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The Structure of η-Intuitionistic Fuzzy Quotient Rings

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Abstract:*In this paper, we present the structure of η-intuitionistic fuzzy quotient rings which is built from the concept of η-intuitionistic fuzzy quotient sets and η-intuitionistic fuzzy ideals. The η-intuitionistic fuzzy quotient sets is built based on sum and product operation in cosets. As a result, an η-intuitionistic fuzzy set A of $\mathbb{Z}_6$ was obtained as an η-intuitionistic fuzzy quotient ring.*
Key Word*:η-intuitionistic fuzzy set;η-intuitionistic fuzzy ideals; η-intuitionistic fuzzy quotient sets; η-intuitionistic fuzzy quotient rings.*

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I. INTRODUCTION

The development of a set associated with the degree of membership was first introduced where the degree of membership of an element is expressed as a number in closed intervals 0 and 1[1]. That theory is developed to the concept of fuzzy sets prompted various studies on modern algebra.One of them is the theory of fuzzy and anti-fuzzy ring theory was built with operators along with ideal properties and fuzzy homomorphism[2].The characteristics of an intuitionistic fuzzy subring from an intuitionistic fuzzy ring are introducedand an intuitionistic fuzzy ring equipped with an operator[3][4]. Further, the intuitionistic fuzzy ideal in ring concept is built from the intuitionistic fuzzy subring in a commutative ring and intuitionistic fuzzy ideal and prime ideal in a ring[5][6].Moreover, several types of ideal algebra are given, including $(\in, \in \vee q)$-intuitionstic fuzzy ideals of BG-algebra, intuitionistic fuzzy ideal of BCK/BCI-algebras, and its anti-intuitionistic fuzzy soft ideal[7][8][9].

In this paper, it is devoted to the development of the structure of η-intuitionistic fuzzy sets and η-intuitionistic fuzzy subgroups [10] into a new structure of the η-intuitionistic fuzzy quotient sets and η-intuitionistic fuzzy quotient rings.

II. PRELIMINARIES

In this section, we consider that theories of the intuitionistic fuzzy quotient ring to build the concept of η-intuitionistic fuzzy quotient ring.

Definition 2.1[11]**:**Let R is a ring. An intuitionistic fuzzy set (IFS) $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in R\}$ is called intuitionistic fuzzy ring (IFR) of R if it satisfies the following conditions:

1. $\mu_A(x + y) \geq \min(\mu_A(x), \mu_A(y))$
2. $\mu_A(xy) \geq \min(\mu_A(x), \mu_A(y))$
3. $\mu_A(x^{-1}) \geq \mu_A(x)$
4. $\nu_A(x + y) \leq \max(\nu_A(x), \nu_A(y))$
5. $\nu_A(xy) \leq \max(\nu_A(x), \nu_A(y))$
6. $\nu_A(x^{-1}) \leq \nu_A(x)$

for all $x, y \in R$.

Definition 2.2 [12]**:** Let R is a ring. An IFS A over R is called intuitionistic fuzzy ideal (IFI) of R if it satisfies the following conditions:

1. $\mu_A(x - y) \geq \min(\mu_A(x), \mu_A(y))$
2. $\mu_A(xy) \geq \max(\mu_A(x), \mu_A(y))$
3. $\nu_A(x - y) \leq \max(\nu_A(x), \nu_A(y))$
4. $\nu_A(xy) \leq \min(\nu_A(x), \nu_A(y))$

for all $x, y \in R$.

Definition 2.3 [12]**:** Let X is an empty set and A is IFI over X. An intuitionistic fuzzy quotient set (IFQS) X of $A$is defined as

$$\hat{X}/A = \{(x + A, \hat{\mu}_A(x + A), \hat{\nu}_A(x + A)) \mid x \in X\}$$

where

$$\hat{\mu}_A(x + A) = \sup_{a \in A}(\mu_A(x + a))$$

and

$$\hat{\nu}_A(x + A) = \inf_{a \in A}(\nu_A(x + a))$$

for every $x \in X$ and $a \in A$.

Definition 2.4 [12]**:** Let X is an empty set and A is IFI. Let $\hat{X}/A$ is a IFQS over A. The sum operation at IFQS $\hat{X}/A$ is defined as

$$\begin{aligned}((x + A), \hat{\mu}_A(x + A), \hat{\nu}_A(x + A)) &\oplus ((y + A), \hat{\mu}_A(y + A), \hat{\nu}_A(y + A)) \\ &= \left(((x + y) + A), \hat{\mu}_A((x + y) + A), \hat{\nu}_A((x + y) + A)\right)\end{aligned}$$

where

$$\hat{\mu}_A((x + y) + A) = \sup_{a \in A}\left(\mu_A((x + y) + a)\right)$$

and

$$\hat{\nu}_A((x + y) + A) = \inf_{a \in A}\left(\nu_A((x + y) + a)\right).$$

Definition 2.5 [12]**:** Let X is an empty set and A is IFI. Let $\hat{X}/A$ is a IFQS over A. The product operation at IFQS $\hat{X}/A$ is defined as

$$\begin{aligned}((x + A), \hat{\mu}_A(x + A), \hat{\nu}_A(x + A)) &\otimes ((y + A), \hat{\mu}_A(y + A), \hat{\nu}_A(y + A)) \\ &= \left(((xy) + A), \hat{\mu}_A((xy) + A), \hat{\nu}_A((xy) + A)\right)\end{aligned}$$

where

$$\hat{\mu}_A((xy) + A) = \sup_{a \in A}\left(\mu_A((xy) + a)\right)$$

and

$$\hat{\nu}_A((xy) + A) = \inf_{a \in A}\left(\nu_A((xy) + a)\right).$$

Theorem 2.6: Let R is a ring, A is IFI, and $\hat{R}/A$ is IFQS, then $\hat{R}/A$ is an intuitionistic fuzzy quotient ring (IFQR).

Proof. Based on Definition 2.1, for any $x, y \in R$, we will show that:

1. $\hat{\mu}_A((x + y) + A) \geq \min(\hat{\mu}_A(x + A), \hat{\mu}_A(y + A))$
2. $\hat{\mu}_A((xy) + A) \geq \min(\hat{\mu}_A(x + A), \hat{\mu}_A(y + A))$
3. $\hat{\mu}_A(-x + A) \geq \hat{\mu}_A(x + A)$
4. $\hat{\nu}_A((x + y) + A) \leq \max(\hat{\nu}_A(x + A), \hat{\nu}_A(y + A))$
5. $\hat{\nu}_A((xy) + A) \leq \max(\hat{\nu}_A(x + A), \hat{\nu}_A(y + A))$
6. $\hat{\nu}_A(-x + A) \leq \hat{\nu}_A(x + A)$.

Thus, we have:

1. $$\begin{aligned}\hat{\mu}_A((x + y) + A) &= \sup_{a \in A}\left(\mu_A((x + y) + a)\right) \\ &\geq \sup_{a \in A}(\min(\mu_A(x + y), \mu_A(a))) \\ &\geq \sup_{a \in A}\left(\min\left(\min(\mu_A(x), \mu_A(y)), \mu_A(a)\right)\right) \\ &= \min\left(\sup_{a \in A}(\min(\mu_A(x), \mu_A(a))), \sup_{a \in A}(\min(\mu_A(y), \mu_A(a)))\right) \\ &= \min\left(\sup_{a \in A}(\mu_A(x + a)), \sup_{a \in A}(\mu_A(y + a))\right) \\ &= \min(\hat{\mu}_A(x + A), \hat{\mu}_A(y + A))\end{aligned}$$
2. $$\begin{aligned}\hat{\mu}_A((xy) + A) &= \sup_{a \in A}\left(\mu_A((xy) + a)\right) \\ &\geq \sup_{a \in A}(\min(\mu_A(xy), \mu_A(a))) \\ &\geq \sup_{a \in A}\left(\min\left(\min(\mu_A(x), \mu_A(y)), \mu_A(a)\right)\right)\end{aligned}$$

$$= \min\left(\sup_{a\in A}\left(\min\left(\mu_A(x),\mu_A(a)\right)\right),\sup_{a\in A}\left(\min\left(\mu_A(y),\mu_A(a)\right)\right)\right)$$
$$= \min\left(\sup_{a\in A}\left(\mu_A(x+a)\right),\sup_{a\in A}\left(\mu_A(y+a)\right)\right)$$
$$= \min\left(\hat{\mu}_A(x+A),\hat{\mu}_A(y+A)\right)$$

3. $\hat{\mu}_A(-x+A) = \sup_{a\in A}\left(\mu_A\left((-x)+a\right)\right)$
$$\geq \sup_{a\in A}\left(\mu_A(x+a)\right)$$
$$= \hat{\mu}_A(x+A)$$

4. $\hat{\nu}_A\left((x+y)+A\right) = \inf_{a\in A}\left(\nu_A\left((x+y)+a\right)\right)$
$$\leq \inf_{a\in A}\left(\max\left(\nu_A(x+y),\nu_A(a)\right)\right)$$
$$\leq \inf_{a\in A}\left(\max\left(\max\left(\nu_A(x),\nu_A(y)\right),\nu_A(a)\right)\right)$$
$$= \max\left(\inf_{a\in A}\left(\max\left(\nu_A(x),\nu_A(a)\right)\right),\inf_{a\in A}\left(\max\left(\nu_A(y),\nu_A(a)\right)\right)\right)$$
$$= \max\left(\inf_{a\in A}\left(\nu_A(x+a)\right),\inf_{a\in A}\left(\nu_A(y+a)\right)\right)$$
$$= \max\left(\hat{\nu}_A(x+A),\hat{\nu}_A(y+A)\right)$$

5. $\hat{\nu}_A\left((xy)+A\right) = \inf_{a\in A}\left(\nu_A\left((xy)+a\right)\right)$
$$\leq \inf_{a\in A}\left(\max\left(\nu_A(xy),\nu_A(a)\right)\right)$$
$$\leq \inf_{a\in A}\left(\max\left(\max\left(\nu_A(x),\nu_A(y)\right),\nu_A(a)\right)\right)$$
$$= \max\left(\inf_{a\in A}\left(\max\left(\nu_A(x),\nu_A(a)\right)\right),\inf_{a\in A}\left(\max\left(\nu_A(y),\nu_A(a)\right)\right)\right)$$
$$= \max\left(\inf_{a\in A}\left(\nu_A(x+a)\right),\inf_{a\in A}\left(\nu_A(y+a)\right)\right)$$
$$= \max\left(\hat{\nu}_A(x+A),\hat{\nu}_A(y+A)\right)$$

6. $\hat{\nu}_A(-x+A) = \inf_{a\in A}\left(\nu_A\left((-x)+a\right)\right)$
$$\leq \inf_{a\in A}\left(\nu_A(x+a)\right)$$
$$= \hat{\nu}_A(x+A)$$

Then, $\hat{R}/A$ is an IFQR is proved.

Definition 2.7 [10]**:** Let $A = \{(x,\mu_A(x),\nu_A(x)) \mid x \in X\}$ and $B = \{(x,\mu_A(x),\nu_A(x)) \mid x \in X\}$ are IFS's of X. The averaging operator of IFS's A and B over X, denoted by $A\$B$, is defined as

$$A\$B = \left\{\left(x,\sqrt{\mu_A(x)\cdot\mu_B(x)},\sqrt{\nu_A(x)\cdot\nu_B(x)}\right) \mid x \in X\right\}$$

which is $\sqrt{\mu_A(x)\cdot\mu_B(x)}$ will be denoted by $\Phi\left(\mu_A(x),\mu_B(x)\right)$ and $\sqrt{\nu_A(x)\cdot\nu_B(x)}$ will be denoted by $\Phi'\left(\nu_A(x),\nu_B(x)\right)$ on the next discussion.

Definition 2.8 [10]**:** Let X is an empty set. Let A is an IFS over X, and $\eta \in [0,1]$. An IFS $A^\eta = \{(x,\mu_{A^\eta}(x),\nu_{A^\eta}(x)) \mid x \in X\}$ which is
1. $\mu_{A^\eta}(x) = \Phi(\mu_A(x),\eta)$
2. $\nu_{A^\eta}(x) = \Phi'(\nu_A(x),1-\eta)$

is called η-intuitionistic fuzzy set (η-IFS) for every $x \in X$.

III. RESULT AND DISCUSSION

In this section will be discussed the definitions and theorems of η-intuitionistic fuzzy quotient sets and η-intuitionistic fuzzy quotient ring based on sum and product operation in cosets.

Definition 3.1: Let R is a ring. An η-intuitionistic fuzzy set (η-IFS) $A^\eta = \{(x,\mu_{A^\eta}(x),\nu_{A^\eta}(x)) \mid x \in R\}$ is called η-intuitionistic fuzzy ring (η-IFR) of R if it satisfies the following conditions:
1. $\mu_{A^\eta}(x+y) \geq \min\left(\mu_{A^\eta}(x),\mu_{A^\eta}(y)\right)$
2. $\mu_{A^\eta}(xy) \geq \min\left(\mu_{A^\eta}(x),\mu_{A^\eta}(y)\right)$
3. $\mu_{A^\eta}(x^{-1}) \geq \mu_{A^\eta}(x)$
4. $\nu_{A^\eta}(x+y) \leq \max\left(\nu_{A^\eta}(x),\nu_{A^\eta}(y)\right)$
5. $\nu_{A^\eta}(xy) \leq \max\left(\nu_{A^\eta}(x),\nu_{A^\eta}(y)\right)$
6. $\nu_{A^\eta}(x^{-1}) \leq \nu_{A^\eta}(x)$

for all $x, y \in R$.

Definition 3.2: Let R is a ring. An η-IFS A^η over R is called η-intuitionistic fuzzy ideal (η-IFI) of R if it satisfies the following conditions:

1. $\mu_{A^\eta}(x-y) \geq \min\left(\mu_{A^\eta}(x), \mu_{A^\eta}(y)\right)$
2. $\mu_{A^\eta}(xy) \geq \max\left(\mu_{A^\eta}(x), \mu_{A^\eta}(y)\right)$
3. $\nu_{A^\eta}(x-y) \leq \max\left(\nu_{A^\eta}(x), \nu_{A^\eta}(y)\right)$
4. $\nu_{A^\eta}(xy) \leq \min\left(\nu_{A^\eta}(x), \nu_{A^\eta}(y)\right)$

for all $x, y \in R$.

Definition 3.3: Let X is an empty set and A^η is η-IFI over X. An η-intuitionistic fuzzy quotient set (η-IFQS) X of A^η is defined as

$$\hat{X}/A^\eta = \left\{\left(x + A^\eta, \hat{\mu}_{A^\eta}(x + A^\eta), \hat{\nu}_{A^\eta}(x + A^\eta)\right) \mid x \in X\right\}$$

where

$$\hat{\mu}_{A^\eta}(x + A^\eta) = \sup_{a \in A^\eta}\left(\mu_{A^\eta}(x + a)\right)$$

and

$$\hat{\nu}_{A^\eta}(x + A^\eta) = \inf_{a \in A^\eta}\left(\nu_{A^\eta}(x + a)\right)$$

for every $x \in X$ and $a \in A^\eta$.

Definition 3.4: Let X is an empty set and A^η is η-IFI. Let $\hat{X}/A^\eta$ is a η-IFQS over A^η. The sum operation at η-IFQS $\hat{R}/A^\eta$ is defined as

$$\begin{aligned}&\left((x + A^\eta), \hat{\mu}_{A^\eta}(x + A^\eta), \hat{\nu}_{A^\eta}(x + A^\eta)\right) \oplus \left((y + A^\eta), \hat{\mu}_{A^\eta}(y + A^\eta), \hat{\nu}_{A^\eta}(y + A^\eta)\right)\\ &= \left(\left((x+y) + A^\eta\right), \hat{\mu}_{A^\eta}\left((x+y) + A^\eta\right), \hat{\nu}_{A^\eta}\left((x+y) + A^\eta\right)\right)\end{aligned}$$

where

$$\hat{\mu}_{A^\eta}\left((x+y) + A^\eta\right) = \sup_{a \in A^\eta}\left(\mu_{A^\eta}\left((x+y) + a\right)\right)$$

and

$$\hat{\nu}_{A^\eta}\left((x+y) + A^\eta\right) = \inf_{a \in A^\eta}\left(\nu_{A^\eta}\left((x+y) + a\right)\right).$$

Definition 3.5: Let X is an empty set and A^η is η-IFI. Let $\hat{X}/A^\eta$ is a η-IFQS over A^η. The product operation at η-IFQS $\hat{X}/A^\eta$ is defined as

$$\begin{aligned}&\left((x + A^\eta), \hat{\mu}_{A^\eta}(x + A^\eta), \hat{\nu}_{A^\eta}(x + A^\eta)\right) \otimes \left((y + A^\eta), \hat{\mu}_{A^\eta}(y + A^\eta), \hat{\nu}_{A^\eta}(y + A^\eta)\right)\\ &= \left(\left((xy) + A^\eta\right), \hat{\mu}_{A^\eta}\left((xy) + A^\eta\right), \hat{\nu}_{A^\eta}\left((xy) + A^\eta\right)\right)\end{aligned}$$

where

$$\hat{\mu}_{A^\eta}\left((xy) + A^\eta\right) = \sup_{a \in A^\eta}\left(\mu_{A^\eta}\left((xy) + a\right)\right)$$

and

$$\hat{\nu}_{A^\eta}\left((xy) + A^\eta\right) = \inf_{a \in A^\eta}\left(\nu_{A^\eta}\left((xy) + a\right)\right).$$

Theorem 3.6: Let R is a ring, A^η is η-IFI, and $\hat{R}/A^\eta$ is η-IFQS, then $\hat{R}/A^\eta$ is an η-intuitionistic fuzzy quotient ring (η-IFQR).

Proof. Based on Definition 3.1, for any $x, y \in R$, we will show that:

1. $\hat{\mu}_{A^\eta}\left((x+y) + A^\eta\right) \geq \min\left(\hat{\mu}_{A^\eta}(x + A^\eta), \hat{\mu}_{A^\eta}(y + A^\eta)\right)$
2. $\hat{\mu}_{A^\eta}\left((xy) + A^\eta\right) \geq \min\left(\hat{\mu}_{A^\eta}(x + A^\eta), \hat{\mu}_{A^\eta}(y + A^\eta)\right)$
3. $\hat{\mu}_{A^\eta}(-x + A^\eta) \geq \hat{\mu}_{A^\eta}(x + A^\eta)$
4. $\hat{\nu}_{A^\eta}\left((x+y) + A^\eta\right) \leq \max\left(\hat{\nu}_{A^\eta}(x + A^\eta), \hat{\nu}_{A^\eta}(y + A^\eta)\right)$
5. $\hat{\nu}_{A^\eta}\left((xy) + A^\eta\right) \leq \max\left(\hat{\nu}_{A^\eta}(x + A^\eta), \hat{\nu}_{A^\eta}(y + A^\eta)\right)$
6. $\hat{\nu}_{A^\eta}(-x + A^\eta) \leq \hat{\nu}_{A^\eta}(x + A^\eta)$.

Thus, we have:

1. $$\begin{aligned}\hat{\mu}_{A^\eta}\left((x+y) + A^\eta\right) &= \sup_{a \in A^\eta}\left(\mu_{A^\eta}\left((x+y) + a\right)\right)\\ &= \sup_{a \in A^\eta}\left(\psi\left(\mu_A\left((x+y) + a\right), \eta\right)\right)\\ &= \sup_{a \in A^\eta}\left(\min\left(\psi(\mu_A(x+y), \eta), \psi(\mu_A(a), \eta)\right)\right)\end{aligned}$$

$$\geq \sup_{a\in A^\eta}\left(\min\left(\min\left(\psi(\mu_A(x),\eta),\psi(\mu_A(y),\eta)\right),\psi(\mu_A(a),\eta)\right)\right)$$
$$= \sup_{a\in A^\eta}\left(\min\begin{pmatrix}\min\left(\psi(\mu_A(x),\eta),\psi(\mu_A(a),\eta)\right),\\ \min\left(\psi(\mu_A(y),\eta),\psi(\mu_A(a),\eta)\right)\end{pmatrix}\right)$$
$$= \sup_{a\in A^\eta}\left(\min\left(\psi(\mu_A(x+a),\eta),\psi(\mu_A(y+a),\eta)\right)\right)$$
$$= \min\left(\sup_{a\in A^\eta}\left(\psi(\mu_A(x+a),\eta)\right),\sup_{a\in A^\eta}\left(\psi(\mu_A(y+a),\eta)\right)\right)$$
$$= \min\left(\sup_{a\in A^\eta}\left(\mu_{A^\eta}(x+a)\right),\sup_{a\in A^\eta}\left(\mu_{A^\eta}(y+a)\right)\right)$$
$$= \min\left(\hat{\mu}_{A^\eta}(x+A^\eta),\hat{\mu}_{A^\eta}(x+A^\eta)\right)$$

2. $\hat{\mu}_{A^\eta}\left((xy)+A^\eta\right) = \sup_{a\in A^\eta}\left(\mu_{A^\eta}\left((xy)+a\right)\right)$
$$= \sup_{a\in A^\eta}\left(\psi\left(\mu_A\left((xy)+a\right),\eta\right)\right)$$
$$= \sup_{a\in A^\eta}\left(\min\left(\psi(\mu_A(xy),\eta),\psi(\mu_A(a),\eta)\right)\right)$$
$$\geq \sup_{a\in A^\eta}\left(\min\left(\min\left(\psi(\mu_A(x),\eta),\psi(\mu_A(y),\eta)\right),\psi(\mu_A(a),\eta)\right)\right)$$
$$= \sup_{a\in A^\eta}\left(\min\begin{pmatrix}\min\left(\psi(\mu_A(x),\eta),\psi(\mu_A(a),\eta)\right),\\ \min\left(\psi(\mu_A(y),\eta),\psi(\mu_A(a),\eta)\right)\end{pmatrix}\right)$$
$$= \sup_{a\in A^\eta}\left(\min\left(\psi(\mu_A(x+a),\eta),\psi(\mu_A(y+a),\eta)\right)\right)$$
$$= \min\left(\sup_{a\in A^\eta}\left(\psi(\mu_A(x+a),\eta)\right),\sup_{a\in A^\eta}\left(\psi(\mu_A(y+a),\eta)\right)\right)$$
$$= \min\left(\sup_{a\in A^\eta}\left(\mu_{A^\eta}(x+a)\right),\sup_{a\in A^\eta}\left(\mu_{A^\eta}(y+a)\right)\right)$$
$$= \min\left(\hat{\mu}_{A^\eta}(x+A^\eta),\hat{\mu}_{A^\eta}(x+A^\eta)\right)$$

3. $\hat{\mu}_{A^\eta}(-x+A^\eta) = \sup_{a\in A^\eta}\left(\mu_{A^\eta}(-x+a)\right)$
$$= \sup_{a\in A^\eta}\left(\psi(\mu_A(-x+a),\eta)\right)$$
$$\geq \sup_{a\in A^\eta}\left(\min\left(\psi(\mu_A(-x),\eta),\psi(\mu_A(a),\eta)\right)\right)$$
$$= \sup_{a\in A^\eta}\left(\min\left(\psi(\mu_A(x),\eta),\psi(\mu_A(a),\eta)\right)\right)$$
$$= \sup_{a\in A^\eta}\left(\psi(\mu_A(x+a),\eta)\right)$$
$$= \sup_{a\in A^\eta}\left(\mu_{A^\eta}(-x+a)\right)$$
$$= \hat{\mu}_{A^\eta}(x+A^\eta)$$

4. $\hat{\nu}_{A^\eta}\left((x+y)+A^\eta\right) = \inf_{a\in A^\eta}\left(\nu_{A^\eta}\left((x+y)+a\right)\right)$
$$= \inf_{a\in A^\eta}\left(\psi'\left(\nu_A\left((x+y)+a\right),1-\eta\right)\right)$$
$$= \inf_{a\in A^\eta}\left(\max\left(\psi'(\nu_A(x+y),1-\eta),\psi'(\nu_A(a),1-\eta)\right)\right)$$
$$\leq \inf_{a\in A^\eta}\left(\max\left(\max\begin{pmatrix}\psi'(\nu_A(x),1-\eta),\\ \psi'(\nu_A(y),1-\eta)\end{pmatrix},\psi'(\nu_A(a),1-\eta)\right)\right)$$
$$= \inf_{a\in A^\eta}\left(\max\begin{pmatrix}\max\left(\psi'(\nu_A(x),1-\eta),\psi'(\nu_A(a),1-\eta)\right),\\ \max\left(\psi'(\nu_A(y),1-\eta),\psi'(\nu_A(a),1-\eta)\right)\end{pmatrix}\right)$$
$$= \inf_{a\in A^\eta}\left(\max\left(\psi'(\nu_A(x+a),1-\eta),\psi'(\nu_A(y+a),1-\eta)\right)\right)$$
$$= \max\left(\inf_{a\in A^\eta}\left(\psi'(\nu_A(x+a),1-\eta)\right),\inf_{a\in A^\eta}\left(\psi'(\nu_A(y+a),1-\eta)\right)\right)$$
$$= \max\left(\inf_{a\in A^\eta}\left(\nu_{A^\eta}(x+a)\right),\inf_{a\in A^\eta}\left(\nu_{A^\eta}(y+a)\right)\right)$$
$$= \max\left(\hat{\nu}_{A^\eta}(x+A^\eta),\hat{\nu}_{A^\eta}(x+A^\eta)\right)$$

5. $\hat{\nu}_{A^\eta}\left((xy)+A^\eta\right) = \inf_{a\in A^\eta}\left(\nu_{A^\eta}\left((xy)+a\right)\right)$
$$= \inf_{a\in A^\eta}\left(\psi'\left(\nu_A\left((xy)+a\right),1-\eta\right)\right)$$
$$= \inf_{a\in A^\eta}\left(\max\left(\psi'(\nu_A(xy),1-\eta),\psi'(\nu_A(a),1-\eta)\right)\right)$$
$$\leq \inf_{a\in A^\eta}\left(\max\left(\max\begin{pmatrix}\psi'(\nu_A(x),1-\eta),\\ \psi'(\nu_A(y),1-\eta)\end{pmatrix},\psi'(\nu_A(a),1-\eta)\right)\right)$$

$$= \inf_{a\in A^\eta}\left(\max\begin{pmatrix}\max(\psi'(\nu_A(x),1-\eta),\psi'(\nu_A(a),1-\eta)),\\ \max(\psi'(\nu_A(y),1-\eta),\psi'(\nu_A(a),1-\eta))\end{pmatrix}\right)$$
$$= \inf_{a\in A^\eta}\left(\max(\psi'(\nu_A(x+a),1-\eta),\psi'(\nu_A(y+a),1-\eta))\right)$$
$$= \max\left(\inf_{a\in A^\eta}\left(\psi'(\nu_A(x+a),1-\eta)\right),\inf_{a\in A^\eta}\left(\psi'(\nu_A(y+a),1-\eta)\right)\right)$$
$$= \max\left(\inf_{a\in A^\eta}\left(\nu_{A^\eta}(x+a)\right),\inf_{a\in A^\eta}\left(\nu_{A^\eta}(y+a)\right)\right)$$
$$= \max(\hat{\nu}_{A^\eta}(x+A^\eta),\hat{\nu}_{A^\eta}(x+A^\eta))$$

6. $\hat{\nu}_{A^\eta}(-x+A^\eta) = \inf_{a\in A^\eta}\left(\nu_{A^\eta}(-x+a)\right)$
$$= \inf_{a\in A^\eta}\left(\psi'(\nu_A(-x+a),1-\eta)\right)$$
$$\le \inf_{a\in A^\eta}\left(\max(\psi'(\nu_A(-x),1-\eta),\psi'(\nu_A(a),1-\eta))\right)$$
$$= \inf_{a\in A^\eta}\left(\max(\psi'(\nu_A(x),1-\eta),\psi'(\nu_A(a),1-\eta))\right)$$
$$= \inf_{a\in A^\eta}\left(\psi'(\nu_A(x+a),1-\eta)\right)$$
$$= \inf_{a\in A^\eta}\left(\nu_{A^\eta}(-x+a)\right)$$
$$= \hat{\nu}_{A^\eta}(x+A^\eta)$$

Then, $\hat{R}/A^\eta$ is an η-IFQR is proved.

Example 3.7: Let $\mathbb{Z}_6$ is a ring. Defined $\mu_A : \mathbb{Z}_6 \to [0,1]$ and $\nu_A : \mathbb{Z}_6 \to [0,1]$, which is

$$\mu_A(x) = \begin{cases}0{,}23 & ,x=\bar{1},\bar{2},\bar{4},\bar{5}\\ 0{,}40 & ,x=\bar{0},\bar{3}\end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases}0{,}77 & ,x=\bar{1},\bar{2},\bar{4},\bar{5}\\ 0{,}25 & ,x=\bar{0},\bar{3}\end{cases}$$

Let $\eta = 0{,}65$, thus $1-\eta = 0{,}35$. We refer to the averaging operation in Definition 2.7, we have $\mu_{A^\eta}(x) = 0{,}39$ and $\nu_{A^\eta}(x) = 0{,}52$ for $x = \{\bar{1},\bar{2},\bar{4},\bar{5}\}$, while $\mu_{A^\eta}(x) = 0{,}51$ and $\nu_{A^\eta}(x) = 0{,}30$ for $x = \{\bar{0},\bar{3}\}$.It can be shown that η-IFQR based on the following description:

1. Let any $x = \bar{1}$,$y = \bar{3}$, and $a = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5}\}$.We have, $\mu_{A^\eta}(\bar{1}+a) = \{0{,}39, 0{,}39, 0{,}51, 0{,}39, 0{,}39, 0{,}51\}$,$\mu_{A^\eta}(\bar{3}+a) = \{0{,}51, 0{,}39, 0{,}39, 0{,}51, 0{,}39, 0{,}39\}$, and $\mu_{A^\eta}((\bar{1}+\bar{3})+a) = \{0{,}39, 0{,}39, 0{,}51, 0{,}39, 0{,}39, 0{,}51\}$. Thus, we obtain $\sup_{a\in A^\eta}(\mu_{A^\eta}(\bar{1}+a)) = 0{,}51$, $\sup_{a\in A^\eta}(\mu_{A^\eta}(\bar{3}+a)) = 0{,}51$, and $\sup_{a\in A^\eta}\left(\mu_{A^\eta}((\bar{1}+\bar{3})+a)\right) = 0{,}51$. Hence, $\hat{\mu}_{A^\eta}((\bar{1}+\bar{3})+A^\eta) \ge \min(\hat{\mu}_{A^\eta}(\bar{1}+A^\eta),\hat{\mu}_{A^\eta}(\bar{3}+A^\eta))$.
2. Similarly, we have $\mu_{A^\eta}((\bar{1}\cdot\bar{3})+a) = \{0{,}51, 0{,}39, 0{,}39, 0{,}51, 0{,}39, 0{,}39\}$. Thus, we obtainsup$_{a\in A^\eta}(\mu_{A^\eta}(\bar{1}+a)) = 0{,}51$, $\sup_{a\in A^\eta}(\mu_{A^\eta}(\bar{3}+a)) = 0{,}51$, and $\sup_{a\in A^\eta}\left(\mu_{A^\eta}((\bar{1}\cdot\bar{3})+a)\right) = 0{,}51$. Hence,$\hat{\mu}_{A^\eta}((\bar{1}\cdot\bar{3})+A^\eta) \ge \min(\hat{\mu}_{A^\eta}(\bar{1}+A^\eta),\hat{\mu}_{A^\eta}(\bar{3}+A^\eta))$.
3. Let any $x = \bar{1}$, thus $-x = \bar{5}$, and $a = \{\bar{0},\bar{1},\bar{2},\bar{3},\bar{4},\bar{5}\}$. We have, $\mu_{A^\eta}(\bar{1}+a) = \{0{,}39, 0{,}39, 0{,}51, 0{,}39, 0{,}39, 0{,}51\}$ and $\mu_{A^\eta}(\bar{5}+a) = \{0{,}39, 0{,}51, 0{,}39, 0{,}39, 0{,}51, 0{,}39\}$. Thus, we obtainsup$_{a\in A^\eta}(\mu_{A^\eta}(\bar{1}+a)) = 0{,}51$andsup$_{a\in A^\eta}\left(\mu_{A^\eta}(\bar{5}+a)\right) = 0{,}51$. Hence,$\hat{\mu}_{A^\eta}(\bar{5}+A^\eta) \ge \hat{\mu}_{A^\eta}(\bar{1}+A^\eta)$.

To show the other axioms of the degree of non membership can use the same method as above.
Then, that can be proved that $\hat{R}/A^\eta = \{(x+A^\eta,\hat{\mu}_{A^\eta}(x+A^\eta),\hat{\nu}_{A^\eta}(x+A^\eta)) \mid x\in\mathbb{Z}_6\}$ is an η-IFQR.

IV. CONCLUSION

Based on the result and discussion, a new structure of η-intuitionistic fuzzy quotient set and η-intuitionistic fuzzy quotient ring are obtained with sum and product operator in cosets. On the next research, it is suggested to build a new structure related to homomorphism and isomorphism mapping and its properties in η-intuitionistic fuzzy quotient ring concepts.

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