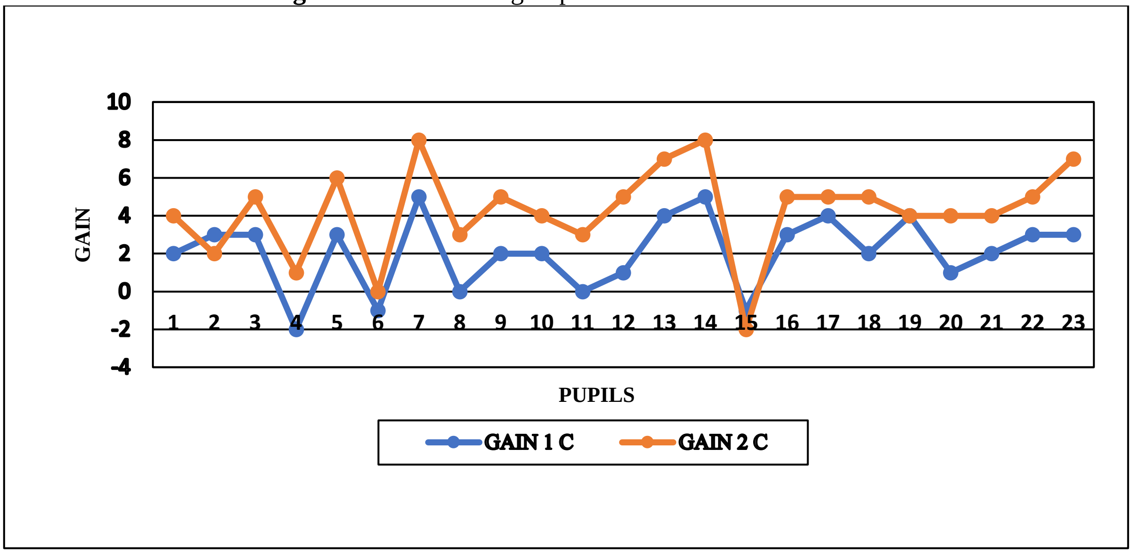

Figure 3 illustrates the contrast between the control group's Gain I and Gain II scores

Figure 3 demonstrates that the control group's Gain II scores exceeded their Gain I level, suggesting that the pupils in the control group understood the subject.

Figure 4: The experimental group's Gain I and Gain II scores

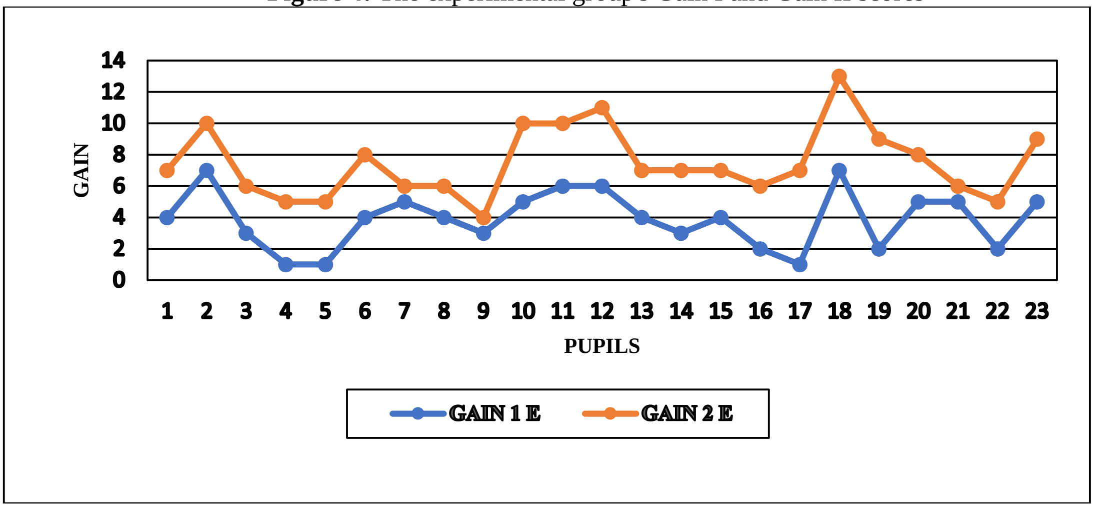

Figure 4 provides a comparison of the experimental group's Gain I and Gain II scores.

Figure 4 illustrates students in the experimental group had better Gain II scores than Gain I scores, indicating that the group's conceptual retention was strong.

As seen in Figures 3 and 4, students from both groups exhibited fair retention. Thus, there was a need to conduct a further analysis to determine if there is a relationship between Gain 1 and Gain II. To do this the researchers employed the Pearson Product Moment Correlation (PPMC) test using the SPSS software.

Table 4: The PPMC results

Group	**N**	Pearson Product Moment Correlation	
Control	23	Gain 1 and Gain II	0.82
Experimental	23		0.72

Table 4 shows the PPMC results of the Gain 1 and Gain II scores

Table 4 showed that there was a 0.72 correlation in the experimental group and a 0.82 correlation in the control group between Gain I and Gain II scores. Strong correlations between Gain I and Gain II were shown by correlation coefficients of 0.82 in the control group and 0.72 in the experimental group, respectively, indicating significant relationships between the two.

Given that both groups showed a strong association, we may conclude that there is no meaningful connection between students' conceptual retention and their use or lack thereof of manipulatives. Therefore, based on the PPMC value analysis, there is no significant correlation between the student's ability to retain information in this experiment and the usage or lack thereof of manipulatives.

The third research question investigated whether or not using manipulatives helps to motivate pupils to complete mathematics. Throughout the six lessons, close observation of the students in the experimental group provided the information needed to answer this question. Students were able to reason abstract ideas, learn new concepts, and create their knowledge when manipulatives were used in lectures. Students were free to express themselves during the therapy. They displayed a relief of tension and panic. They accepted the use of manipulatives with ease. Students who participated in the class also showed excitement and curiosity.
The real identities of the students were withheld to maintain the anonymity of the anecdotes.

Motivation

"I enjoyed class today." "Sir can you give us some more." "Please give us some for to do home." "Don't do it yet sir, give me couple minutes." "Thank you, Sir. This is fun and exciting stuff." "I can do this all day."

The impetus behind students' mathematical endeavors is known as motivation. Students who are driven are eager to contribute to class activities and put in a lot of effort to reach their objectives. The researchers saw that pupils were very driven to complete their arithmetic assignments during the treatment. They were excited to investigate objects and find solutions.

According to Cain-Caston (1996), mathematics classes have evolved from the tedious pencil and paper tasks they once were to a joyful and engaging activity that many students now look forward to. The teaching of the same topics has, for some reason, turned into an enjoyable activity rather than a work. Manipulatives enable instructors and students to convey their ideas by providing them with something tangible (Ball, 1992; Kilpatrick, Swafford, & Findel, 2001; Moyer, 2001).

The researchers offered words of encouragement when activities were successfully finished, and they were urged to try again when they didn't succeed. According to Moch (2002), pupils who had not been highly interested in mathematics before were suddenly eager to participate and acquire new concepts. The children looked forward to future opportunities to explore additional concepts and loved the chance to explore and ponder via exercises utilizing manipulatives.

Self-directed inquiry in particular, when combined with manipulatives, produces a dynamic learning environment and helps children develop a positive attitude toward learning. Making learning enjoyable was one of the advantages that multiple researchers discovered when utilizing manipulatives (Moch, 2002).

Discovery

"Wow. Me never knew calculus was so simple." "Sir, I now get it." "I never knew this before." "These materials gimme a clearer picture."

Because they are naturally curious, students are eager to investigate new ideas and circumstances. They can create knowledge that sticks in their memories longer by learning new mathematical concepts on their own. Students were given manipulatives by the researchers to work with to learn new ideas.

According to research, students are more likely to understand and retain what is taught if they are engaged in the process of learning actively rather than just watching (Gallenstein, 2003). According to Gardener (1983), the degree to which an individual's intellect develops depends in large part on the quantity of exposure to intelligence-related items. Furthermore, Singer & Revenson (1997) state that a learner must experience something to fully comprehend it—rather than merely being taught about it.

Positive Mindset

"We done Sir, challenge us." "Is this the big calculus that everybody frighten?" "I got all right."

According to Good, Aronson, and Inzlicht (2003), it has been discovered that certain mindsets improve pupils' mathematical academic performance. In addition, to improve students' attitudes toward mathematics, manipulatives can be used in education to assist students retain information and enhancing test performance by using concrete materials (Sowell, 1989).

Throughout the treatment, the researchers gave the students chances to succeed, which helped them develop a positive outlook. Furthermore, the researcher allowed students to work in groups so that less capable pupils might receive the support they needed to succeed. When students finished assigned tasks, they felt a sense of accomplishment. Students were able to experience the thrill of discovery, a positive mindset, and motivation by employing manipulatives in mathematics lectures. This made them enthusiastic about studying mathematics.

V. Conclusion

The results showed that using manipulatives helped pupils perform better while tackling calculus-related mathematical tasks. For both groups (those exposed to manipulatives and those who were not exposed) the post-test scores showed improvement. On the post-test, students who learned mathematics through manipulatives performed better than those who learned it through typical lecture techniques. Thus, the usage of manipulatives affected the performance of the pupils. Based on the analysis done after the retest, students exposed to manipulatives retained the concepts almost as well as those who were not. Therefore, there is no discernible relationship between the use of manipulatives or not and the student's retention of the material in this experiment. Pupils who learned using manipulatives demonstrated enthusiasm and curiosity. They developed a positive mindset; they were motivated and they experienced the joy of discovery. Thus, using manipulatives encourages students to do mathematics.

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