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Queueing Model for PoA based Blockchain system

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Abstract: *For many Blockchain applications, Proof of Authority (PoA) is a desirable replacement for PoW-based protocols due to its great performance and energy efficiency. In this work, a Markovian queue-based quantitative analysis is framed to represent authentic PoA-based Blockchains. Consensus process of Blockchain is analysed theoretically using queueing model* M^b */ M / 1. Primary performance measures such as expected time of transaction in system (transaction confirmation time), expected time of transaction in queue, expected number of transactions in system and expected number of transactions in queue, throughput are examined numerically with the help of python.*

Keywords: *Blockchain, Consensus, Proof of Authority, Queueing model, Throughput*

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I. Introduction

Industry is more interested in Blockchain technology, has gain widespread acceptance and became independent of cryptocurrencies. In 2008, Satoshi Nakamoto proposed an electronic cash system for bitcoin. It is peer-to-peer (P2P) technology, allowing online payments to be initiated directly by sender to receiver without going through any financial foundations. This idea gave rise to the term "Blockchain". The networks like Bitcoin, Ethereum have garnered public attention to Blockchain technology during the last decade.

Decentralization, persistence, anonymity and audibility are among the key features of Blockchain technology. Multiple core technologies including the integrated password hash, the digital signature (based on asymmetric encryption), and the distributed consensus mechanism can work in a decentralized framework to implement Blockchain.

All validating nodes participating in the Blockchain network are required to complete the verification of transactions according to the consensus mechanisms theory. To hold tamper-proof transaction records and smart contracts, public chains using consensus protocol like Proof of Work (PoW) have created decentralized databases or ledgers.

Consensus process is the foundation of Blockchain technology, that needs to perform properly. Because of its perceived shortcomings, the Proof of Work (PoW) consensus is not widely utilized in many distributed applications. For many Blockchain applications, Proof of Authority (PoA) is a desirable replacement for PoW-based protocols due to its great performance and energy efficiency.

Unfortunately, it appears that as Blockchain technology evolves, performance faults other than the core issue of trust, such as scalability and speed limitations, will emerge. They are quickly becoming the major obstacle to success. The ability of raising performance to the next level to meet market needs and criteria is essential for this technology to succeed. Analytical model to analyse and ensure the performance of concern is established. Quantitatively, it is highly efficient during the early design cycle. It is prerequisite to the performance demands and requirements.

In this paper mathematical analysis of Proof-of-Authority (PoA) consensus has been done using Queueing theory. One of the BFT algorithms called Proof-of-Authority (PoA) has recently gained popularity due to the performance and fault tolerance it offers. Markovian Queueing model $M^b/\ M\ /\ 1$ is analysed to establish theoretical consensus process for Blockchain framework. Bulk arrivals of transactions in Poisson manner are assumed.

From the basic performance measures like average transaction confirmation time i.e., block posting time and throughput in terms of number of transactions in a block processed per time is demonstrated. This can easily and effectively solve the queueing aspect of transaction flow and block posting. It eventually prevents high performance Blockchain system. PoA makes a strong premise that authorities are trusted. So, it is only suitable for private implementation.

II. Literature Review

Angelis et. al.[2] (2018) derived the functioning of two prominent consensus algorithms for permissioned Blockchains based on the PoA paradigm, namely Aura and Clique. A qualitative comparison of them with respect to PBFT in terms of consistency, availability and performance were provided. Also, a qualitative latency analysis based on message rounds was reported using it. As per their study, byzantine nodes are used for internet based PoA for permissioned Blockchains. It does not offer sufficient consistency guarantee for situations when data integrity is crucial.

Liu. et. al.[7] (2019) investigated PoA protocol's security and scalability. A flexible quantitative analysis framework based on the Markov Decision Process was put forward. The outline can be used to indicate security in terms of adversarial optimal strategies. A formal model of PoA and its attack model were given to capture the action tactics of an enemy. To validate the modelling process, they discussed the security provisions of a PoA consensus as a case study.

Wu et. al.[11] (2020) suggested a hybrid consensus algorithm which combines advantages of the PoS and PBFT algorithms. It was divided into two parts: sortition and witness. The new algorithm reduced the number of consensus nodes to a constant value by verifiable pseudorandom sortition and performs transaction witness between nodes. From the experiments they obtained that the improved hybrid consensus algorithm was significantly superior to the previous single algorithms for its excellent scalability, throughput, and low latency.

Seol et.al.[9] (2020) proposed a novel model of the form $M^{1,n}/M^n/1$ embedded Markovian queue to build a theoretical framework for creating a Blockchain-based system. They denoted $M^{1,n}$ variable bulk arrivals of transactions with a Poisson distribution, where n is the total number of slots for all mined transactions and M^n denoted static bulk service of transactions in exponential time. The three main performance indicators monitored are throughput, which is defined as the average number of slots to be processed per time, average waiting time per slot, and average slot count regardless of how many transactions are mined under the assumption that there are n maximum slots per block. Numerical simulations were done in MATLAB to show the model's effectiveness.

Lian et. al.[6] (2020) analyzed the principle and basic knowledge of the Markov chain and the queuing theory. They proposed a new stock trading method based on the Blockchain. They established a queuing model with priority-based queue on the Blockchain. They used the Markov chain to schedule resources for the queuing model. Optimized and improved stock trading showed efficient results which can shorten the settlement time and improve the liquidity of funds.

Joshi[5] (2021) explored the feasibility of Proof of Authority Consensus algorithm in a Blockchain network. Because of its inherent advantages, PoA offers a viable alternative for the drawbacks of PoW and PoS consensus techniques. PoA is a good alternative for developing and maintaining DApps, private Blockchains, and other decentralized applications because it compromises decentralization to get high throughput and scalability.

Manolache et. al.[8] (2022) proposed the method of decision-making system in Blockchain. In which the ranking of a decision was determined by all the participants, while a superior voting power to the most specialized and experienced participant is provided. Which took key concepts from the Blockchain such as transparency and incorruptibility. Different results were analysed, by showcasing some test scenarios which will run in the Quorum network, and detailing. The paper was designed with a new approach when it comes to decision making, using the strengths of different technologies, and merging them in a new system.

Vora and Gor[10] (2022) analysed M/G/1 queue with discrete time Markov chain (DTMC) in Blockchain system. Queueing theory was theoretically analysed for the block-generation and Blockchain building processes. In this, the sum of the block-generation and block-building times was considered as the transaction-confirmation time of a block. The Blockchain system was examined at arrival as well as departure point. Average transaction confirmation time, number of transactions in queue at arrival point and the number of transactions at departure points etc. were observed with the help of python.

Consensus Protocol Proof of Authority:[6],[2]

Here, Proof of Authority Consensus algorithm is described. Permissioned and permissionless Blockchain are the two categories that can be used to categorize Blockchain systems. Depending on whether a certain block's production is accessible by everyone or just available to a specific set of authority nodes. A new family of BFT algorithms called the Proof of Authority (PoA) protocol is designed for consortiums where only approved nodes are permitted to submit transactions and build the Blockchain.

The authorities are a group of N trusted nodes that are essential to the PoA algorithms' consensus process. A distinct Id is provided to every authority (node) in the network. There must be $\frac{N}{2}+1$ honest authorities (node) in the network. The consensus process can be divided into following parts.

- **Leader election:** In the current round t, a leader l is chosen using a mining rotation scheme[1]. Depending on how it is implemented, the election has different procedure. This leader has the authority to group the transactions sent by clients and generate a block b_t.
- **Block Proposal:** Following the election, the leader l can arrange the transactions and group them into the block b_t. This must be linked to the previous block b_{t-1} of his local Blockchain view to serve as the parent block. After that, he will add his signature on the block b_t before sending it out to the network.
- **Delivery:** The other authorities will determine whether to accept or reject the proposed block b_t after comparing its signature and parent block's information to the results (such as the leader's index in the authorities set) they computed during the election process. A block b_t will be sent to the peers it was connected to whenever an authority accepts it.
- **Block Committing:** The chain selection process can be done using GHOST in Clique [2] or by such other techniques. Else, when the majority of authorized nodes acquire the same block b_t, those nodes will add the block b_t to their local storage. Otherwise, they will retain the previous version.

III. Model Description

Blockchain systems are basically bulk queuing systems. Markovian single server exponential queueing system $M^x / M / 1$ or $M^b / M / 1$ is considered to describe the consensus process.

Transactions arrive (in the form of block) in the system for consensus process/service. They exit from the system when a block is added to the Blockchain. A block is made by the batch of identical transactions and then sent in batches for processing. A situation like this can be modelled as a queue with mass arrivals.

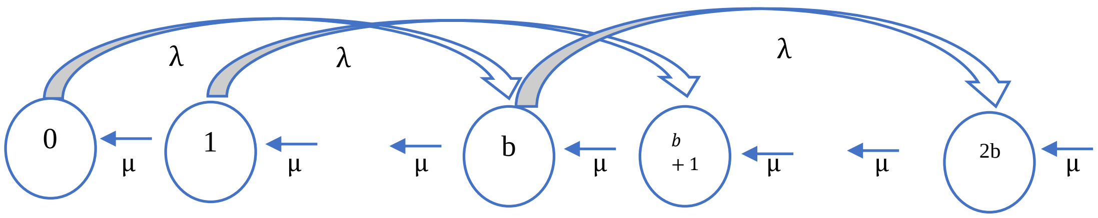
- **Arrival process:** Transactions arrive in the system for consensus process/service in batches. The leader l can arrange and group the arrived transactions into the block b_t.They arrive in the form of block according to Poisson process with arrival rate λ.
- **Service process:** By grouping all transactions into a block, the leader forms the block. After signing the block, the leader put it in the network. The remaining authorities will decide whether to accept or reject it immediately after reviewing the proposed block b_t's signature and parent block's data. A block is added to the Blockchain when most of authorized nodes receive it.

  The service is provided within a specific timeframe, but no transaction is anticipated to occur in the interim. The Service time is the time required for agreement. And the process for consensus is considered as service process. The terms utilized in this study are displayed in Table 1.

Table 1: Terminology

λ	Arrival rate of a transaction
μ	Service rate of transaction
b	The group of transactions. i.e., batch size
ρ	Utility factor of Blockchain system
$N(t)$	Total Number of transactions in the system at time t
$N_q(t)$	Number of transactions in queue at time t
$N_s(t)$	Number of transactions in system at time t
T_q	Time a transaction spends to wait in a queue
T	Total time a transaction spends in the system
$E[N]$	Expected number of transactions
$E[N_q]$	Expected number of transactions in queue
$E[T_q]$	Expected waiting time in queue
$E[T]$	Expected waiting time in the system

IV. Analysis



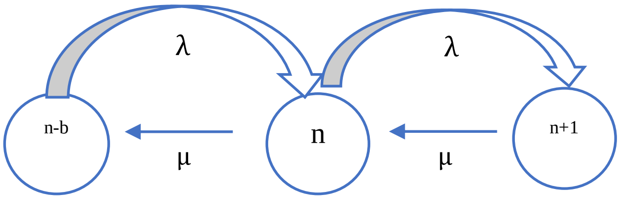


Figure 1: Transition Diagrams

Let P_n $(n = 0,1,2 \dots n)$ be steady-state probability that there are n transactions in the Blockchain queueing system. The state balance equation can be written with the help of diagram as:

$$(\lambda + \mu)P_n = \text{Flow rate out of } n \text{ state}$$
$$\lambda P_{n-b} + \mu P_{n+1} = \text{Flow rate into } n \text{ state}$$

By comparing,

$$(\lambda + \mu)P_n = \lambda P_{n-b} + \mu P_{n+1}; \qquad n \geq b \qquad \dots(1)$$
$$(\lambda + \mu)P_n = \mu P_{n+1}; \qquad n < b, n = 1,2,3, \dots, b-1 \qquad \dots(2)$$

From (2), $\lambda P_0 = \mu P_1$

i.e. $\mu P_1 - \lambda P_0 = 0; \qquad n = 0 \qquad \dots(3)$

Multiplying (1) by z^n under the summation from b to ∞, and (2) by z^n under the sum from $n = 1$ to $b - 1$ and combining with (3),

$$\sum_{n=b}^{\infty} \lambda P_{n-b} z^n + \sum_{n=b}^{\infty} \mu P_{n+1} z^n - \sum_{n=b}^{\infty} (\lambda + \mu) P_n z^n + \sum_{n=1}^{b-1} \mu P_{n+1} z^n - \sum_{n=1}^{b-1} (\lambda + \mu) P_n z^n + (\mu P_1 - \lambda P_0) = 0$$

$$\Rightarrow z^b \sum_{n=b}^{\infty} \lambda P_{n-b} z^{n-b} + \sum_{n=1}^{\infty} \mu P_{n+1} z^n - \sum_{n=1}^{\infty} (\lambda + \mu) P_n z^n + (\mu P_1 - \lambda P_0) = 0$$

$$\Rightarrow \lambda z^b \sum_{n=b}^{\infty} P_{n-b} z^{n-b} + \mu z^{-1} \left[\sum_{n=-1}^{\infty} P_{n+1} z^{n+1} - P_0 z^0 - P_1 z^1 \right] - \sum_{n=0}^{\infty} (\lambda + \mu) P_n z^n + (\lambda + \mu) P_0 + \mu P_1 - \lambda P_0 = 0$$

Generating function of P_n can be defined as:

$$G(z) = \sum_{n=0}^{\infty} P_n z^n = P_0 z^0 + \sum_{n=1}^{\infty} P_n z^n$$

i.e., $\quad G(z) - P_0 = \sum_{n=1}^{\infty} P_n z^n \qquad \dots(4)$

$$\Rightarrow \lambda z^b G(z) + \mu z^{-1} [G(z) - P_0 - P_1 z^1] - G(z)(\lambda + \mu) + (\lambda + \mu) P_0 + \mu P_1 - \lambda P_0 = 0$$
$$\Rightarrow G(z)[\lambda z^b + \mu z^{-1} - (\lambda + \mu)] + \mu P_0 - \mu z^{-1} = 0$$
$$\Rightarrow G(z) \frac{\lambda z^{b+1} + \mu - (\lambda + \mu) z}{z} + \mu \left(1 - \frac{1}{z}\right) P_0 = 0$$
$$\Rightarrow G(z) = \frac{\mu(1-z)P_0}{\mu - (\lambda + \mu) z + \lambda z^{b+1}} = \frac{\mu(1-z)P_0}{\mu - \mu z - \lambda z + \lambda z z^b}$$
$$= \frac{\mu(1-z)P_0}{\mu(1-z) - \lambda z(1-z)[1 + z + z^2 + \dots + z^{b-1}]} = \frac{\mu P_0}{\mu - \lambda z \sum_{j=0}^{b-1} z^j}$$
$$\Rightarrow G(z) = \frac{\mu P_0}{\mu - \lambda \sum_{j=1}^{b} z^j} \qquad \dots(5)$$

For $z = 1, G(1) = \frac{\mu P_0}{\mu - \lambda \sum_{j=0}^{b} (1)^j} = \frac{\mu P_0}{\mu - \lambda \text{b}}$

From $G(z) = \sum_{n=0}^{\infty} P_n z^n \Rightarrow G(1) = \sum_{n=0}^{\infty} P_n (1)^n = 1$

$$1 = \frac{\mu P_0}{\mu - \lambda \text{b}}$$

The utilization of the system ρ can be defined as $\rho = \frac{\lambda \text{b}}{\mu}$

Therefor, $1 = \frac{P_0}{1 - \frac{\lambda \text{b}}{\mu}} = \frac{P_0}{1 - \rho}$

$$P_0 = 1 - \rho$$

$G(z)$ becomes,

$$G(Z) = \frac{\mu(1-\rho)}{\mu - \lambda \sum_{j=1}^{b} z^j}$$

Performance Measures of the Blockchain System:

1. Expected number of transactions in the system:

$$E[N] = \sum_{n=0}^{\infty} nP_n$$

$$= \left[\frac{d}{dz} G(z)\right]_{z=1} = \left[\frac{d}{dz}\left[\frac{\mu(1-\rho)}{\mu - \lambda \sum_{j=1}^{b} z^j}\right]\right]_{z=1}$$

$$= \left[\mu(1-\rho)(-1)\left[\mu - \lambda \sum_{j=1}^{b} z^j\right]^{-2} . (-\lambda) \sum_{j=1}^{b} jz^{j-1}\right]_{z=1}$$

$$= \left[\frac{\mu(1-\rho)\lambda \sum_{j=1}^{b} jz^{j-1}}{[\mu - \lambda \sum_{j=1}^{b} z^j]^2}\right]_{z=1} = \frac{\lambda\mu(1-\rho)\frac{b(b+1)}{2}}{(\mu - \lambda b)^2}$$

$$E[N] = \frac{\rho(1+b)}{2(1-\rho)} \qquad \dots (6)$$

2. Expected waiting time of transactions in the system using Little's result $E[N] = \lambda b E[T]$:

$$E[T] = \frac{E[N]}{\lambda b} = \frac{1+b}{2\mu(1-\rho)}$$

$$= \frac{(1+b)}{2\mu(1-\rho)} \qquad \dots (7)$$

3. Expected number of transactions in the queue:

$$E[N_q] = E[N] - \rho = \frac{\rho(1+b)}{2(1-\rho)} - \rho$$

$$= \frac{\rho(b-1+2\rho)}{2(1-\rho)} \qquad \dots (8)$$

4. Expected waiting time of transaction in queue:

$$E[T_q] = E[T] - \frac{1}{\mu} = \frac{(1+b)}{2\mu(1-\rho)} - \frac{1}{\mu}$$

$$= \frac{b-1+2\rho}{2\mu(1-\rho)} \qquad \dots (9)$$

5. Throughput: Throughput is to measure a system's ability in handling issues, requests, and transactions per unit time. It is also an important indicator to measure the system's concurrency. Here, TPS (transaction per second) is applied to represent it. The throughput in the Blockchain application refers to that the total number of transactions recorded into the Blockchain divided by the time from transaction processing. The formula is as follows:

$$TPS = \frac{TransactionSum}{T} \qquad \dots (10)$$

Where TransactionSum denotes the number of transactions processed and T denotes transaction processing time.

V. Numerical Experiments

The simulation's main goal is to demonstrate various primary parameters for the relevant Blockchain technology model in the context of a queueing system. To analyse performance measures such as an expected transaction confirmation time, expected number of transactions in the system and queue, throughput etc. of Blockchain system, numerical experiments of theoretical results are carried out in python.

Performance measures and throughput are calculated for the different values of μ and various batch size. Table 2 shows performance measures and throughput for $\lambda = 0.025$ and $\mu = 3.5$. And same for $\lambda = 0.025$ and $\mu = 2.5$, $\lambda = 0.025$ and $\mu = 1.8$ in table 3 and 4 respectively.

Figure 2 and 3 shows the graph of Batch size versus Expected transaction confirmation time (transaction processing time) for system and queue respectively.

For fixed value of λ, it is observed that as batch size increases transaction confirmation time also increases. As μ increases, the growth of expected transaction confirmation time decreases with respect to batch size. For larger value of μ, the deviation of expected transaction confirmation time remains almost constant. On the other hand, for lower value of μ, the deviation of expected transaction confirmation time monotonically increases. Smaller batch size is not affected by the different values of μ.

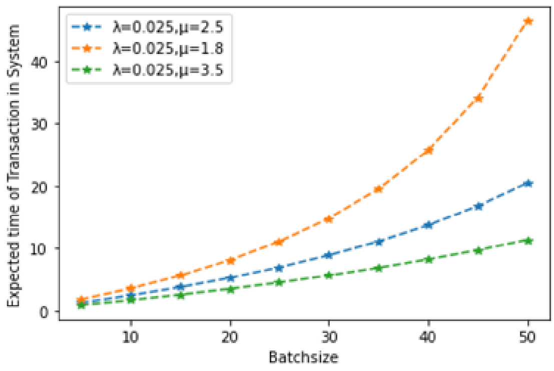


Figure2: Batchsize vs. Expected transaction confirmation time

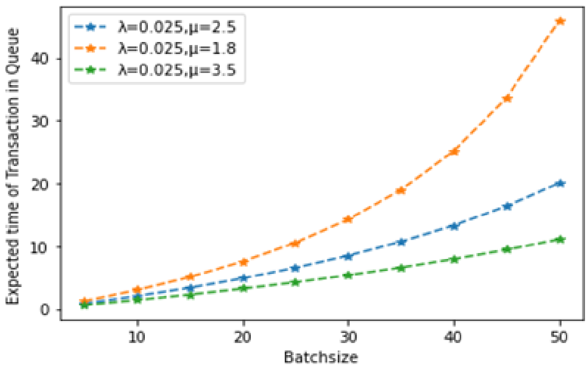


Figure 3: Batch size vs. Expected time of transactions in Queue

Table 2. Values for $\lambda = 0.025$ and $\mu = 3.5$

b	ρ	$E[N_q]$	$E[N]$	$E[T_q]$	$E[T]$	TPS
5	0.036	0.075	0.111	0.603	0.889	5.625
10	0.071	0.352	0.423	1.406	1.692	5.909
15	0.107	0.853	0.96	2.274	2.56	5.859
20	0.143	1.607	1.75	3.214	3.5	5.714
25	0.179	2.648	2.826	4.236	4.522	5.529
30	0.214	4.013	4.227	5.351	5.636	5.323
35	0.25	5.75	6.00	6.571	6.857	5.104
40	0.286	7.914	8.2	7.914	8.2	4.878
45	0.321	10.573	10.895	9.398	9.684	4.647
50	0.357	13.810	14.167	11.048	11.333	4.118

Table 3. Values for $\lambda = 0.025$ and $\mu = 2.5$

b	ρ	$E[N_q]$	$E[N]$	$E[T_q]$	$E[T]$	TPS
5	0.05	0.108	0.158	0.863	1.263	3.958
10	0.1	0.511	0.611	2.044	2.444	4.091
15	0.15	1.262	1.412	3.365	3.765	3.984
20	0.2	2.425	2.625	4.85	5.25	3.810
25	0.25	4.083	4.333	6.533	6.933	3.606
30	0.3	6.343	6.643	8.457	8.857	3.387
35	0.35	9.342	9.692	10.677	11.077	3.160
40	0.4	13.267	13.667	13.267	13.667	2.927
45	0.45	18.368	18.818	16.327	16.727	2.690
50	0.5	25	25.5	20	20.4	2.451

Table 4. Values for $\lambda = 0.025$ and $\mu = 1.8$						
b	ρ	$E[N_q]$	$E[N]$	$E[T_q]$	$E[T]$	TPS
5	0.0694	0.154	0.224	1.235	1.791	2.792
10	0.139	0.748	0.887	2.992	3.548	2.818
15	0.208	1.897	2.105	5.058	5.614	2.672
20	0.278	3.761	4.038	7.521	8.077	2.476
25	0.347	6.568	6.914	10.508	11.064	2.260
30	0.417	10.655	11.071	14.206	14.762	2.032
35	0.486	16.541	17.027	18.903	19.459	1.799
40	0.556	25.069	25.625	25.069	25.625	1.561
45	0.625	33.519	38.333	33.519	34.074	1.321
50	0.694	45.808	57.955	45.808	46.364	1.078

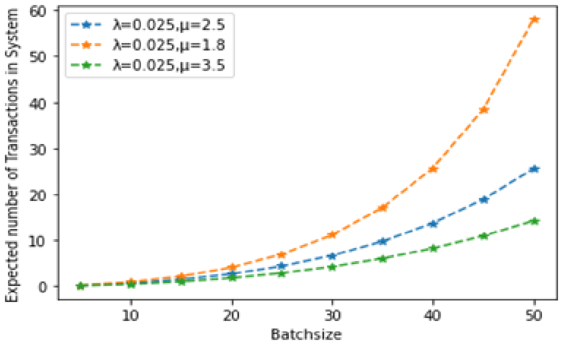


Figure 4: Batch size vs. Expected Number of transactions in System

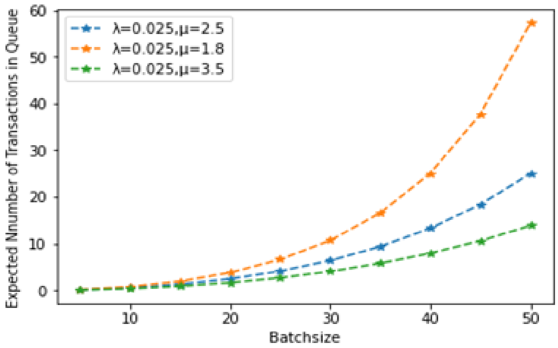


Figure 5: Batch size vs. Expected Number of transactions in Queue

Figure4 and 5 shows the graph of Batch size vs. Expected number of transactions in System and Queue respectively.

For fixed value of λ, it is observed that as batch size increases transaction confirmation time also increases. As μ increases, the growth in expected number of transactions decreases with respect to batch size. For larger value of μ, the deviation in the expected number of transactions remains almost constant. On the other hand, for lower value of μ, the deviation in expected number of transactions monotonically increases. Smaller batch size is not affected by the different values of μ.

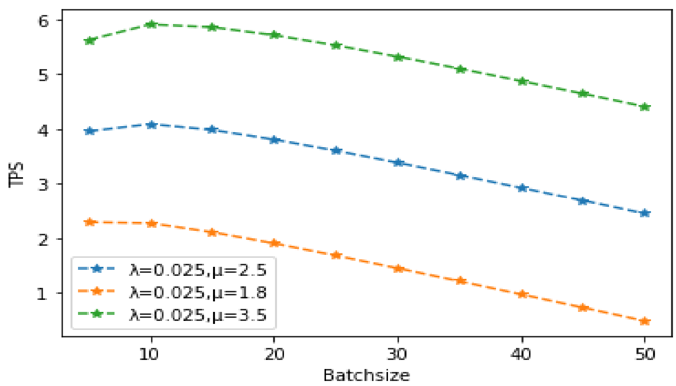


Figure 6: Batch size vs. TPS

Figure 6 shows the graph of Batch size versus TPS. TPS indicates transactions per second i.e., throughput. It is observed that as batch size increases throughput decreases.
For smaller μ, throughput decreases with increases in batch size. For larger μ, throughput increases with increase in batch size. At this point, optimal throughput is obtained. After this throughput decreases with increases in batch size.

Observations:

- When number of transactions in a block i.e., batch size of transactions in a block is less, the required time for consensus is decreased and throughput rate increases. Hence block is confirmed rapidly.
- For the maximum number of transactions in a block, consensus process takes more time. Thus, block confirmation time increases and throughput rate decreases.

VI. Conclusion

In order to lay the theoretical groundwork for developing a concrete Proof of Authority (PoA)-based Blockchain system that focuses on the stochastic behavior of the transactions, consensus process and throughput, this study has presented an embedded Markovian queueing model of the type $M^b/M/1$. The model assumes that transactions arrive in batches with a Poisson distribution. PoA consensus protocol has been studied and analyzed using queueing model. Numerical experiments performed in Python are provided to show the validity and baseline simulation model in the context of a queueing system. It is observed that for the small amount of transactions in a block, optimal throughput rate is obtained. Primary performance measures such as expected time of transaction in system (transaction confirmation time), expected time of transaction in queue, expected number of transactions in system and expected number of transactions in queue are also examined. The outcomes support the validity and effectiveness of the model as expected, allowing for assertion.

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