

# Mathematical prototype for the control of malaria by interrupting the life cycle of the Anopheles mosquito through the use of biological enemies in the larva, pupa and adult stages

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## Abstract

Mathematical prototype to fight malaria by interrupting the life cycle of the Anopheles mosquito through the use of biological enemies in the larval, pupal and adult stages has been derived to eradicate larvae, pupae and adult Anopheles mosquitoes using natural predators. The new model is a control flowchart of the predator-prey interaction model in the mosquito life cycle, considering an open population of mosquitoes and predators. These models provide a solid understanding of malaria control in our environment, especially when models are based on vector population ecology and a solid understanding of transmission-relevant parameters and variables. Model equations were derived using parameters and variables from the model. Stability analysis of free equilibrium states was analyzed simultaneously using equilibrium point, Maple software, elimination and substitution methods. The number of larvae that pupate is almost zero, and the number of pupae that turn into adults is minimal, and the number of adults that escape to the vector stage is negligible. which means that the life cycle could be disrupted at larval, pupal and adult stages with the introduction of natural enemies, with the natural implication there will be no adult Anopheles mosquito for transmission of the malaria. The contribution of this research to knowledge is to produce the model or the mathematical formula and the biologically sound methods that will contribute to the eradication of the adult Anopheles mosquito, which will also lead to the eradication of malaria in our society.

**Keywords:** Mathematical prototype; Biological enemies; Anopheles; Control; Malaria

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## I. Introduction

The *Anopheles* vector system in Nigeria and of course in sub-Saharan Africa is probably the strongest that exists for human *Plasmodium*. Contact with human vectors, particularly *An. gambiae* s.l., shows remarkable stability and flexibility, resulting in extremely high vaccination rates under different seasonal and geographic ecological conditions (Mokuolu et al., 2018). Malaria remains a leading cause of death and disease in most tropical regions of the world, where it is endemic in 106 countries. In 2010, out of a total of 216 million cases of malaria, around 81% occurred in Africa and 13% in Southeast Asia. The majority (91%) of the estimated 665,000 malaria deaths occur in Africa and primarily affect children under the age of five (86%). In America in 2010 there were more than 670,000 confirmed cases of malaria with 133 deaths from malaria. The transmission is active in 21 countries and puts approximately 20% of the US population at risk. Malaria severely limits economic development and is a cause of poverty in most countries where the disease is endemic. Malaria remains an ongoing problem in sub-Saharan Africa, and while great strides have been made over the past 15 years, millions of people are still at risk of contracting the parasite (Patouillard et al., 2017).

Africa offers a stable and ecologically diverse ecosystem and hosts the world's highest vectors of malaria (Bernard et al., 2020) and is expected to remain so in the future. Climate change (Adigun et al., 2015). The main vectors of *Anopheles* malaria in sub-Saharan Africa are *Anopheles funestus* s.s. and three members of the *Anopheles gambiae* complex: *An. Gambiae* s.s., *Anopheles coluzzii* and *Anopheles arabiensis* (Molinario et al., 2015), which play a role in the transmission of malaria in their distribution area, e.g. the groups *Anopheles moucheti* and *Anopheles nili* (Rajeswari, 2017) and another of secondary or random vectors (Antonio-nkondjio et al., 2006). Considering that the genus *Anopheles* includes more than 500 species worldwide, of which only a few are considered important species for the transmission of malaria (Garcia Guerra et al., 2014). The morphological identification of species is crucial for allocating scarce resources solely to the fight against

malaria vectors. Species groups and species complexes are common within the genus *Anopheles* (Harbach & Besansky, 2014) and this complicates vector control because not all species in a complex share similar behaviors or similar roles in transmission malaria disease (Vanelle et al., 2012a).

Mosquitoes of the family *Culicidae* are considered a nuisance and a major public health problem because their females feed on human blood and therefore transmit extremely harmful diseases such as malaria, yellow fever and *filariasis* (Tsoka-Gwegweni & Okafor, 2014). They are estimated to transmit diseases to more than 700 million people each year and are responsible for the death of around 1 in 17 people (“Malaria Policy Advisory Committee to the WHO: Conclusions and Recommendations of Eighth Biannual Meeting (September 2015),” 2016). Effective transmission of mosquito-borne diseases requires successful contact between female mosquitoes and their hosts (Vanelle et al., 2012b). Among *Anopheles*, members of the genus *Anopheles* are best known for their role in the global transmission of malaria and *filariasis* (“Malaria Policy Advisory Committee to the WHO: Conclusions and Recommendations of Fifth Biannual Meeting (March 2014),” 2014). Among these diseases, malaria, caused by the *Plasmodium* parasite, is one of the deadliest diseases in the world (“Malaria Policy Advisory Committee to the WHO: Conclusions and Recommendations of Sixth Biannual Meeting (September 2014),” 2015). (“Malaria Vaccine: WHO Position Paper, January 2016 – Recommendations,” 2018) reported approximately 207 million cases of malaria in 2012, of which 200 million (80.0%) were on the affected continent. Patterns of disease spread, transmission, and intensity depend on the degree of urbanization and distance from vector breeding sites (MCNAMARA, 2005). The endemicity of malaria in each region is determined, among other things, by native *Anopheles* mosquitoes, their abundance, diet, resting behavior and *Plasmodium* infectivity (Atta & Reeder, 2014). The Federal Ministry of Health in Abuja reported that at least 50.0% of Nigerians suffer from some form of malaria, making it the most significant health problem in Nigeria (UM & AN, 2016). The high transmission rate and prevalence of malaria is the result of the various mosquito breeding sites, including convenient water reservoirs such as cans, old tires, tree holes, cisterns, open pools, drains, streams and ponds (McKenzie, 2014). Part of the fight is the official observance of April 25 each year, beginning in 2008, as World Malaria Day (CDC Weekly, 2020). Arms-only people face a variety of barriers when assessing malaria prevention, particularly with respect to knowledge of mathematical modeling and vector biology (Emmanuel et al., 2020).

## II. Materials and method

In table below, variables and parameters used in the new model are defined below

**Table 1: Variables and Parameters Defined**

| Variables  | Description   |
|------------|---|
| $A(t)$     | Number of adult mosquitoes at time(t)                                   |
| $E(t)$     | Number of eggs at time(t)   |
| $L(t)$     | Number of larvae at time(t)   |
| $P(t)$     | Number of pupae at time(t)  |
| $N(t)$     | Total population  |
| $C_p(t)$   | Number of natural Predator for larva at time(t) (Copepods)              |
| $T_p(t)$   | Number of natural Predator for pupa at time(t)(Tadpoles)                |
| $P_m(t)$   | Number of natural Predator for Adult at time(t) (Purple Martins)        |
| Parameters | Description   |
| $b_1$      | Natural birth rate of adult class                                       |
| $b_2$      | Natural birth rate of copepods class                                    |
| $b_3$      | Natural birth rate of tadpoles' class                                   |
| $b_4$      | Natural birth rate of purple martins' class                             |
| $\mu_1$    | Natural death rate of adult class                                       |
| $\mu_2$    | Natural death rate of egg class   |
| $\mu_3$    | Natural death rate of larva class                                       |
| $\mu_4$    | Natural death rate of pupa class  |
| $\mu_5$    | Natural death rate of purple martins' class                             |
| $\mu_6$    | Natural death rate of copepods class                                    |
| $\mu_7$    | Natural death rate of tadpoles' class                                   |
| $\beta_1$  | Induce death rate of adult due to chemical and environmental conditions |
| $\beta_2$  | Induce death rate of egg due to chemical and environmental conditions   |
| $\beta_3$  | Induce death rate of larva due to chemical and environmental conditions |
| $\beta_4$  | Induce death rate of pupa due to chemical and environmental conditions  |

|           |   |
|-----------|---|
| $\beta_5$ | Induce death rate of purple martins' due to chemical and environmental conditions |
| $\beta_6$ | Induce death rate of copepods due to chemical and environmental conditions        |
| $\beta_7$ | Induce death rate of tadpoles' due to chemical and environmental conditions       |
| $\eta$    | The incidence rate (the rate at which adult mosquitoes oviposit)                  |
| $\sigma$  | The proportion at which egg harsh to larva  |
| $\lambda$ | The proportion of larva that transform to pupa                                    |
| $\pi$     | The proportion of pupa that transform to adult                                    |
| $\alpha$  | The probability at which mosquito larva are eaten up by copepods                  |
| $\omega$  | The probability at which mosquito pupa are eaten up by tadpoles                   |
| $\gamma$  | The probability at which mosquito adult are eaten up by purple matins             |
| $C$       | The average temperature of the water culture                                      |
| $N_L$     | Number of larva been eaten up by copepods at time(t)                              |
| $N_P$     | Number of pupa been eaten up by tadpoles at time(t)                               |
| $N_A$     | Number of adult been eaten up by purple martins at time(t)                        |

**Model Assumptions**

When formulating the model, the following assumptions were made

- 1) The total population of Anopheles mosquitoes consists of four populations such as egg, larva, pupa and adult.
- 2) The total population of natural predators consists of three populations such as copepods, tadpoles and purple martins.
- 3) The parasite of a mosquito, transmitted from one mosquito to another, is transmitted only through the host, this is called horizontal transmission.
- 4) Predators can consume infinite amounts of prey.
- 5) Emigration and immigration of the Anopheles mosquito population does not occur in this population; however, the population increases only by the natural birth rate and decreases only by the natural death rate and also due to environmental factors.
- 6) The prey population grows exponentially when the predator is absent.
- 7) The Anopheles mosquito is thought to transmit malaria only through direct contact.
- 8) The predator population will starve in the absence of the prey population

The following diagram describes the flux control of the predator-prey interaction; It will be useful in formulating models.

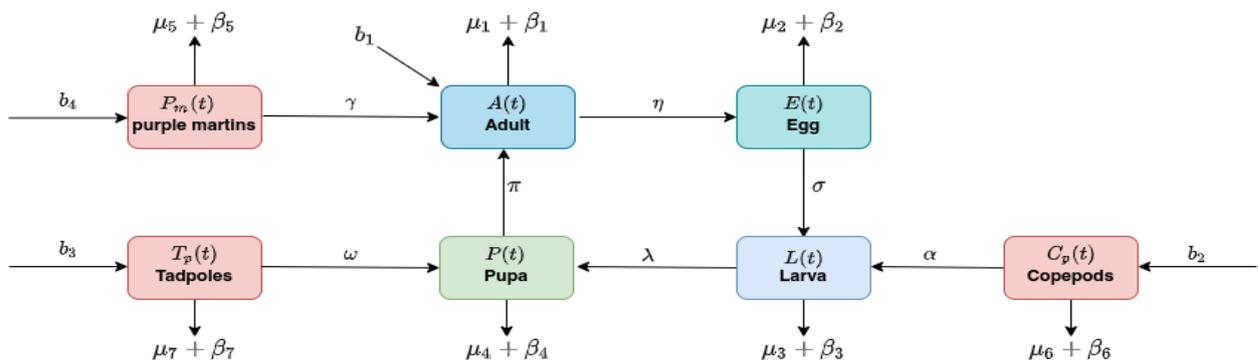


Figure 1: Flow Control Diagram of Predator-prey Interaction Model in Mosquito Life-Cycle

**Description of the Model**

The new model is a control flowchart of the predator-prey interaction model in the mosquito life cycle that considers an open population of mosquitoes and predators. The population is subdivided according to the life cycle of mosquitoes and natural predators. In the life cycle of a mosquito, the population is divided into four compartments: Egg compartment  $E(t)$ , Larva compartment  $L(t)$ , Pupa compartment  $P(t)$ , Adult compartment  $A(t)$  and natural Predator divided into three divisions. Copepods  $C_P(t)$ , Tadpoles  $T_P(t)$ , and Purple Martins  $P_M(t)$ .

Mathematical models provide a solid understanding of planning and risk controls in heterogeneous settings, especially when the models are based on vector population ecology and a solid understanding of entomological parameters relevant to transmission. Research conducted by (Killeen & Chitnis, 2014) that mathematical models have also played an important role in understanding the epidemiology of malaria and other infectious diseases; that mathematical models also provide an accurate quantitative description of complex nonlinear processes and a method to relate the individual infection process to the incidence of disease or

infection in a population over time, yielding insights important on the introduction of natural predator to increase the interruption of the life cycle of the Anopheles mosquito at the larval, pupal and Adult stages, thereby reducing or eradicating the mosquitoes. This introduction of natural enemies reduces malaria by the biting vector. They work by reducing the intensity of malaria transmission or eradicating malaria. The classification of a natural enemy as predator or parasite largely depends on the number of prey or hosts attacked or consumed the reproductive strategy and other details of the system, in which there are many similarities in characteristics of the natural predator and in the model, to study them. Mathematical modeling of malaria is a challenging area of applied mathematics due to its peculiarities in Africa and particularly in Nigeria. Millions of people die of malaria every year. Mosquitoes are resistant to most vaccines we have today. It is important to develop preventives/methods to fight against malaria and mosquitoes in general.

Therefore, each of the two population compartments above is divided into classes below;

$A(t)$  = Number of adult mosquitoes at time(t)

$L(t)$  = Number of larvae at time(t)

$P(t)$  = Number of pupae at time(t)

$C_p(t)$  = Number of natural predator for larva (Copepods)

$T_p(t)$  = Number of natural predator for pupa (Tadpoles)

$P_m(t)$  = Number of natural predator for purple martins (Purple martins)

$N_1$  = Total population for mosquitoes at time t,  $N_1 = A(t) + L(t) + P(t) + E(t)$

$N_2$  = Total population for predator at time t,  $N_2 = P_m(t) + C_p(t) + T_p(t)$

$N_2$  = Total population at time t,  $N(t) = N_1(t) + N_2(t)$

An adult female mosquito interact sexually with males or vice versa at a rate called the incidence rate, given by  $\eta$ .  $b_1$  is the natural birth rate of the adult class,  $\beta_1$  is the induced mortality rate of copepods due to chemical and environmental conditions of the adult class and  $\mu_1$  is the natural mortality rate of the adult class.  $\sigma$  is the fraction in which the egg is harsh to larva,  $\beta_2$  is the induced mortality rate of the egg due to the chemical and environmental conditions of the egg class, and  $\mu_2$  is the natural mortality rate of the compartment to eggs.  $\lambda$  is the fraction at which the larvae transform to pupate,  $\beta_3$  is the induced mortality rate of the larvae due to the chemical and environmental conditions of the larval class, and  $\mu_3$  is the natural mortality rate of the larvae.  $\pi$  is the fraction at which the pupa transforms into an adult,  $\beta_3$  is the induced death rate of the pupa due to chemical and environmental conditions of the pupal class, and  $\mu_3$  is the natural death rate of the pupal compartment.  $b_2$  is the natural birth rate of the copepod class,  $\beta_6$  is the induced mortality rate of copepods due to chemical and environmental conditions of the copepod class,  $\mu_6$  is the natural death rate of the copepod compartment and  $\alpha$  is the probability at which mosquito larvae eaten by copepods.  $b_3$  is the natural birth rate of the tadpole class,  $\beta_7$  is the induced mortality rate of the tadpoles due to the chemical and environmental conditions of the tadpole class,  $\mu_7$  is the natural death rate of tadpoles compartment and  $\omega$  is the probability at which adult mosquito are eaten up by purple martins.  $b_4$  is the natural birth rate of purple martins class,  $\beta_5$  is the induce death rate of purple martins due to chemical and environment conditions of purple martins class and  $\mu_5$  is the natural death rate of purple martins compartment and  $\gamma$  is the probability at which mosquito adult are eaten up by purple martins.

### The Model Equations

From the above assumptions and flowchart, the following equations were derived

#### Model Equations for Mosquito Life-Cycle

$$\frac{dA}{dt} = b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) \quad \dots (1)$$

$$\frac{dE}{dt} = \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) \quad \dots (2)$$

$$\frac{dL}{dt} = \sigma E(t) + \alpha C_p(t) - (\mu_3 + \beta_3 + \lambda)L(t) \quad \dots (3)$$

$$\frac{dP}{dt} = \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) \quad \dots (4)$$

#### Model Equations for Natural Predators

$$\frac{dC_p}{dt} = b_2 - (\mu_6 + \beta_6 + \alpha)C_p(t) \quad \dots (5)$$

$$\frac{dT_p}{dt} = b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) \quad \dots (6)$$

$$\frac{dP_m}{dt} = b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) \quad \dots (7)$$

#### Model Equations for Total Population

$$N_1 = A(t) + L(t) + P(t) + E(t)$$

$$\begin{aligned}
 N_2 &= P_m(t) + C_p(t) + T_p(t) \\
 N(t) &= N_1(t) + N_2(t) \\
 N(t) &= A(t) + L(t) + P(t) + E(t) + P_m(t) + C_p(t) + T_p(t) \\
 \frac{dN}{dt} &= b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) + \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) + \sigma E(t) + \alpha C_p(t) - \\
 &(\mu_3 + \beta_3 + \lambda)L(t) + \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) + b_2 - (\mu_6 + \beta_6 + \alpha)C_p(t) + b_3 - (\mu_7 + \beta_7 + \\
 &\omega T_p(t) + b_4 - \mu_5 + \beta_5 + \gamma P_m(t) \quad \dots \quad (8)
 \end{aligned}$$

### III. Results

#### Existence and Uniqueness of the Disease-Free Steady State of the model

Here, we would determine the model-free steady-state stability (MFE) by considering the model variables and parameters and using the model equations. Since we have nonlinear eight systems of equations or deterministic ordinary differential equations, we know that it is almost impossible to obtain an analytical solution of these systems. Therefore, we used the idea of equilibrium point, Beltrami and Diekmann conditions and we also used Maple software to graph the result.

The Mosquito Free Equilibrium (MFE) state of the model by zeroing the left-hand sides of equations (1-8), the following model equations associated are given below;

$$b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) = 0 \quad \dots \quad (9)$$

$$\eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) = 0 \quad \dots \quad (10)$$

$$\sigma E(t) + \alpha C_p(t) - (\mu_3 + \beta_3 + \lambda)L(t) = 0 \quad \dots \quad (11)$$

$$\lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) = 0 \quad \dots \quad (12)$$

$$b_2 - (\mu_6 + \beta_6 + \alpha)C_p(t) = 0 \quad \dots \quad (13)$$

$$b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) = 0 \quad \dots \quad (14)$$

$$b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) = 0 \quad \dots \quad (15)$$

$$\begin{aligned}
 b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) + \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) + \sigma E(t) + \alpha C_p(t) \\
 - (\mu_3 + \beta_3 + \lambda)L(t) + \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) + b_2 - (\mu_6 + \beta_6 + \alpha)C_p(t) \\
 + b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) + b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) = 0 \quad \dots \quad (16)
 \end{aligned}$$

Looking at the system of equations, it is now clear that if we have eight unknowns whose values are to be determined; it must be possible to express them in two equations where the two unknowns can be related. To solve a pair of simultaneous equations, two main methods are used as follows:

#### Method 1: Substitution Method

In the substitution method, one of the two unknowns becomes the subject of the formula in one of the equations. This will then be substituted into the second equation to have a simple equation with one unknown. The equation is then solved linearly to obtain a value, and so the obtained value is substituted to obtain the other unknown.

#### Method 2: Elimination Method

In the elimination method, one of the two unknowns (which is not present) is eliminated by adding or subtracting the two equations. Note that each unknown to be eliminated must have the same (same) coefficient to facilitate (allow) addition or subtraction.

At this point, we make  $C_p(t)$ ,  $T_p(t)$  and  $P_m(t)$  the subject of the formula from equation (13)-(15)

From equation, 13 we have

$$\begin{aligned}
 \frac{dC_p}{dt} &= b_2 - (\mu_6 + \beta_6 + \alpha)C_p(t) \\
 \Rightarrow b_2 - (\mu_6 + \beta_6 + \alpha)C_p(t) &= 0 \\
 \Rightarrow (\mu_6 + \beta_6 + \alpha)C_p(t) &= b_2 \\
 \Rightarrow C_p(t) &= \frac{b_2}{(\mu_6 + \beta_6 + \alpha)} \quad \dots \quad (17)
 \end{aligned}$$

From equation 14, we have

$$\begin{aligned}
 \frac{dT_p}{dt} &= b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) \\
 \Rightarrow b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) &= 0 \\
 \Rightarrow (\mu_7 + \beta_7 + \omega)T_p(t) &= b_3 \\
 \Rightarrow T_p(t) &= \frac{b_3}{(\mu_7 + \beta_7 + \omega)} \quad \dots \quad (18)
 \end{aligned}$$

From equation 15, we have

$$\begin{aligned} \frac{dP_m}{dt} &= b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) \\ \Rightarrow b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) &= 0 \\ \Rightarrow (\mu_5 + \beta_5 + \gamma)P_m(t) &= b_4 \\ \Rightarrow P_m(t) &= \frac{b_4}{(\mu_5 + \beta_5 + \gamma)} \quad \dots (19) \end{aligned}$$

At this point, the substitution method is used to solve the system of equations (9-12 and 16).

From equation 9, we have

$$\begin{aligned} \frac{dA}{dt} &= b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) \\ \Rightarrow b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) &= 0 \\ \Rightarrow (\mu_1 + \beta_1 + \eta)A(t) &= b_1 + \gamma P_m(t) + \pi P(t) \\ \Rightarrow (\mu_1 + \beta_1 + \eta)A(t) - \pi P(t) &= b_1 + \gamma P_m(t) \end{aligned}$$

where  $P_m(t) = \frac{b_4}{(\mu_5 + \beta_5 + \gamma)}$

let  $M_1 = (\mu_1 + \beta_1 + \eta)$  and  $M_5 = (\mu_5 + \beta_5 + \gamma)$ , we have

$$\begin{aligned} \Rightarrow M_1 A(t) - \pi P(t) &= b_1 + \gamma M_1 \left\{ \frac{b_4}{M_5} \right\} \\ \Rightarrow M_1 M_5 A(t) - M_5 \pi P(t) &= M_5 b_1 + \gamma M_1 b_4 \quad \dots (20) \end{aligned}$$

From equation 10, we have

$$\begin{aligned} \frac{dE}{dt} &= \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) \\ \Rightarrow \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) &= 0 \\ \text{Let } M_2 &= (\mu_2 + \beta_2 + \sigma), \text{ we have} \\ \Rightarrow \eta A(t) - M_2 E(t) &= 0 \quad \dots (21) \end{aligned}$$

From equation 11, we have

$$\begin{aligned} \frac{dL}{dt} &= \sigma E(t) + \alpha C_p(t) - (\mu_3 + \beta_3 + \lambda)L(t) \\ \Rightarrow \sigma E(t) + \alpha C_p(t) - (\mu_3 + \beta_3 + \lambda)L(t) &= 0 \\ \text{where } C_p(t) &= \frac{b_2}{(\mu_6 + \beta_6 + \alpha)}, M_3 = (\mu_3 + \beta_3 + \lambda) \text{ and } M_6 = (\mu_6 + \beta_6 + \alpha), \text{ we have} \\ \Rightarrow \sigma E(t) - M_3 L(t) &= -\alpha \frac{b_2}{M_6} \\ \Rightarrow M_6 \sigma E(t) - M_3 M_6 L(t) &= -\alpha b_2 \quad \dots (22) \end{aligned}$$

From equation 12, we have

$$\begin{aligned} \frac{dP}{dt} &= \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) \\ \Rightarrow \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) &= 0 \\ \text{where } T_p(t) &= \frac{b_3}{(\mu_7 + \beta_7 + \omega)}, M_4 = (\mu_4 + \beta_4 + \pi) \text{ and } M_7 = (\mu_7 + \beta_7 + \omega), \text{ we have} \\ \Rightarrow \lambda L(t) + \omega \frac{b_3}{(\mu_7 + \beta_7 + \omega)} - (\mu_4 + \beta_4 + \pi)P(t) &= 0 \\ \Rightarrow \lambda L(t) + \omega \frac{b_3}{M_7} - M_4 P(t) &= 0 \\ \Rightarrow M_7 \lambda L(t) + \omega b_3 - M_4 M_7 P(t) &= 0 \\ \Rightarrow M_7 \lambda L(t) - M_4 M_7 P(t) &= -\omega b_3 \quad \dots (23) \end{aligned}$$

From equation 16, we have

$$\begin{aligned} \frac{dN}{dt} &= b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) + \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) + \sigma E(t) + \alpha C_p(t) \\ &\quad - (\mu_3 + \beta_3 + \lambda)L(t) + \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) + b_2 - (\mu_6 + \beta_6 + \alpha)C_p(t) \\ &\quad + b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) + b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) \\ \Rightarrow b_1 + \gamma P_m(t) + \pi P(t) - (\mu_1 + \beta_1 + \eta)A(t) + \eta A(t) - (\mu_2 + \beta_2 + \sigma)E(t) + \sigma E(t) + \alpha C_p(t) \\ &\quad - (\mu_3 + \beta_3 + \lambda)L(t) + \lambda L(t) + \omega T_p(t) - (\mu_4 + \beta_4 + \pi)P(t) + b_2 - (\mu_6 + \beta_6 + \alpha)C_p(t) \\ &\quad + b_3 - (\mu_7 + \beta_7 + \omega)T_p(t) + b_4 - (\mu_5 + \beta_5 + \gamma)P_m(t) &= 0 \end{aligned}$$

where  $P_m(t) = \frac{b_4}{(\mu_5 + \beta_5 + \gamma)}$ ,  $C_p(t) = \frac{b_2}{(\mu_6 + \beta_6 + \alpha)}$ ,  $T_p(t) = \frac{b_3}{(\mu_7 + \beta_7 + \omega)}$ ,  $M_1 = (\mu_1 + \beta_1 + \eta)$ ,  $M_2 = (\mu_2 + \beta_2 + \sigma)$ ,  $M_3 = (\mu_3 + \beta_3 + \lambda)$ ,  $M_4 = (\mu_4 + \beta_4 + \pi)$ ,  $M_5 = (\mu_5 + \beta_5 + \gamma)$ ,  $M_6 = (\mu_6 + \beta_6 + \alpha)$  and  $M_7 = (\mu_7 + \beta_7 + \omega)$  we have

$$\begin{aligned} \Rightarrow b_1 - (\mu_1 + \beta_1)A(t) - (\mu_2 + \beta_2)E(t) - (\mu_3 + \beta_3)L(t) - (\mu_4 + \beta_4)P(t) + b_2 - (\mu_6 + \beta_6)\frac{b_2}{(\mu_6 + \beta_6 + \alpha)} \\ + b_3 - (\mu_7 + \beta_7)\frac{b_3}{(\mu_7 + \beta_7 + \omega)} + b_4 - (\mu_5 + \beta_5)\frac{b_4}{(\mu_5 + \beta_5 + \gamma)} = 0 \\ \Rightarrow b_1 + b_2 + b_3 + b_4 - (\mu_1 + \beta_1)A(t) - (\mu_2 + \beta_2)E(t) - (\mu_3 + \beta_3)L(t) - (\mu_4 + \beta_4)P(t) \\ - (\mu_6 + \beta_6)\frac{b_2}{(\mu_6 + \beta_6 + \alpha)} - (\mu_7 + \beta_7)\frac{b_3}{(\mu_7 + \beta_7 + \omega)} - (\mu_5 + \beta_5)\frac{b_4}{(\mu_5 + \beta_5 + \gamma)} = 0 \end{aligned}$$

let  $M_1^* = (\mu_1 + \beta_1)$ ,  $M_2^* = (\mu_2 + \beta_2)$ ,  $M_3^* = (\mu_3 + \beta_3)$ ,  $M_4^* = (\mu_4 + \beta_4)$ , we have

$$\Rightarrow b_1 + b_2 + b_3 + b_4 - M_1^*A(t) - M_2^*E(t) - M_3^*L(t) - M_4^*P(t) - M_6^*\frac{b_2}{M_6} - M_7^*\frac{b_3}{M_7} - M_5^*\frac{b_4}{M_5} = 0$$

let  $b_1 + b_2 + b_3 + b_4 = b_0^*$ , we have

$$\begin{aligned} \Rightarrow M_1^*A(t) + M_2^*E(t) + M_3^*L(t) + M_4^*P(t) = b_0^* + M_5^*\frac{b_4}{M_5} + M_6^*\frac{b_2}{M_6} + M_7^*\frac{b_3}{M_7} \\ \Rightarrow M_1^*M_5M_6M_7A(t) + M_2^*M_5M_6M_7E(t) + M_3^*M_5M_6M_7L(t) + M_4^*M_5M_6M_7P(t) \\ = b_0^* + M_5^*M_6M_7b_4 + M_6^*M_5M_7b_2 + M_7^*M_5M_6b_3 \end{aligned} \dots (24)$$

In synopsis we have the following equations

$$C_p(t) = \frac{b_2}{(\mu_6 + \beta_6 + \alpha)} = \frac{b_2}{M_6} \dots (17)$$

$$T_p(t) = \frac{b_3}{(\mu_7 + \beta_7 + \omega)} = \frac{b_3}{M_7} \dots (18)$$

$$P_m(t) = \frac{b_4}{(\mu_5 + \beta_5 + \gamma)} = \frac{b_4}{M_5} \dots (19)$$

$$\Rightarrow M_1M_5A(t) - M_5\pi P(t) = M_5b_1 + \gamma M_1b_4 \dots (20)$$

$$\Rightarrow \eta A(t) - M_2E(t) = 0 \dots (21)$$

$$\Rightarrow M_6\sigma E(t) - M_3M_6L(t) = -\alpha b_2 \dots (22)$$

$$\Rightarrow M_7\lambda L(t) - M_4M_7P(t) = -\omega b_3 \dots (23)$$

$$\Rightarrow M_1^*M_5M_6M_7A(t) + M_2^*M_5M_6M_7E(t) + M_3^*M_5M_6M_7L(t) + M_4^*M_5M_6M_7P(t) \\ = b_0^* + M_5^*M_6M_7b_4 + M_6^*M_5M_7b_2 + M_7^*M_5M_6b_3 \dots (24)$$

From equation 21, we make A(t) the subject of the formula and substitute into equation 20, we have

$$\eta A(t) - M_2E(t) = 0$$

$$A(t) = \frac{M_2E(t)}{\eta}$$

$$M_1M_5A(t) - M_5\pi P(t) = M_5b_1 + \gamma M_1b_4$$

$$\Rightarrow M_1M_5\frac{M_2E(t)}{\eta} - M_5\pi P(t) = M_5b_1 + \gamma M_1b_4$$

$$\Rightarrow M_1M_2M_5E(t) - M_5\pi P(t) = M_5b_1\eta + \gamma M_1b_4\eta \dots (25)$$

From equation 23, we make L(t) the subject of the formula and substitute to equation 22, we have

$$M_7\lambda L(t) - M_4M_7P(t) = -\omega b_3$$

$$M_7\lambda L(t) = M_4M_7P(t) - \omega b_3$$

$$L(t) = \frac{M_4M_7P(t) - \omega b_3}{M_7\lambda}$$

$$\Rightarrow M_6\sigma E(t) - M_3M_6L(t) = -\alpha b_2$$

$$\Rightarrow M_6\sigma E(t) - M_3M_6\left(\frac{M_4M_7P(t) - \omega b_3}{M_7\lambda}\right) = -\alpha b_2$$

$$\Rightarrow M_6M_7\lambda\sigma E(t) - M_3M_4M_6M_7P(t) - M_3M_6\omega b_3 = -M_7\lambda\alpha b_2$$

$$\Rightarrow M_6M_7\lambda\sigma E(t) - M_3M_4M_6M_7P(t) = M_3M_6\omega b_3 - M_7\lambda\alpha b_2 \dots (26)$$

Solve equation 25 and 26 simultaneously.

Multiply equation 25 by  $M_6M_7\lambda\sigma$ , we have

$$M_1M_2M_5E(t) - M_5\pi P(t) = M_5b_1\eta + \gamma M_1b_4\eta$$

$$\begin{aligned} &\Rightarrow M_6 M_7 \lambda \sigma \{M_1 M_2 M_5 E(t) - M_5 \pi \eta P(t) = M_5 b_1 \eta + \gamma M_1 b_4 \eta\} \\ &\Rightarrow M_1 M_2 M_5 M_6 M_7 \lambda \sigma E(t) - M_5 M_6 M_7 \lambda \sigma \pi \eta P(t) = M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta \dots (a) \\ &\text{Multiply equation 26 by } M_1 M_2 M_5, \text{ we have} \\ &\Rightarrow M_6 M_7 \lambda \sigma E(t) - M_3 M_4 M_6 M_7 P(t) = M_3 M_6 \omega b_3 - M_7 \lambda \alpha b_2 \\ &\Rightarrow M_1 M_2 M_5 \{M_6 M_7 \lambda \sigma E(t) - M_3 M_4 M_6 M_7 P(t) = M_3 M_6 \omega b_3 - M_7 \lambda \alpha b_2\} \\ &\Rightarrow M_1 M_2 M_5 M_6 M_7 \lambda \sigma E(t) - M_1 M_2 M_5 M_3 M_4 M_6 M_7 P(t) = M_1 M_2 M_5 M_3 M_6 \omega b_3 - M_1 M_2 M_5 M_7 \lambda \alpha b_2 \dots (b) \\ &\text{Subtract equation (a) from equation (b), we have} \\ &\Rightarrow M_1 M_2 M_5 M_6 M_7 \lambda \sigma E(t) - M_5 M_6 M_7 \lambda \sigma \pi \eta P(t) = M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta \dots a \\ &\Rightarrow M_1 M_2 M_5 M_6 M_7 \lambda \sigma E(t) - M_1 M_2 M_5 M_3 M_4 M_6 M_7 P(t) = M_1 M_2 M_5 M_3 M_6 \omega b_3 - M_1 M_2 M_5 M_7 \lambda \alpha b_2 \dots b \\ &\Rightarrow M_1 M_2 M_5 M_3 M_4 M_6 M_7 P(t) - M_5 M_6 M_7 \lambda \sigma \pi \eta P(t) \\ &\quad = M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta + M_1 M_2 M_5 M_7 \lambda \alpha b_2 - M_1 M_2 M_5 M_3 M_6 \omega b_3 \\ &\Rightarrow \{M_1 M_2 M_5 M_3 M_4 M_6 M_7 - M_5 M_6 M_7 \lambda \sigma \pi \eta\} P(t) \\ &\quad = M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta + M_1 M_2 M_5 M_7 \lambda \alpha b_2 + M_1 M_2 M_5 M_3 M_6 \omega b_3 \end{aligned}$$

$$P(t) = \frac{M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta + M_1 M_2 M_5 M_7 \lambda \alpha b_2 + M_1 M_2 M_5 M_3 M_6 \omega b_3}{M_5 M_6 M_7 (M_1 M_2 M_3 M_4 - \lambda \sigma \pi \eta)}$$

Put P(t) into equation (a), we have

$$M_1 M_2 M_5 M_6 M_7 \lambda \sigma E(t) - M_5 M_6 M_7 \lambda \sigma \pi \eta P(t) = M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta$$

$$\begin{aligned} &\Rightarrow M_1 M_2 M_5 M_6 M_7 \lambda \sigma E(t) \\ &- M_5 M_6 M_7 \lambda \sigma \pi \eta \left( \frac{M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta + M_1 M_2 M_5 M_7 \lambda \alpha b_2 + M_1 M_2 M_5 M_3 M_6 \omega b_3}{M_1 M_2 M_5 M_3 M_4 M_6 M_7 - M_5 M_6 M_7 \lambda \sigma \pi \eta} \right) \\ &= M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta \\ &\Rightarrow M_1 M_2 M_5 M_6 M_7 \lambda \sigma E(t) \\ &- M_5 M_6 M_7 \lambda \sigma \pi \eta \left( \frac{M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta + M_1 M_2 M_5 M_7 \lambda \alpha b_2 + M_1 M_2 M_5 M_3 M_6 \omega b_3}{M_1 M_2 M_5 M_3 M_4 M_6 M_7 - M_5 M_6 M_7 \lambda \sigma \pi \eta} \right) \\ &= M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta \\ &E(t) = \frac{M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 \eta + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2}{M_1 M_2 M_5 M_3 M_4 M_6 M_7 - M_5 M_6 M_7 \lambda \sigma \pi \eta} \end{aligned}$$

$$E(t) = \frac{\eta (M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2)}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)}$$

Substitute E(t) in equation 21, we have

$$\begin{aligned} &\eta A(t) - M_2 E(t) = 0 \\ &\Rightarrow \eta A(t) - M_2 \left( \frac{\eta (M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2)}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right) = 0 \\ &\Rightarrow \eta A(t) = M_2 \left( \frac{\eta (M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2)}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right) \\ &\Rightarrow A(t) = M_2 \left( \frac{M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right) \end{aligned}$$

Settle E(t) into equation 22, we have

$$\begin{aligned} &M_6 \sigma E(t) - M_3 M_6 L(t) = -\alpha b_2 \\ &\Rightarrow M_6 \sigma \left( \frac{\eta (M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2)}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right) - M_3 M_6 L(t) = -\alpha b_2 \\ &\Rightarrow M_6 \sigma \left( \frac{\eta (M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2)}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right) + \alpha b_2 = M_3 M_6 L(t) \end{aligned}$$

$$\Rightarrow M_3 M_6 L(t) = M_6 \sigma \left( \frac{\eta(M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2)}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right) + \alpha b_2$$

$$\Rightarrow L(t) = \frac{M_4 M_6 M_7 \gamma \eta \sigma b_4 + M_4 M_5 M_6 M_7 \eta \sigma b_1 + M_5 M_6 \omega \pi \sigma b_3 + M_1 M_2 M_4 M_5 M_7 \alpha b_2}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)}$$

Therefore, the total population becomes

$$M_1^* M_5 M_6 M_7 A(t) + M_2^* M_5 M_6 M_7 E(t) + M_3^* M_5 M_6 M_7 L(t) + M_4^* M_5 M_6 M_7 P(t) \\ = b_0^* + M_5^* M_6 M_7 b_4 + M_6^* M_5 M_7 b_2 + M_7^* M_5 M_6 b_3$$

Insert A(t), E(t), L(t), P(t),  $C_p(t)$ ,  $T_p(t)$  and  $P_m(t)$ , into equation 4.1.16, we have

$$M_1^* M_5 M_6 M_7 A(t) + M_2^* M_5 M_6 M_7 E(t) + M_3^* M_5 M_6 M_7 L(t) + M_4^* M_5 M_6 M_7 P(t) \\ = b_0^* + M_5^* M_6 M_7 b_4 + M_6^* M_5 M_7 b_2 + M_7^* M_5 M_6 b_3$$

The following results were obtained manually.

$$\text{where } A(t) = M_2 \left( \frac{M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right)$$

$$E(t) = \frac{\eta(M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2)}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)}$$

$$L(t) = \frac{M_4 M_6 M_7 \gamma \eta \sigma b_4 + M_4 M_5 M_6 M_7 \eta \sigma b_1 + M_5 M_6 \omega \pi \sigma b_3 + M_1 M_2 M_4 M_5 M_7 \alpha b_2}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)}$$

$$P(t) = \frac{M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta + M_1 M_2 M_5 M_7 \lambda \alpha b_2 + M_1 M_2 M_5 M_3 M_6 \omega b_3}{M_5 M_6 M_7 (M_1 M_2 M_3 M_4 - \lambda \sigma \pi \eta)}$$

$$C_p(t) = \frac{b_2}{(\mu_6 + \beta_6 + \alpha)} = \frac{b_2}{M_6}$$

$$T_p(t) = \frac{b_3}{(\mu_7 + \beta_7 + \omega)} = \frac{b_3}{M_7}$$

$$P_m(t) = \frac{b_4}{(\mu_5 + \beta_5 + \gamma)} = \frac{b_4}{M_7}$$

Total population is giving as;

$$M_1^* M_5 M_6 M_7 \left\{ M_2 \left( \frac{M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right) \right\} \\ + M_2^* M_5 M_6 M_7 \left\{ \frac{\eta(M_3 M_4 M_6 M_7 \gamma b_4 + M_3 M_4 M_5 M_6 M_7 b_1 + M_5 M_6 \omega \pi b_3 + M_5 M_7 \lambda \alpha \pi b_2)}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right\} \\ + M_3^* M_5 M_6 M_7 \left\{ \frac{M_4 M_6 M_7 \gamma \eta \sigma b_4 + M_4 M_5 M_6 M_7 \eta \sigma b_1 + M_5 M_6 \omega \pi \sigma b_3 + M_1 M_2 M_4 M_5 M_7 \alpha b_2}{M_5 M_6 M_7 (M_1 M_2 M_3 - \lambda \sigma \pi \eta)} \right\} \\ + M_4^* M_5 M_6 M_7 \left\{ \frac{M_5 M_6 M_7 \lambda \sigma b_1 \eta + M_1 M_6 M_7 \lambda \gamma \sigma b_4 \eta + M_1 M_2 M_5 M_7 \lambda \alpha b_2 + M_1 M_2 M_5 M_3 M_6 \omega b_3}{M_5 M_6 M_7 (M_1 M_2 M_3 M_4 - \lambda \sigma \pi \eta)} \right\} \\ = b_0^* + M_5^* M_6 M_7 b_4 + M_6^* M_5 M_7 b_2 + M_7^* M_5 M_6 b_3$$

Based on the idea of elimination and substitution method, we conclude that if the natural predators introduced are large, the number of larvae leading to pupae will be almost zero and the number of pupae developing into adults will be zero, which will prolong the life cycle of the interrupted Anopheles mosquito. Therefore, in our society, there will be no adult Anopheles mosquitoes for the transmission of malaria parasites.

### List of Numerical Experiments of the Model

The following experiments are carried out

**Experiment 1:** Effect of introducing one natural predator, purple martins on mosquitoes' adult ( $P_m = 130, C_p = 0, \text{ and } T_p = 0$ ).

**Experiment 2:** Effect of introducing two natural predators, copepod and purple martins on mosquitoes' larva and adult respectively ( $C_p = 500, T_p = 0 \text{ and } P_m = 130$ ).

**Experiment 3:** Comparison of the effect of introducing one, two and three natural predator on adult.

**Experiment 4:** Comparison of the effect of introducing two and three natural predator on adult.

**Experiment 5:** Effect of introducing low rate of natural predators, copepod, on mosquitoes' larva ( $C_p = 500$ ).

**Experiment 6:** Effect of introducing high rate of natural predators, copepod, on mosquitoes' larva ( $C_p = 2000$ ).

**Experiment 7:** comparison of the effect of introducing low and high rate of natural predators, copepod, on mosquitoes' larva.

**Experiment 8:** Effect of introducing low rate of natural predator, copepod, on mosquitoes' pupa ( $T_p = 2000$ ).

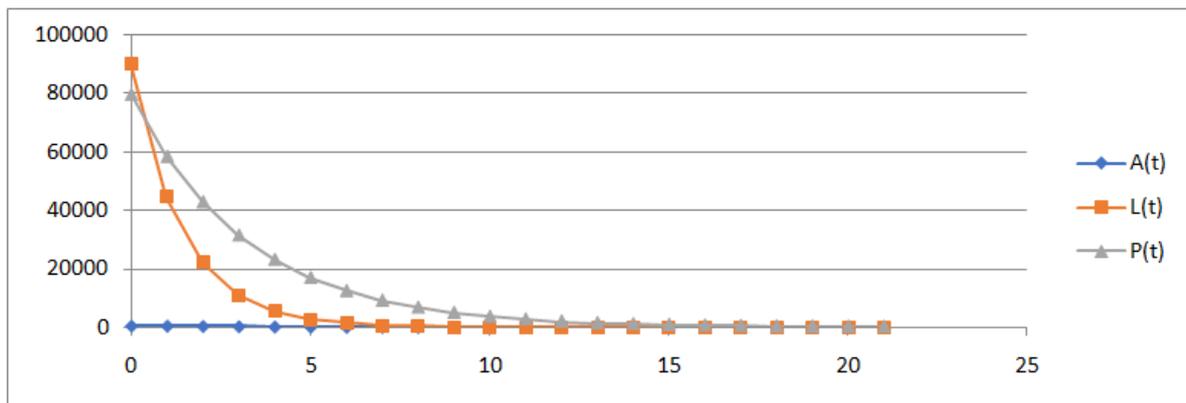
**Experiment 9:** Effect of introducing high rate of natural predator, tadpole, on mosquitoes' pupa ( $T_p = 2000$ )

**Experiment 10:** Comparison of the effect of introducing low and high rate of natural predators, tadpole on mosquitoes' pupa.

**Table 2: Numerical values of the variables and parameters**

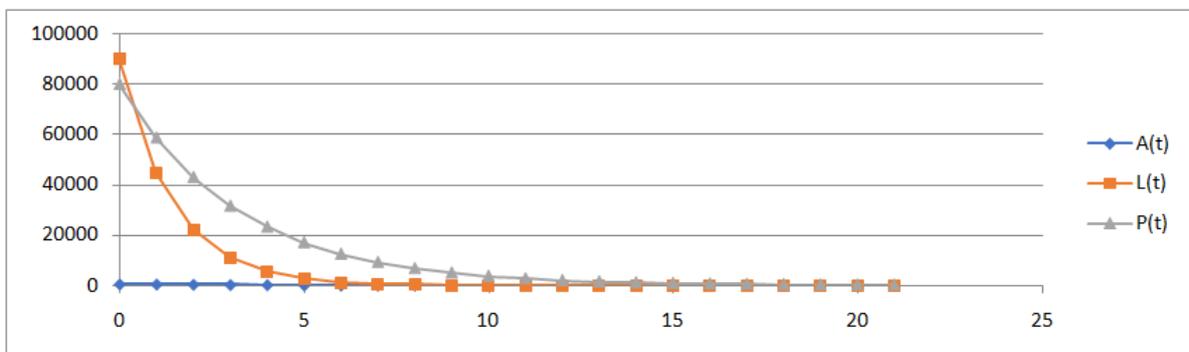
| Variables/Parameters | Values               | Source                |
|----------------------|----------------------|-----------------------|
| A(t)                 | 500                  | Assumed               |
| E (t)                | 100000               | Guerra, (2014)        |
| L(t)                 | 90000                | Assumed               |
| P(t)                 | 80000                | Assumed               |
| N(t)                 | 270000               | Assumed               |
| $C_p(t)$             | 500                  | Practical             |
| $T_p(t)$             | 500                  | Practical             |
| $P_m(t)$             | 130                  | Assumed               |
| $b_1$                | 0.02                 | Olivier, (202)        |
| $b_2$                | 0.21                 | Gearty, (2021)        |
| $b_3$                | 0.9                  | Calef, (1973)         |
| $b_4$                | 0.5                  | Joshua,( 1971)        |
| $\mu_1$              | 0.4                  | Mathews, (2020)       |
| $\mu_2$              | 0.3                  | Clements, (1981)      |
| $\mu_3$              | 0.2                  | Couret, (2014)        |
| $\mu_4$              | 0.1                  | Mondragon, (2020)     |
| $\mu_5$              | 0.5                  | Jervis, (2019)        |
| $\mu_6$              | 0.02                 | Charyl, (2011)        |
| $\mu_7$              | 0.01                 | Szekely, (2022)       |
| $\beta_1$            | $40^\circ C(0.3)$    | Beck-Johnson,, (2013) |
| $\beta_2$            | $37^\circ C(0.57)$   | Sukiato, (2019)       |
| $\beta_3$            | $28^\circ C(0.0110)$ | Adam, (2014)          |
| $\beta_4$            | $28^\circ C(0.0110)$ | Adam, (2014)          |
| $\beta_5$            | $25^\circ C (0.13)$  | Fred, (2014)          |
| $\beta_6$            | $40^\circ C(0.01)$   | Jiang, (2014)         |
| $\beta_7$            | $35^\circ C(0.02)$   | Halsbank-Lenk,(2014)  |
| $\eta$               | 0.002                | Practical             |
| $\sigma$             | 0.00004              | Practical             |
| $\lambda$            | 0.00005              | Practical             |
| $\pi \pi$            | 0.01                 | Practical             |
| $\alpha$             | 0.5                  | Practical             |
| $\omega$             | 0.5                  | Practical             |
| $\gamma$             | 0.9                  | Practical             |

**Experiment 1:** Effect of introducing one natural predator, purple martins on mosquitoes' adult.



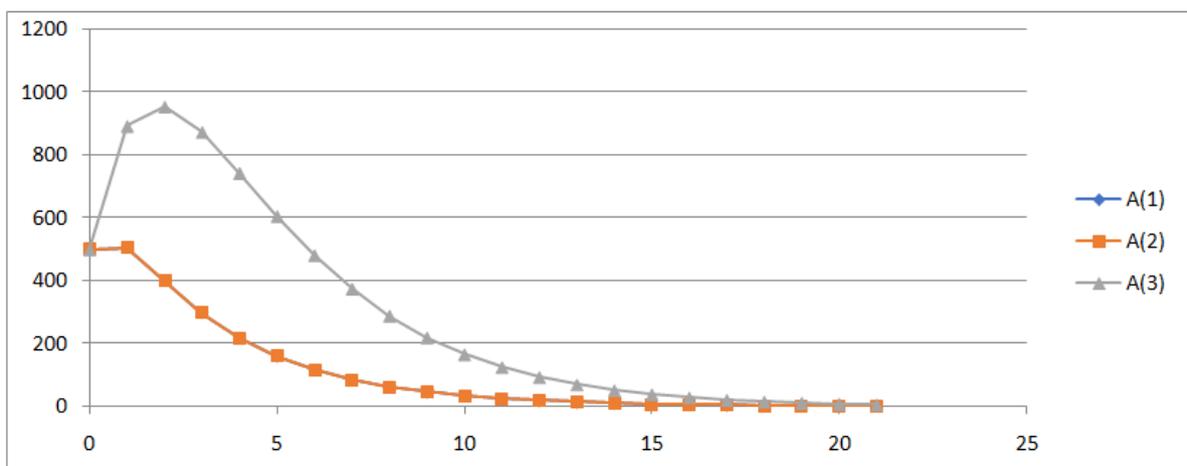
**Figure 3:** Number of mosquitoes' adult when one natural predator, purple martins was introduced to mosquito adult ( $P_m = 130, C_p = 0, T_p = 0, \gamma = 5, \mu_5 = 0.5, \beta_5 = 0.13$  and  $b_4 = 0.5$ ).

**Experiment 2:** Effect of introducing two natural predators, copepod and purple martins on mosquitoes' larva and adult respectively.



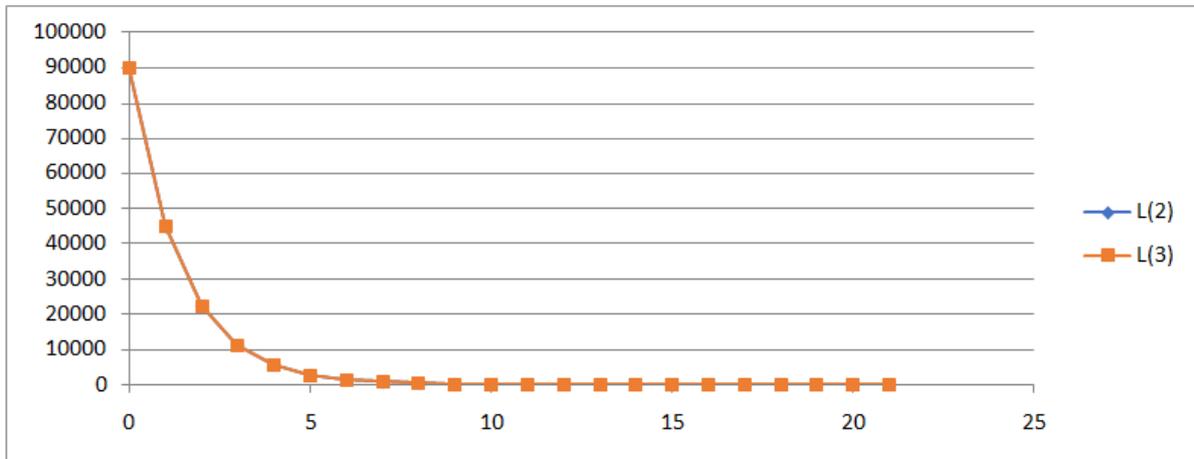
**Figure 4:** Number of mosquitoes' larva and adult when two natural predators, copepod and purple martins are introduced respectively ( $C_p = 500, P_m = 130, T_p = 0, \alpha = 0.5, \mu_6 = 0.02, \beta_6 = 0.01, b_2 = 0.21, \gamma = 5, \mu_5 = 0.5, \beta_5 = 0.13$  and  $b_4 = 0.5$ ).

**Experiment 3:** Comparison of the effect of introducing one, two and three natural predator on adult.



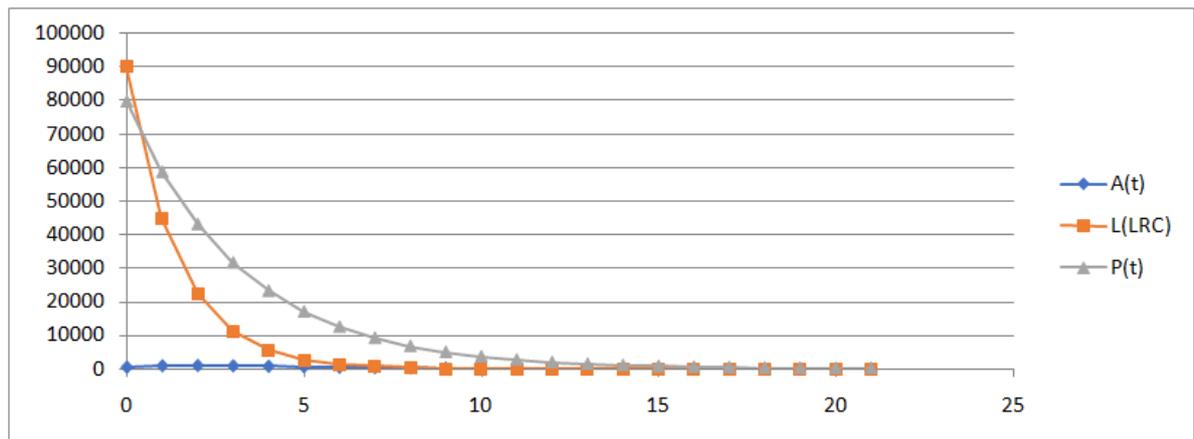
**Figure 5:** Number of mosquitoes' adult's when one, two and three natural predator, purple martins are compared respectively ( $P_{1,2\&3} = 130, \omega = 0.5, \mu_7 = 0.01, \beta_7 = 0.02,$  and  $b_3 = 0.9$ ).

**Experiment 4:** Comparison of the effect of introducing two and three natural predator, copepod on larva.



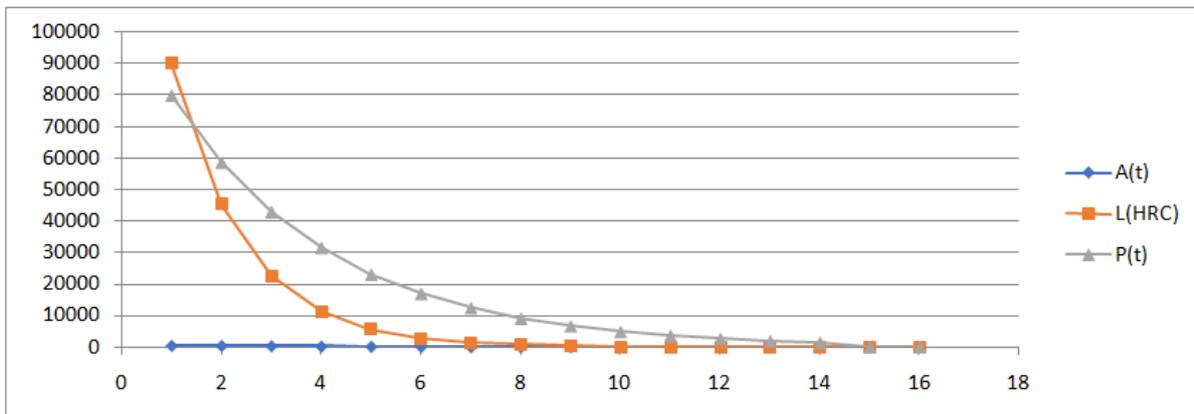
**Figure 6:** Number of mosquitoes' larva when, two and three natural predators are compared respectively ( $C_{2\&3} = 500$ ,  $\omega = 0.5$ ,  $\mu_7 = 0.01$ ,  $\beta_7 = 0.02$  and  $b_3 = 0.9$ ).

**Experiment 5:** Effect of introducing low rate of natural predators, copepod on mosquitoes' larva ( $C_p = 200$ ).



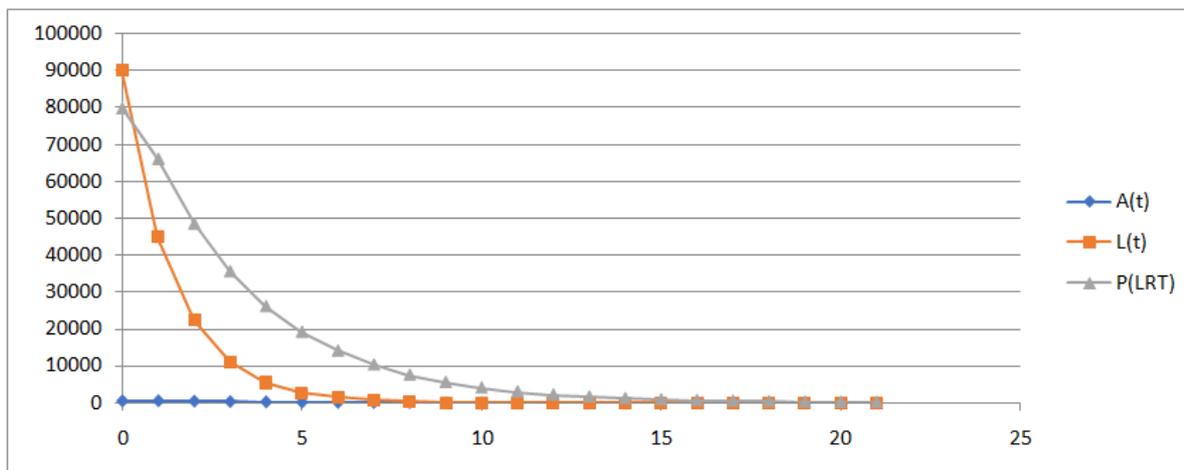
**Figure 7:** Number of mosquitoes' larva when low rate of natural predator, copepod was introduced to mosquitoes' larva ( $C_p = 200$ ,  $T_p = 0$ ,  $P_m = 0$ ,  $\alpha = 0.5$ ,  $\mu_6 = 0.02$ ,  $\beta_6 = 0.01$  and  $b_2 = 0.21$ ).

**Experiment 6:** Effect of introducing high rate of natural predator, copepod on mosquitoes' larva ( $C_p = 2000$ ).



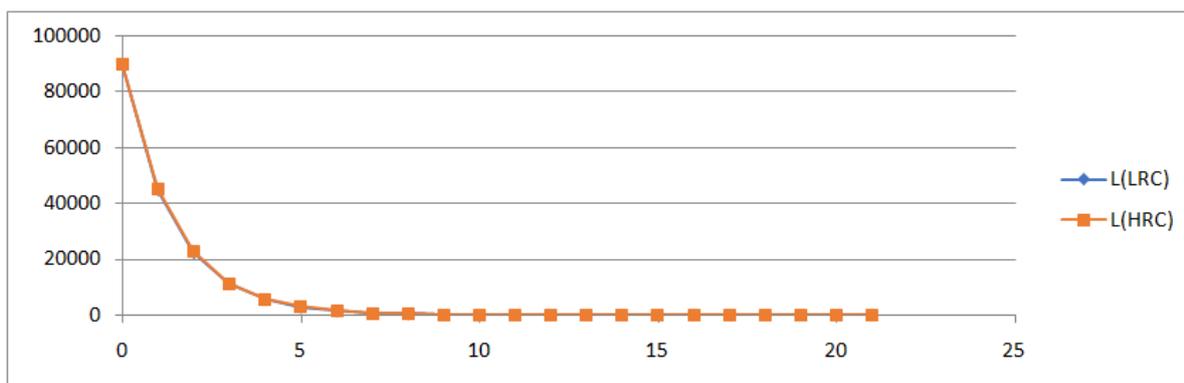
**Figure 8:** Number of mosquitoes' larva when high rate of natural predator, copepod was introduced to mosquitoes' larva ( $C_p = 2000$ ,  $\alpha = 0.5$ ,  $\mu_6 = 0.02$ ,  $\beta_6 = 0.01$  and  $b_2 = 0.21$ ).

**Experiment 7:** Effect of introducing low rate of natural predator, tadpole on mosquitoes' pupa ( $T_p = 200$ ).



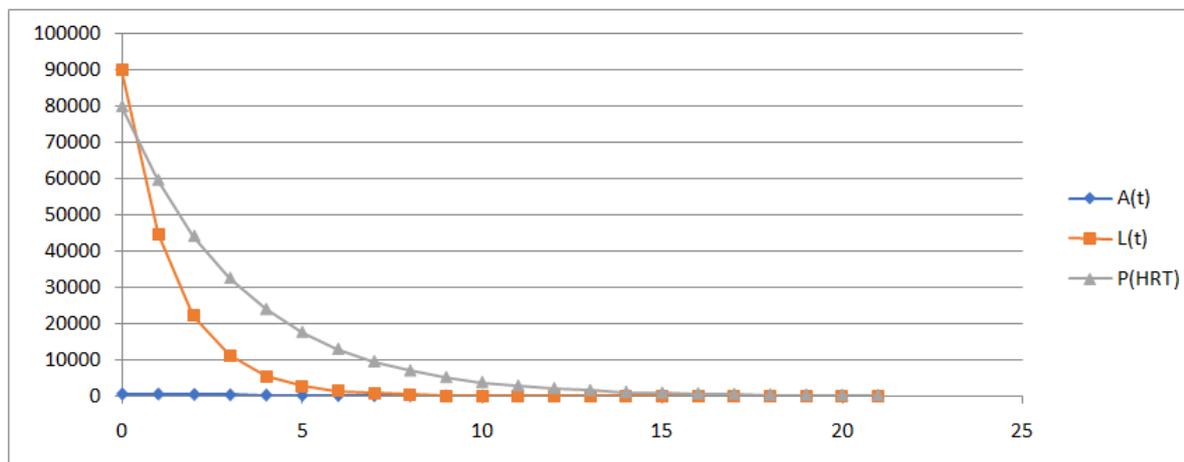
**Figure 9:** Number of mosquitoes' pupa when low rate of natural predator, tadpole was introduced ( $T_p = 200$ ,  $\omega = 0.5$ ,  $\mu_7 = 0.01$ ,  $\beta_7 = 0.02$  and  $b_3 = 0.9$ ).

**Experiment 8:** Comparison of the effect of introducing low and high rate of natural predator, copepod on mosquitoes' larva.



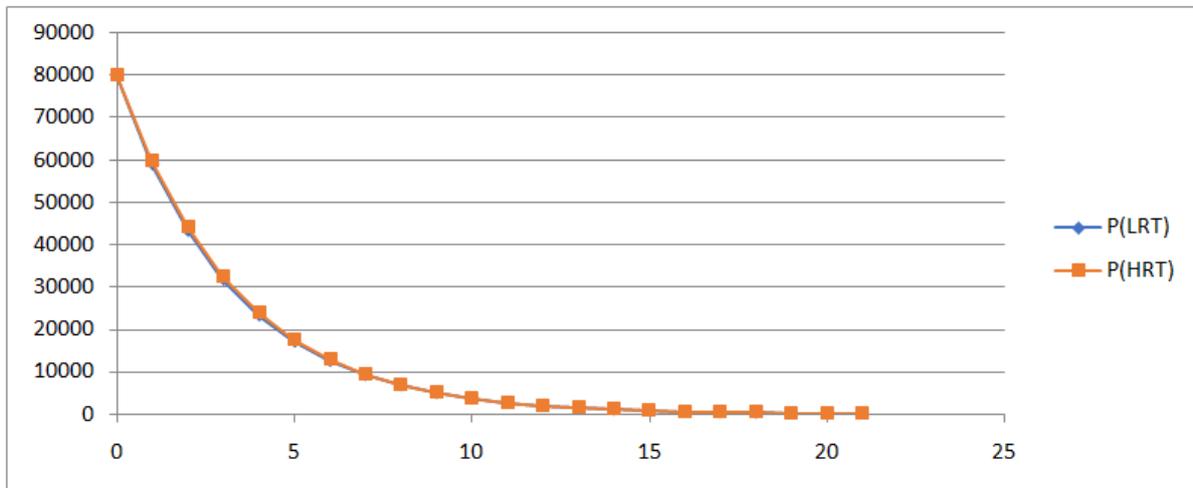
**Figure 10:** Number of mosquitoes' larva when low and high rate of natural predator, copepod are compared respectively ( $C_{L\&H} = 200\&2000$ ,  $\alpha = 0.5$ ,  $\mu_6 = 0.02$ ,  $\beta_6 = 0.01$  and  $b_2 = 0.21$ ).

**Experiment 9:** Effect of introducing high rate of natural predator, tadpole on mosquitoes' pupa ( $T_p = 2000$ ).



**Figure 11:** Number of mosquitoes' pupa when high rate of natural predator, tadpole was introduced to mosquitoes' pupa ( $T_p = 2000$ ,  $\omega = 0.5$ ,  $\mu_7 = 0.01$ ,  $\beta_7 = 0.02$  and  $b_3 = 0.9$ ).

**Experiment 10:** Comparison of the effect of introducing low and high rate of natural predator, tadpole on mosquitoes' pupa.



**Figure 12:** Number of mosquitoes' pupa when low and high rate of natural predator, tadpole is compared respectively ( $T_{L\&H} = 200\&2000, \omega = 0.5, \mu_7 = 0.01, \beta_7 = 0.02$  and  $b_3 = 0.9$ )

#### IV. Discussion of Results

A prototype of a mathematical model for the control of malaria by interrupting the life cycle of the Anopheles mosquito through the use of biological enemies at the larval, pupal and adult stages is presented. In the introduction, we discussed the prevalence of mosquitoes in our society, where two million deaths are due to malaria parasites in sub-Saharan Africa in general and Nigeria in particular, one third of which are children.

In Material and Methods, we define model variables and parameters, make assumptions, and present the model showing the flow control diagram of predator-prey interaction in model. Three natural predators (copepods, tadpoles, and crimson swallows) were introduced into the model at larval, pupal, and adult stages, and model equations for mosquito and predator life cycles were derived.

Accordingly, we performed the disease-free steady-state stability analysis of the model using equilibrium points, elimination and substitution ideas and also used Maple software for the solution symbolically, numerically and plotted the results showing the effects of introducing three natural predators (copepods, copepods, tadpoles and swallows) in the larva, pupa and adult stages. From the results, we see that the stability analysis of the free equilibrium state is stable. With natural implication, there will be no adult female Anopheles mosquitoes to transmit malaria in our society.

The new model used the parameters and variables shown in Table 1. These parameters and variables are chosen with the thresholds obtained in the steady-state disease-free stability analysis of the model. In the analytical output, the model analysis showed the existence of a single disease-free steady state that is locally and asymptotically stable. These threshold parameters and variables mentioned in Table 2 above should be considered when implementing the above model to provide control measures aimed at reducing the prevalence of the malaria parasite in our society and consequently eradicating mosquito disease in Nigeria. Regarding the numerical results, numerical experiments performed using the variables and parameter values in Table 2 and applying disease-free steady-state stability conditions yield the following results:

In Experiment 1, the effect of the introduction of a natural predator, purple swallow, on adult mosquitoes was studied, and the numerical values of variables and parameters were analyzed as shown in Table 2, resolved and numerical simulation. The result shown in Figure 3 after the introduction of a natural predator, the purple swallow, is quite stagnant in adult Anopheles mosquitoes and the transmission rate is very weak.

In Experiment 2, the effect of the introduction of two natural predators, copepods and purple swallows, on mosquito larvae and adults, respectively, was examined, and the numerical values of variables and parameters were examined, as shown in Tables 2. Solve and run a numerical simulation with a graphical representation of the result shown in Figure 4 when two natural predators, copepods and purple swallows, are introduced. Infection in the adult Anopheles mosquito population is significantly slowed down and thus eradicated, and the probability of transmission from the pupa to the adult Anopheles mosquito population is very low.

In Experiment 3, the comparison of the effect of introducing one, two and three natural predators on adult mosquitoes and the numerical diagram in Table 2 were analyzed and solved, and a numerical simulation was performed with a graphical representation of the result as indicated in figure 5 at the introduction of one,

two or three natural predators. The result shows that the infection rate in Figure 5 decreases significantly to prevent new malaria infection.

In Experiment 4, the effect of the introduction of two and three natural predators on mosquito larvae was compared with the numerical values of the variables and parameters presented in Table 2, and a numerical simulation with graphical representation was analyzed, resolved and carried out results shown in Figure 6, when two and three natural predators were introduced, respectively. The result shows that the infection rate in Figure 6 decreases to prevent malaria infection.

In Experiment 5, the effect of introducing a low rate of natural predators, copepods, on mosquito larvae was investigated, and the numerical values of variables and parameters were presented in Tables 2, analyzed, solved and a numerical simulation was played with a graphical representation of the result as shown in Figure 7. Low rate of natural predators, copepods have been introduced. Infection in adult Anopheles mosquitoes has decrease and the percentage of transmission is low.

In Experiment 6, the effect of introducing a high level of natural predators, copepods, on mosquito larvae was studied, and the numerical values of variables and parameters were analyzed, solved and executed. as shown in Table 2, a numerical simulation with a graphical representation of the result, as shown in Figure 8. High levels of natural predators, copepods, have been introduced. Infection in adult mosquitoes of the Anopheles family is fairly stagnant and is therefore well on the way to eliminating malaria infection.

In Experiment 7, the effect of introducing a low number of natural predators, tadpoles, into the mosquito pupa was investigated, and the numerical values of variables and parameters are shown in Tables 2, and the graphical result shown in Figure 9. Low rate of natural predators, introduction of copepods reduced infection in adult Anopheles mosquitoes.

In experiment 8, the effects of the introduction of low and high rate of natural predators, copepods, on mosquito larvae were studied; analyzed, solved and numerical simulations were carried out with numerical values of variables and parameters, as shown in Table 2. The representation of the resulting result in Figure 10 is shown when low and high levels of natural predators, copepods, were introduced. The infection in adult mosquitoes of the Anopheles family is quite stagnant due to the low and high rate of natural predators introduced at the same time, therefore in the process of elimination, and the percentage of transmission is almost nil.

In Experiment 9, the effect of introducing a high rate of natural predators, tadpoles, into the mosquito pupa and the numerical values of the variables and parameters presented in Table 2, were analyzed and resolved, and performed a numerical simulation performed in the graphical representation of Figure 11 when a high level of the natural predator, the tadpole, was introduced. Infection in adult mosquitoes of the Anopheles family was almost nil.

In Experiment 10, the comparison of the effects of introducing low and high numbers of natural predators, tadpoles, into the mosquito pupa and the numerical values of the variables and parameters as shown in Table 2 and the Graphical result in Figure 12 shows that low and high rates of natural predators, copepods, were introduced. Infection in adult mosquitoes of the Anopheles family is fairly stagnant, and therefore on the way to elimination, and the percentage of transmission is low.

When assessing the total population, the effect of introducing two natural predators, one and two, on the adult, larva and adult (swallow and copepod) respectively (compare Figure 3, Figure 4 with Figure 5 and Figure 6). The infectious agent content is greatly reduced and the infection of the egg, larva and pupa is eradicated, but persists at a low level in the adult Anopheles mosquito.

When examining the total population, the effect of introducing low and high rate natural predators (copepods) on the larvae was introduced and studied (compare Figure 7, Figure 8 with Figure 9 and Figure 10). The infectious agent content is greatly reduced and the infection of the egg, larva and pupa is eradicated, but persists at a low level in the adult Anopheles mosquito.

When analyzing the total population, the impact on the introduction of pupae of a high and low rate of natural predators, one (tadpole) was introduced and examined (compare Figure 11 with Figure 12). The infectivity of the adult Anopheles mosquito remains at a low level.

Finally, to understand the effects of introducing three natural enemies (copepods, tadpoles and house swallows) on larvae, pupae and adults when three natural enemies are introduced each, Figures 3,4, 5, ...12 specify the representations to deliver. It could be clearly observed that the transmission speed was reduced to the indispensable minimum. This could be achieved since research should focus on formulating models that capture preventive strategies based on stability analysis to prevent the onset of the disease and thus eradicate it.

## **V. Conclusion**

We state that based on the ideas of existence and uniqueness, elimination, substitution, and maple, we conclude that if the introduced natural predators are large, the number of larvae that will pupate will be close to zero and the number of pupae that will develop into adults will be zero, which will lengthen the life cycle of the

interrupted Anopheles mosquito. Therefore, in our society there will be no adult Anopheles mosquitoes that transmit malaria pathogens.

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