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Solving Fuzzy Transportation Problem by Newly Approach Ranking Method of Trapezoidal Fuzzy Number

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Abstract:
In this paper, we propose a new method of ranking a fuzzy number using the simple idea that two nonparallel lines intersect at a unique point. We consider an unbalance fuzzy transportation problem where cost, requirement and availability are a trapezoidal fuzzy number. Using the purpose of ranking fuzzy numbers, the fuzzy transportation problem is converted into a crisp transportation problem. The method for solving the transportation problem are the linear programming technique and Vogal Approximation Method. Numerical examples are used to demonstrate the effectiveness and accuracy of this method.

Keywords: *Fuzzy Transportation Problem, Trapezoidal Fuzzy Number, Ranking Fuzzy Number. Linear Programming Technique, Vogel Approximation Method*
2010 AMS Subject Classification: 90C08, 03E72

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I. Introduction

The Transportation Problem is a branch of the Linear programming Problem, a technique of finding optimum shipping patterns between the supplier and the demands. The objective of the Transportation Problem is concerned with a minimum cost of transportation by satisfying the supply and demand constraint. The Fuzzy Transportation Problem is also a Transportation Problem in which the transportation cost, demand and supply are in fuzzy quantities. The objective of the fuzzy transportation problem is also finding the optimum shipping pattern to minimize the fuzzy transportation cost while satisfying the fuzzy supply and demand limits.

For the first time, in 1940 Hitchcock [1] introduced the basic concept of transportation problems. In 1965 Zadeh [2] introduced the notion of fuzziness that was encouraged by Bellman and Zadeh [3], by introducing the concept of decision-making in a fuzzy environment. Zimmermann, Fuzzy set theory and its applications [22] and Fuzzy programming and linear programming with several objective functions [23]. For more understanding refer [4],[5].

Several methods of solving fuzzy transportation were introduced by many authors Liu and Kao [18], Chanas et al [20], Pandian et al [21] and many others. Defuzzification is the technique of converting the fuzzy number or fuzzy set into a crisp value or number. Ranking fuzzy numbers are related to many problems in real life, such as decision-making, optimization, forecasting, socioeconomic systems, control and certain other fuzzy application systems. Numerous methods for ranking fuzzy numbers have been proposed by many researchers. However, all those methods have limitations, and currently, there is no general model for the ranking process. By converting the fuzzy parameter into the crisp parameter, various models have been developed. Chen and Chen[17] discussed the ranking of generalized trapezoidal fuzzy numbers. P.PhaniBushan Rao and N. Ravi Shankar[14] presented Ranking Fuzzy Numbers with a Distance Method using the Circumcenter of Centroids and an Index of Modality. P. PhaniBushan Rao and N. Ravi Shankar [24] proposed Ranking Fuzzy Numbers with an Area Method Using Circumcenter of Centroids. Kumar and Subramanian [13] presented a solution to Fuzzy Transportation Problems with Trapezoidal Fuzzy Numbers using the Robust Ranking Methodology. Ying-Ming Wang et al [9] proposed the centroids of fuzzy numbers. Maheswari and Ganesan[6] solved fully fuzzy transportation problems using pentagonal fuzzy numbers. M.K.Purushothkumar and M.Ananathanarayanan[12] have shown the Fuzzy Transportation problem of Trapezoidal Fuzzy numbers with the New Ranking Technique. C.H.Chen [19] obtained a new approach for ranking fuzzy numbers by distance method. Saini, Prakash and Atul [10] solved Unbalanced Transportation Problems in Fuzzy Environment using Centroid Ranking Technique.

This paper is an effort to remedy this by suggesting a new method for ranking fuzzy numbers and applying it to solve the fuzzy transportation problem. First by converting the fuzzy parameter such as fuzzy cost, fuzzy supply and fuzzy demand into crisp parameters and solving the transportation problem by the method of linear programming technique.

II. Preliminaries

This section aims to present some notations, notions and results which are useful for our further consideration.

Definition 2.1

A fuzzy set $\widetilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}}: R \to [0,1]$ has the following characteristics:

(i) $\mu_{\tilde{A}}$ is convex i.e. $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, \lambda \in [0,1] and \ \forall\ x_1, x_2 \in R$

(ii) $\mu_{\tilde{A}}$ is normal, i.e., there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$

(iii) $\tilde{A}$ is upper semi-continuous.

(iv) $\sup(\tilde{A})$ is bounded in R

***Definition* 2.2**

A fuzzy number $\tilde{A}$ is said to be a non-negative fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ satisfies $\mu_{\tilde{A}}(x) = 0\ \forall x < 0$.

***Definition* 2.3**

A fuzzy number $\tilde{A} = (a, b, c, d)$ where $a<b<c<d$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & b < x < c \\ \frac{d-x}{d-c}, & c \leq x < d \\ 0, & x > d \end{cases}$$

***Definition* 2.4**

A trapezoidal fuzzy number $\tilde{A}= (a,b,c,d)$ is said to be a non-negative (non-positive) trapezoidal fuzzy number. i.e $A \geq 0 (A \leq 0)$ if and only if $a \geq 0 (c \leq 0)$. A trapezoidal fuzzy number is said to be a positive (negative) trapezoidal fuzzy number i.e. $A > 0 (A < 0)$ if and only if $a > 0 (c < 0)$.

***Definition* 2.5**

Two trapezoidal fuzzy numbers$\widetilde{A_1} = (a, b, c, d)$ and $\widetilde{A_2} = (e, f, g, h)$are said to be equal. i.e. $\widetilde{A_1} = \widetilde{A_2}$if and only if *a=e, b=f, c=g, d=h.*

***Definition* 2.6**

Let $\widetilde{A_1} = (a, b, c, d)$ and $\widetilde{A_2} = (e, f, g, h)$be two non-negative trapezoidal fuzzy number then

(i) $\widetilde{A_1} \oplus \widetilde{A_2} = (a,b,c,d) \oplus (e,f,g,h) = (a+e,b+f,c+g,d+h)$

(ii)$\widetilde{A_1} \ominus \widetilde{A_2} = (a,b,c,d) \ominus (e,f,g,h) = (a-h,b-g,c-f,d-e)$

(iii) $-\widetilde{A_1} = -(a,b,c,d) = (-d,-c,-b,-a)$

(iv) $\widetilde{A_1} \otimes \widetilde{A_2} = (a,b,c,d) \otimes (e,f,g,h) = (ae,bf,cg,dh)$

(v) $\frac{1}{\widetilde{A_1}} \cong (\frac{1}{d}, \frac{1}{c}, \frac{1}{b}, \frac{1}{a})$

III. Proposed Ranking Methods

In this section, we present a new approach to ranking a trapezoidal fuzzy number and a triangular fuzzy number. The ranking method proposed in this paper is based on the simple idea that two non-parallel lines intersect at a unique point.

Proposition 1: A ranking function is a function $R : F(R) \to R$ which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (a, b, c, d)$ is a trapezoidal fuzzy number then $R(\tilde{A}) = \frac{a^2+ab-cd-d^2}{3a+b-c-3d}$

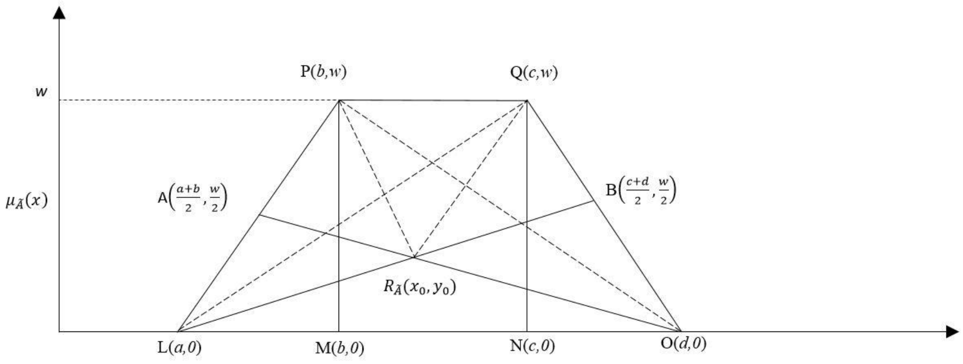

Figure: 1

A trapezoidal fuzzy number represents a trapezoid. Let us suppose L(*a,0*), O(*d,0*), Q(*c,w*), and P(*b,w*) be the vertices of the trapezium LOQP. In ΔLOP, OA is median on LP and in ΔLOQ, LB is median on OQ. Using the simple fact that every line has a unique mid-point. The point of intersection of two medians is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for choosing this point as the reference point is that the point of intersection $R_{\tilde{A}}(x_0, y_0)$lies in the region where the membership function is 1 and also the reference point is the common vertex of two triangles ΔRPL and ΔRQO having the same area with base PL and QO respectively.

Consider a generalized trapezoidal fuzzy number$\tilde{A} = (a, b, c, d; w)$. In the above plane figure, $A\left(\frac{a+b}{2}, \frac{w}{2}\right)$ is the midpoint of LP and $B\left(\frac{c+d}{2}, \frac{w}{2}\right)$ is the midpoint of QO. The equation of line $\overleftrightarrow{AO}$ is given by $wx - (c + d - 2a)y - wa = 0$. The equation of line$\overleftrightarrow{LB}$ is given by $wx - (a + b - 2d)y - wd = 0$

We defined $R_{\tilde{A}}(x_0, y_0)$ as the point of reference of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ given by

$$R_{\tilde{A}}(x_0, y_0) = \left(\frac{\mathrm{a}^2 + \mathrm{ab} - \mathrm{cd} - \mathrm{d}^2}{3\mathrm{a} + \mathrm{b} - \mathrm{c} - 3\mathrm{d}}, \frac{(a-d)w}{3\mathrm{a} + \mathrm{b} - \mathrm{c} - 3\mathrm{d}}\right)$$

And let us use this result in ranking a trapezoidal fuzzy number. Let $\tilde{A} = (a, b, c, d; w)$ is a trapezoidal fuzzy number then $R(\tilde{A}) = \frac{\mathrm{a}^2+\mathrm{ab}-\mathrm{cd}-\mathrm{d}^2}{3\mathrm{a}+\mathrm{b}-\mathrm{c}-3\mathrm{d}}$

Proposition 2: As a special case, When b = c the trapezoidal fuzzy number becomes a triangular fuzzy number. Let $\widetilde{\mathrm{A}} = (a, b, c; w)$ is a triangular fuzzy number then

$$R_{\tilde{A}}(x_0, y_0) = \left(\frac{\mathrm{a}^2 + \mathrm{ab} - \mathrm{bc} - \mathrm{c}^2}{3\mathrm{a} - 3\mathrm{c}}, \frac{(a-c)w}{3\mathrm{a} - 3\mathrm{c}}\right)$$

And let us use this result in ranking a triangular fuzzy number. Let $\tilde{A} = (a, b, c; w)$ is a triangular fuzzy number then $R(\tilde{A}) = \frac{\mathrm{a}^2+\mathrm{ab}-\mathrm{bc}-\mathrm{c}^2}{3\mathrm{a}-3\mathrm{c}}$

IV. Fuzzy transportation problems [6]

The Fuzzy Transportation problems deal with the Transportation of a single product from several sources to several destination. In general, let there be *m* sources $S_1,S_2,S_3,\ldots,S_m$ with $\tilde{a}$ *(i = 1,2,...,m)*available supplies or capacity at each source *i* , to be allocated among n destinations $D_1,D_2,D_3,\ldots,D_n$ with $\tilde{b}$ *(i = 1,2,...,n)*specified requirements at each destination *j* . Let $\widetilde{c_{ij}}$be the cost of shipping one from *i* to destination *j* for each route. Then, if $\widetilde{x_{ij}}$be the units shipped per route from source *i* to destination *j* , the problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying the supply and demand condition.

The fuzzy transportation problem is explicitly represented by the following fuzzy transportation table:

…	D_1	D_2	…	D_n	Supply
S_1	$\widetilde{c_{11}}$	$\widetilde{c_{12}}$		$\widetilde{c_{1n}}$	$\widetilde{a_1}$
S_2	$\widetilde{c_{12}}$	$\widetilde{c_{22}}$		$\widetilde{c_{2n}}$	$\widetilde{a_2}$
…			…		…
S_m	$\widetilde{c_{m1}}$	$\widetilde{c_{m2}}$		$\widetilde{c_{mn}}$	$\widetilde{a_m}$
Demand	$\widetilde{b_1}$	$\widetilde{b_2}$	…	$\widetilde{b_m}$	

The problem may as stated as follows:

$$Minimize\ \tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c_{\iota\jmath}} \widetilde{x_{\iota\jmath}}$$

Subject to the constraints:

$$\widetilde{x_{\iota 1}} + \widetilde{x_{\iota 2}} + \cdots + \widetilde{x_{\iota n}} \leq \widetilde{a_\iota}\ ; \quad i = 1,2,\ldots,m$$
$$\widetilde{x_{1\jmath}} + \widetilde{x_{2\jmath}} + \cdots + \widetilde{x_{m\jmath}} \geq \widetilde{b_\jmath}\ ; \quad j = 1,2,\ldots,n$$
$$and\ \widetilde{x_{\iota\jmath}} \geq 0, for\ all\ i\ and\ j$$

Vogal Approximation method (VAM): This method is preferred over the NWCM or LCM, because the initial basic feasible solution obtained by this method is either the optimal solution or nearer to the optimal solution.
Algorithm of Vogal Approximation method (VAM):

1) For an unbalanced transportation problem (the sum of the availability constraint is not equal to the sum of the requirement constraint), convert to a balance transportation problem by adding a dummy row or dummy column so that sum of the availability constraint is equal to the sum of the requirement constraint.
2) Find the cells having the smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in the row penalty.
3) Find the cells having the smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in column penalty.
4) Select the row or column with the maximum penalty and find a cell that has the least cost in the selected row or column. Allocate as much as possible in this cell. If there is a tie in the value of penalties then select the cell where maximum allocation can be possible.
5) Adjust the supply and demand and cross out the satisfied row and column
6) Repeat these steps until supply and demand values are 0.

A feasible solution to a transportation problem is said to be a basic feasible solution if the number of allocated cells is equal to $m + n - 1$ where *m* is no of rows and *n* is no of columns of the transportation problem.
Once the basic feasible solution is obtained, MODI Method is applied to check whether the solution is optimal or not.

Algorithm of MODI Method:

1) After finding an initial basic feasible solution, find U_i and V_j for rows and columns. To start
a) Assign 0 to $U_i$or V_j where the maximum number of allocations is a row or column respectively.
b) Calculate other U_i's and V_j's using $C_{ij} = U_i + V_j$, for all occupied cells.
2) For all unoccupied cells, calculate $\Delta_{ij} = C_{ij} – (U_i + V_j)$
3) Check the sign of Δij
a) If $\Delta_{ij} > 0$, then the current basic feasible solution is optimal and stops the procedure.
b) If $\Delta_{ij} \geq 0$, then an alternative solution exists, with a different set of allocations and the same transportation cost. Now stop the procedure.
c) If $\Delta_{ij} < 0$, then the given solution is not an optimal solution and further improvement in the solution is possible.
4) Select the unoccupied cells with the largest negative value of Δ_{ij}, and include the next solution.
5) Draw the closed path or loop from the unoccupied cell (selected in the previous step). The right angle turn in the path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell.
6)
a. Select the minimum value from cells marked with (-) sign of the closed path.
b. Assign this value to the selected unoccupied cell. (so unoccupied become occupied cell)

c. Add this value to the either occupied cells marked (+) sign.
d. Subtract this value to the other occupied cells marked with a (-) sign.
7) Repeat step2 to step7 optimal solution is obtained.

V. Numerical example

In this section, the fuzzy linear programming problem in the paper [11] is solved using the proposed ranking method.

Consider the fuzzy transportation problem. The following table shows all the necessary information on the availability of supply to each warehouse, the following requirement of each market and unit transportation cost in (Rupee) from each warehouse to each market. In the following table cost value and supplies and demands are in the trapezoidal fuzzy number. The problem given below is an unbalanced transportation problem in which the supplies are greater than the demands. Though the objective is to minimize the cost of transportation the problem has an optimum solution.

Table 1: Data of transportation problem in trapezoidal fuzzy number:

	D1	D2	D3	Supply
S1	(5,6,8,10)	(7,8,10,12)	(16,18,20,22)	(17,18,20,25)
S2	(37,38,40,42)	(28,29,30,32)	(52,52,55,57)	(45,47,50,55)
S3	(18,19,20,22)	(22,23,25,27)	(32,33,35,27)	(46,46,50,55)
Demand	(8,9,10,12)	(5,6,7,8)	(12,13,14,16)	

Table 2:Application of the purpose ranking method and conversion to crisp form:

	D1	D2	D3	Supply
S1	7.352	9.352	19	20.384
S2	39.352	29.846	54.352	49.545
S3	19.846	24.352	34.352	49.774
Demand	9.846	6.5	13.846	

Thus the problem can be solved using linear programming problem:

The objective function:

$$\text{Minimize}=7.352X_{11}+9.352X_{12}+19X_{13}+39.352X_{21}+29.846X_{22}+54.352X_{23}+19.846X_{31}+24.352X_{32}+34.352X_{33}$$

And constraint are

$$X_{11} + X_{12} + X_{13} \leq 20.384$$
$$X_{21} + X_{22} + X_{23} \leq 49.545$$
$$X_{31} + X_{32} + X_{33} \leq 49.774$$
$$X_{11} + X_{21} + X_{31} \geq 9.846$$
$$X_{12} + X_{22} + X_{32} \geq 6.5$$
$$X_{13} + X_{23} + X_{33} \geq 13.846$$
$$X_{ij} \geq 0;\ i=1,2,3;\ j=1,2,3$$

The problem is solved using the software called LINGO.

The optimal solution is obtained as the transportation cost is Rs518.79 and $X_{11} = 0.038$, $X_{12} = 6.5$, $X_{13} = 13.846$, $X_{31} = 9.808$.

Also, the problem is solved by VAM (Vogal Approximation Method)

Table 3: Solution of Vogal Approximation Method:

	D1	D2	D3	D4	SUPPLY	P1	P2	P3	P4	P5
S1	0.038 7.352	6.5 9.352	13.846 19	0	~~20.384~~ ~~6.538~~ ~~0.038~~ 0	7.352	7.352	2	2	7.352
S2	39.352	29.846	54.352	49.545 0	~~49.545~~ 0	29.846↑				
S3	9.808 19.846	24.352	34.352	39.966 0	~~49.774~~ ~~9.808~~ 0	19.846	19.846↑	4.506	4.506	19.846↑
DEMAND	~~9.846~~ ~~0.038~~ 0	~~6.5~~ 0	~~13.846~~ 0	~~89.511~~ ~~39.966~~ 0						
P1	12.494	15	15.352	0						
P2	12.494	15	15.352	0						
P3	9.846	15	15.352↑							
P4	9.846	15↑								
P5	9.846	15								

Since the m+n-1 = 6 which is equal to the no of occupied cells. Therefore it is a basic feasible solution. For finding the optimum solution we applied the Modified Distribution Method.

Table 4: Solution of MODI method

	D1	D2	D3	D4	U_i
S1	0.038 7.352	6.5 9.352	13.846 19	0	U_1=0
S2	39.352	29.846	54.352	49.545 0	U_2=12.494
S3	9.808 19.846	24.352	34.352	39.966 0	U_3=12.494
V_i	V_1=7.352	V_2=9.352	V_3=19	V_4=-12.494	

The value of Δ_{ij} is given below:

$\Delta_{14} = 0 - \{0 + (-12.494)\} = 12.494$

$\Delta_{21} = 39.352 - (12.494 + 7.352) = 19.506$

$\Delta_{22} = 29.846 - (12.494 + 9.352) = 8$

$\Delta_{23} = 54.352 - (12.494 + 19) = 22.858$

$\Delta_{32} = 24.352 - (12.494 + 9.352) = 2.506$

$\Delta_{33} = 34.352 - (12.4994 + 19) = 2.858$

Since $\Delta_{ij} > 0$, it is the optimal stage and the above table gives the optimum solution.
The transportation cost so obtained is Rs518.79

$\therefore X_{11} = 0.038$ $\quad X_{12} = 6.5$

$X_{13} = 13.846$ $\quad X_{31} = 9.808$

And the following table is the result of the above example based on the fuzzy ranking number from the published paper.

Table 5: Comparison results of various ranking methods:

Robust Ranking [13],[7]	$R(\tilde{A}) = \int_0^1 0.5[(b-a)s + a, d - (d-c)s]ds$	Z = 517.69	$x_{12}=6.25$ $x_{13}=13.75$ $x_{31}=9.75$ $x_{32}=0.25$
Ranking [9],[12]	$R(\tilde{A}) = 1/3[a+b+c+d - \frac{dc-ab}{(d+c)-(b+a)}]$	Z = 518.09	$x_{12}=6.4$ $x_{13}=13.8$ $x_{31}=9.8$ $x_{32}=0.10$
Ranking [10] [11]	$R(\tilde{A}) = \frac{2a+7b+7c+2d}{18}$	Z = 516.31	$x_{12}=5.83$ $x_{13}=13.61$ $x_{31}=9.61$ $x_{32}=0.66$
Ranking [8], [14], [11]	$R(\tilde{A}) = \frac{a+2b+2c+d}{6}$	Z = 516.82	$x_{12}=6$ $x_{13}=13.66$ $x_{31}=9.66$ $x_{32}=0.5$
Ranking [15], [16]	$R(\tilde{A}) = \frac{a+b+c+d}{4}$	Z = 510.81	$x_{11}=0.25$ $x_{12}=6.5$ $x_{13}=13.75$ $x_{31}=9.5$
Proposed ranking	$R(\tilde{A}) = \frac{a^2+ab-cd-d^2}{3a+b-c-3d}$	Z = 518.79	$x_{11}=0.038$ $x_{12}=6.5$ $x_{13}=13.846$ $x_{31}=9.808$

VI. Conclusion

The purpose ranking technique converted the trapezoidal fuzzy number into a crisp number as shown in the example. This method can be used for all trapezoidal fuzzy numbers to convert them into crisp numbers. The transportation costs obtained in the example are realistic and general. The ranking process is simple and efficient though it has a large computational effort. Thus this method can be used to solve the fuzzy transportation problem which is a trapezoidal fuzzy number as well as a triangular fuzzy number.

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