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Variable-Scale Methods For Seismic Wave Propagation

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ABSTRACT
We present a Variable-scale methods (VSM) with finite-difference methods (FDM) for modelling 2-D acoustic wave propagation in homogeneous media. We used the variable function as follows: $\xi=\alpha \tanh(\beta x)$ where α, β positive constants. In this method the stability condition is given by $\frac{c\Delta t}{h}=\frac{1}{\sqrt{2}A(\xi)}$ *, $A(\xi)$ is locally constant. We present one medium, two media and multimedia result. These two method seismograms show acceptable results. Variable-scale method is useful for one borehole geometry.*

Key words : ***F****inite-difference methods, Seismograms, seismic wave propagation, variable scale methods.*

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I. INTRODUCTION

Various numerical methods for modelling seismic wave propagation problems have been improved, including the FDM (Alterman and Karal (1968); Kelly et al. (1976)), FEM (Chen (1984)), Fourier or pseudospectral methods (Kosloff (1986)), and Hybrid methods (Gazdag (1981)). The FDM is a most powerful simple direct method, and well understood in solving the equations of motion widely used in seismic wave propagation problems. We are required to choose the correct grid parameters for the numerical solution so that unacceptable results can be avoided. We must pay attention to stability, consistency, and convergence and grid dispersion (Alford et al. (1974); Boore (1972); Virieux (1986)). The seismic wave propagation problems are normally solved for an infinite medium, but because of the finite core of computers and finite time available the FDM solution can only be obtained at a finite number of points, thus it is necessary to introduce domain boundaries to obtain a finite model. We employ well known absorbing boundary conditions (ABCs) which are namely the Reynolds ABCs. The specific numerical schemes that will be employed for the numerical results of wave propagation problems. The methods apply to acoustic wave propagation displacement waves.

Absorbing boundaries based on the formulations of Reynolds (1978) and Clayton and Engquist (1977) were used to reduce reflections from the boundaries. They compared the sharp interface model with a discrete-wavenumber model. Staggered grid models were developed by Virieux (1986) in 2-D, and used on Weathered-layer and Corner-edge problems to model P-SV waves in an inhomogeneous plane. Staggered grid algorithms are recognised as having better experimental stability and accuracy than centred ones as witnessed by Levander (1988). A further useful development of the staggered grid approach is given by Luo & Schuster (1990) who introduced a 2-D formulation in which the stress-velocity, (p-v), fields are basically included as functions of a single field of vector displacement values. Improvement in accuracy and stability over a conventional centred scheme is demonstrated, but, a smaller number of values needs to be stored (at the expense of more calculation at each time-step) to make the numerical simulation more memory effective.

II. VARIABLE SCALE METHOD

In this section we employ a new method for the FDM, which is useful for boreholes in Cartesian and cylindrical coordinates. It is inconvenient to solve the borehole problems with the standard FDM, because the borehole radius is very small. Therefore, we develop the variable scale method (VSM) that is superior to all other methods in terms of taking a small grid interval near the borehole and larger grids further from it. We obtained satisfactory results using this method for standard problems. We employ the variable function as follows.

$$\xi=\alpha \tanh(\beta x) \qquad (1)$$

where α, β positive constants.

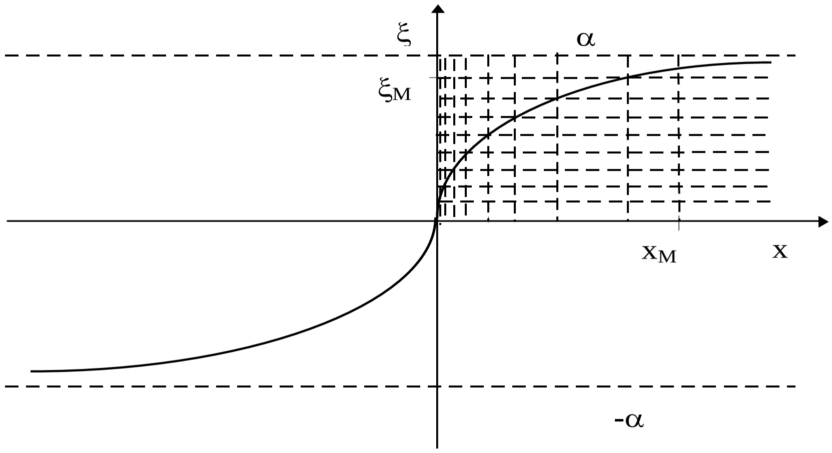

Figure 1. Variable scale geometry

The 2-D wave equation is converted to the scaling coordinates and is now given as

$$\rho\frac{\partial^2 \mathrm{u}}{\partial \mathrm{t}^2} = \mathrm{v}_\mathrm{p}^2 \frac{\partial}{\partial \xi}\left(\mathrm{A}(\xi)\frac{\partial \mathrm{u}}{\partial \xi}\right) + \mathrm{v}_\mathrm{s}^2 \frac{\partial^2 \mathrm{u}}{\partial \mathrm{z}^2} + \left(\mathrm{v}_\mathrm{p}^2 - \mathrm{v}_\mathrm{s}^2\right)\mathrm{A}(\xi)\frac{\partial}{\partial \xi}\frac{\partial \mathrm{w}}{\partial \mathrm{z}} \tag{2}$$

where

$A(\xi) = dx/d\xi = (\beta/\alpha)(\alpha^2-\xi^2)$

In this method the stability condition is given by

$$\frac{c\Delta t}{h} \le \frac{1}{\sqrt{2}A(\xi)} \tag{3}$$

based on an approximate von Neumann stability and assuming $A(\xi)$ is locally constant. To solve the partial differential equations that is approximated using finite differences with VSM. Returning to the Cauchy problem involving the wave equation (2), The VSM replacement of equation (2) on a rectangular grid is

$$\begin{aligned} u_{i,j}^{n+1} &= 2u_{i,j}^{n} - u_{i,j}^{n-1} + q^2 A_i\left(A_{i+1}u_{i+1,j}^{n} - (A_{i+1}+A_i))u_{i,j}^{n} + A_i u_{i-1,j}^{n}\right) \\ &+ 2q^2 B_j\left[B_{j+1}u_{i,j+1}^{n} - (B_{j+1}+B_j)u_{i,j}^{n} + B_j u_{i,j-1}^{n}\right] + q^2 f(t) \end{aligned} \tag{4}$$

The approximate absorbing boundary conditions for rectangular grid is

Thus at $\mathrm{x} = \pm \mathrm{a}$:

$$\frac{p}{v_p^2}\frac{\partial^2 u}{\partial t^2} \pm A(\mp a)\left(\frac{p+1}{v_p}\frac{\partial^2 u}{\partial t \partial x} + \frac{\partial^2 u}{\partial x^2}\right) = 0 \tag{5}$$

and at $\mathrm{z} = \pm \mathrm{a}$:

$$\frac{p}{v_p^2}\frac{\partial^2 u}{\partial t^2} \pm B(\mp a)\left(\frac{p+1}{v_p}\frac{\partial^2 u}{\partial t \partial z} + \frac{\partial^2 u}{\partial z^2}\right) = 0 \tag{6}$$

There are modifications for the VSM boundary conditions. Using finite difference formulas for equations (5) and (6)

$$\begin{aligned} &\mathrm{u}_{0,\mathrm{j}}^{\mathrm{n+1}} = \mathrm{u}_{0,\mathrm{j}}^{\mathrm{n}} + \mathrm{u}_{1,\mathrm{j}}^{\mathrm{n}} - \mathrm{u}_{1,\mathrm{j}}^{\mathrm{n-1}} + \mathrm{A}(0)\mathrm{q}\left(\mathrm{u}_{1,\mathrm{j}}^{\mathrm{n}} - \mathrm{u}_{0,\mathrm{j}}^{\mathrm{n}} - \left(\mathrm{u}_{2,\mathrm{j}}^{\mathrm{n-1}} - \mathrm{u}_{1,\mathrm{j}}^{\mathrm{n-1}}\right)\right) \\ &1 \le \mathrm{j} \le \mathrm{N},\ 1 \le \mathrm{n} \le \mathrm{J},\ \mathrm{A}(0) = 0.15963 \\ &\mathrm{u}_{\mathrm{M},\mathrm{j}}^{\mathrm{n+1}} = \mathrm{u}_{\mathrm{M},\mathrm{j}}^{\mathrm{n}} + \mathrm{u}_{\mathrm{M-1},\mathrm{j}}^{\mathrm{n}} - \mathrm{u}_{\mathrm{M-1},\mathrm{j}}^{\mathrm{n-1}} - \mathrm{A}(\mathrm{M})\mathrm{q}\left(\mathrm{u}_{\mathrm{M},\mathrm{j}}^{\mathrm{n}} - \mathrm{u}_{\mathrm{M-1},\mathrm{j}}^{\mathrm{n}} - \left(\mathrm{u}_{\mathrm{M-1},\mathrm{j}}^{\mathrm{n-1}} - \mathrm{u}_{\mathrm{M-2},\mathrm{j}}^{\mathrm{n-1}}\right)\right) \\ &1 \le \mathrm{j} \le \mathrm{N},\ 1 \le \mathrm{n} \le \mathrm{J},\ \mathrm{A}(\mathrm{N}) = 0.14966 \end{aligned}$$

(7)

$$
\begin{aligned}
&u_{i,0}^{n+1} = u_{i,0}^{n} + u_{i,1}^{n} - u_{i,1}^{n-1} + B(0)q\left(u_{i,1}^{n} - u_{i,0}^{n} - \left(u_{i,2}^{n-1} - u_{i,1}^{n-1}\right)\right) \\
&1 \le i \le M,\ 1 \le n \le J,\ B(0) = 0.15963 \\
&u_{i,N}^{n+1} = u_{i,N}^{n} + u_{i,N-1}^{n} - u_{i,N-1}^{n-1} - B(N)q\left(u_{i,Nj}^{n} - u_{i,N-1}^{n} - \left(u_{i,N-1}^{n-1} - u_{i,N-2}^{n-1}\right)\right) \\
&1 \le i \le M,\ 1 \le n \le J,\ B(N) = 0.14966
\end{aligned} \tag{8}
$$

III. COMPARISON OF THE FDM AND VSM RESULTS

In this section, we compare directly the FDM, VSM, Reynolds and Clayton-Engquist ABCs. Accuracy of the FDM seismograms is mainly affected by grid dispersion. Figure (2) shows the synthetic seismogram with Dirichlet boundary conditions calculated by the FDM and VSM for single layer model. There is a close match of the two waveforms; wave traces are nearly identical, and there is overall good agreement between the two results. Figure (3) presents the synthetic seismogram calculated for Reynolds and Clayton-Engquist ABCs by the FDM and VSM. There is an excellent match between the two methods, with only significant difference being a slight amplitude difference in the direct wave pulses in the Clayton-Engquist ABCs.

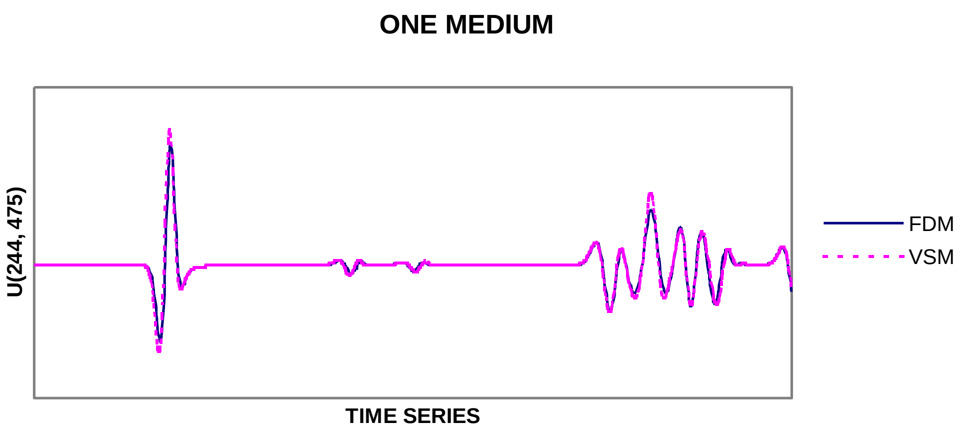

Figure 2. Synthetic seismograms obtained from the standard FDM and VSM with Dirichlet boundary conditions solutions for single medium.

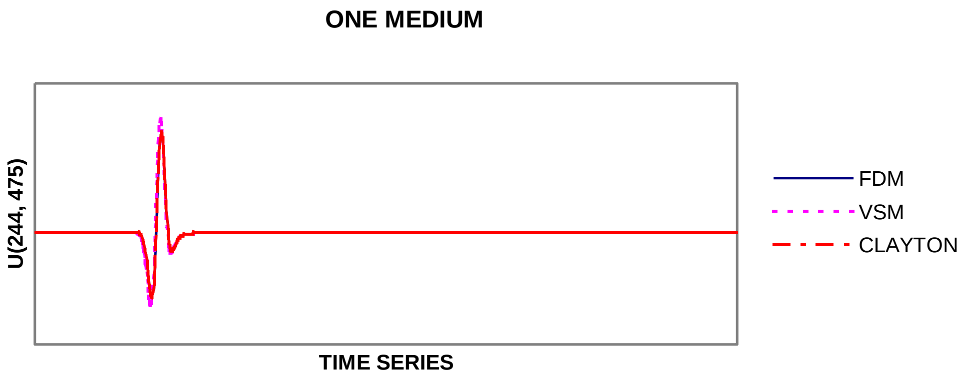

Figure 3. Synthetic seismograms obtained from the standard FDM and VSM with ABCs solutions for single medium.

Figure 4. shows the synthetic seismograms calculated by the standard FDM and VSM with Dirichlet boundary conditions for two media. There is good agreement again between the seismograms generated by the two different methods. We now compare with the standard FDM, VSM for ABCs in Figure 5. We find there is a good agreement between FDM, VSM and Clayton-Engquist ABCs but the amplitude difference exhibits the main contrast.

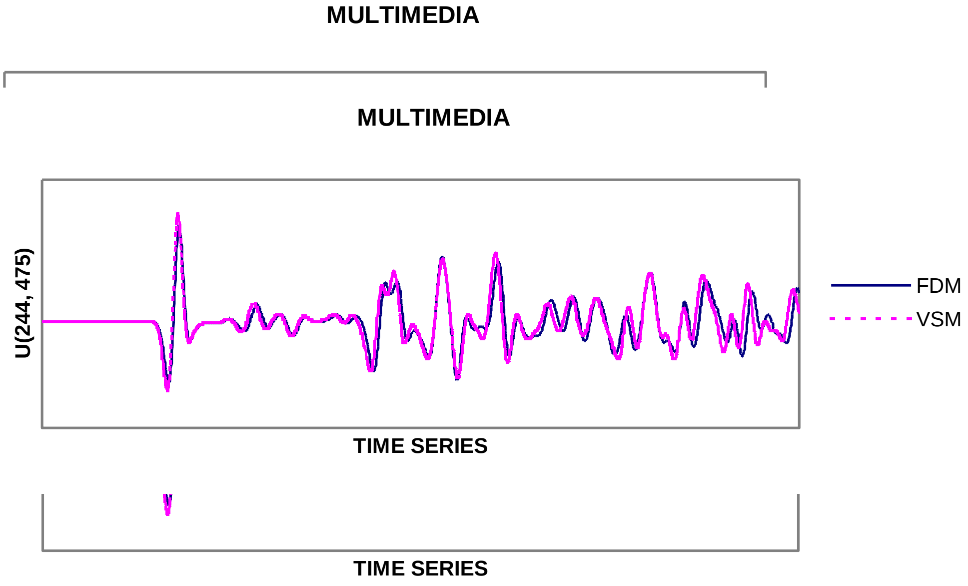

Figure 4. Synthetic seismograms obtained from the standard FDM and VSM with Dirichlet boundary condition solutions for two media.

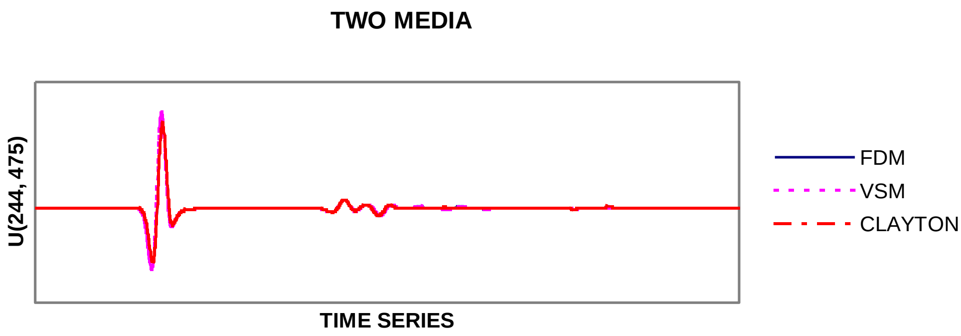

Figure 5. Synthetic seismograms obtained from the standard FDM and VSM with ABCs solutions for two media.

Figure 6. shows a comparison of the synthetic seismograms generated by the standard FDM and VSM for Dirichlet boundary conditions for multimedia.

Figure 6. Synthetic seismograms obtained from the standard FDM and VSM with Dirichlet boundary conditions solutions for multimedia.

The comparison, in Figure 7. shows the FDM, VSM for ABCs for the multimedia. There is excellent agreement between the FDM and VSM and Clayton-Engquist ABCs apart from amplitude difference.

IV. CONCLUSION

We solved the 2-D acoustic wave propagation problem numerically with Dirichlet and absorbing boundary conditions in Cartesian coordinates for single layer, two layered, and multi layered media based on the grid / wavelength criteria for best effect. The VSM results were found to be as good as those from the standard FDM results. The Reynolds ABCs were modified for Cartesian coordinates and was found to be most effective.

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