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Some Special Operators On Bipolar Intuitionistic Fuzzy α-Ideal and Bipolar Intuitionistic Anti Fuzzy α-Ideal of a BP-Algebra

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Abstract:
The concept of a bipolar intuitionistic fuzzy α-ideal and bipolar intuitionistic anti fuzzy α-ideal are a new algebraic structure of BP-algebra and to use special operators. The purpose of this study is to implement the fuzzy set theory and ideal theory of a BP-algebra. The relation between the operation of special operators $P_{\alpha,\alpha',\beta,\beta'}$, $Q_{\alpha,\alpha',\beta,\beta'}$ and $G_{\alpha,\alpha',\beta,\beta'}$ on bipolar intuitionistic fuzzy α-ideal and bipolar intuitionistic anti fuzzy α-ideal are established.

Keywords:
BP-algebra, fuzzy ideal, bipolar fuzzy ideal, bipolar intuitionistic fuzzy α-ideal, bipolar intuitionistic anti fuzzy α-ideal, $P_{\alpha,\alpha',\beta,\beta'}$, $Q_{\alpha,\alpha',\beta,\beta'}$ and $G_{\alpha,\alpha',\beta,\beta'}$.

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I. Introduction

The concept of fuzzy sets was initiated by I.A.Zadeh [11] then it has become a vigorous area of research in engineering, medical science, graph theory. S.S.Ahn [2] gave the idea of BP-algebra. Bipolar valued fuzzy sets was introduced by K.J.Lee [4] are an extension of fuzzy sets whose positive membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the positive membership degree (0, 1] indicates that elements somewhat satisfies the property and the negative membership degree [-1, 0) indicates that elements somewhat satisfies the implicit counter property. The author W.R.Zhang [12] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1998. K.Chakrabarthy and Biswas R.Nanda [3] investigated note on union and intersection of intuitionistic fuzzy sets. A.Rajeshkumar [10] was analyzed fuzzy groups and level subgroups. K.Gunasekaran, S.Nandakumar and S.Sivakaminathan [13] introduced the definition of bipolar intuitionistic fuzzy α-ideal of a BP-algebra. S.Sivakaminathan, K.Gunasekaran and S.Nandakumar [14] analyzed some operations on bipolar intuitionistic fuzzy α-ideal and bipolar intuitionistic anti fuzzy α-ideal of a BP-algebra.

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II. Preliminaries

Definition: 1

Let A and B be any two bipolar intuitionistic fuzzy set A = $(\mu^P_{\alpha_A}, \mu^N_{\alpha_A}, \nu^P_{\alpha_A}, \nu^N_{\alpha_A})$ and B = $(\mu^P_{\alpha_B}, \mu^N_{\alpha_B}, \nu^P_{\alpha_B}, \nu^N_{\alpha_B})$ in X, we define

(i) A ∩ B ={(x, min($\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)$), max($\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x)$), max($\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)$), min($\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x)$)) /x ∈ X}

(ii) A ∪ B ={(x, max($\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)$), min($\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x)$), min($\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)$), max($\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x)$)) /x ∈ X}

(iii) $\bar{A}$= {(x,$\nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x), \mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x)$) / x ∈ X}.

Definition: 2

A bipolar intuitionistic fuzzy set A = {(x, $\mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x), \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x)$) / x ∈ X}, of BP-algebra X is called a bipolar intuitionistic fuzzy α-ideal of X if it satisfies the following conditions:

(i) $\mu^P_{\alpha_A}(0) \geq \mu^P_{\alpha_A}(x)$ and $\mu^N_{\alpha_A}(0) \leq \mu^N_{\alpha_A}(x)$

(ii) $\mu^P_{\alpha_A}(y * z) \geq \min\{\mu^P_{\alpha_A}(x * z), \ \mu^P_{\alpha_A}(x * y)\}$

(iii) $\mu^N_{\alpha_A}(y * z) \leq \max\{\mu^N_{\alpha_A}(x * z), \ \mu^N_{\alpha_A}(x * y)\}$

(iv) $\nu^P_{\alpha_A}(0) \leq \nu^P_{\alpha_A}(x)$ and $\nu^N_{\alpha_A}(0) \geq \nu^N_{\alpha_A}(x)$

(v) $\nu^P_{\alpha_A}(y * z) \leq \max\{\nu^P_{\alpha_A}(x * z), \ \nu^P_{\alpha_A}(x * y)\}$

(vi) $\nu^N_{\alpha_A}(y * z) \geq \min\{\nu^N_{\alpha_A}(x * z), \ \nu^N_{\alpha_A}(x * y)\}$, for all x, y, z ∈ X.

Definition: 3

A bipolar intuitionistic fuzzy set A = {(x, $\mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x), \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x)$) / x ∈ X}, of BP-algebra X is called a bipolar intuitionistic anti fuzzy α-ideal of X if it satisfies the following conditions:

(i) $\mu^P_{\alpha_A}(0) \leq \mu^P_{\alpha_A}(x)$ and $\mu^N_{\alpha_A}(0) \geq \mu^N_{\alpha_A}(x)$

(ii) $\mu^P_{\alpha_A}(y * z) \leq \max\{\mu^P_{\alpha_A}(x * z), \ \mu^P_{\alpha_A}(x * y)\}$

(iii) $\mu^N_{\alpha_A}(y * z) \geq \min\{\mu^N_{\alpha_A}(x * z), \ \mu^N_{\alpha_A}(x * y)\}$

(iv) $\nu^P_{\alpha_A}(0) \geq \nu^P_{\alpha_A}(x)$ and $\nu^N_{\alpha_A}(0) \leq \nu^N_{\alpha_A}(x)$

(v) $\nu^P_{\alpha_A}(y * z) \geq \min\{\nu^P_{\alpha_A}(x * z), \ \nu^P_{\alpha_A}(x * y)\}$

(vi) $\nu^N_{\alpha_A}(y * z) \leq \max\{\nu^N_{\alpha_A}(x * z), \ \nu^N_{\alpha_A}(x * y)\}$, for all x, y, z ∈ X.

Definition: 4

Let A is a bipolar intuitionistic fuzzy set of X, then

$P_{\alpha,\alpha',\beta,\beta'}$(A) = { (x, max($\alpha, \mu^P_{\alpha_A}(x)$), min($\alpha', \mu^N_{\alpha_A}(x)$), min($\beta, \nu^P_{\alpha_A}(x)$), max($\beta', \nu^N_{\alpha_A}(x)$) /x ∈ X },

for α, β ∈ [0, 1], α', β' ∈ [-1, 0] and $\alpha + \beta \leq 1$, $\alpha' + \beta' \geq$ -1.

Definition: 5

Let A is a bipolar intuitionistic fuzzy set of X, then

$Q_{\alpha,\alpha',\beta,\beta'}$(A) = { (x, min($\alpha, \mu^P_{\alpha_A}(x)$), max($\alpha', \mu^N_{\alpha_A}(x)$), max($\beta, \nu^P_{\alpha_A}(x)$), min($\beta', \nu^N_{\alpha_A}(x)$) /x ∈ X},

for α, β ∈ [0, 1], α', β' ∈ [-1, 0] and $\alpha + \beta \leq 1$, $\alpha' + \beta' \geq$ -1.

Definition: 6

Let A is a bipolar intuitionistic fuzzy set of X, then

$G_{\alpha,\alpha',\beta,\beta'}$(A) = { (x, $\alpha\mu^P_{\alpha_A}(x)$, $\alpha'\mu^N_{\alpha_A}(x)$, $\beta\nu^P_{\alpha_A}(x)$, $\beta'\nu^N_{\alpha_A}(x)$) / x ∈ X},

for α, β ∈ [0, 1], α', β' ∈ [-1, 0] and $\alpha + \beta \leq 1$, $\alpha' + \beta' \geq$ -1.

III. SPECIAL OPERATORS ON BIPOLAR INTUITIONISTIC FUZZY α-IDEAL

Theorem: 1

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $P_{\alpha,\alpha',\beta,\beta'}$(A) is a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z ∈ A.

(i) Now $\mu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \max(\alpha, \mu^P_{\alpha_A}(0))$

$\geq \max(\alpha, \mu^P_{\alpha_A}(x))$

$= \mu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\mu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \geq \mu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Now $\mu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \min(\alpha', \ \mu^N_{\alpha_A}(0))$

$\leq \min(\alpha', \mu^N_{\alpha_A}(x))$

$= \mu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \leq \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

(ii) Now $\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \max(\alpha, \mu^{P}_{\alpha_A}(y*z))$

$\geq \max(\alpha, \min\{\mu^{P}_{\alpha_A}(x*z), \mu^{P}_{\alpha_A}(x*y)\})$

$= \min\{\max(\alpha, \mu^{P}_{\alpha_A}(x*z)), \max(\alpha, \mu^{P}_{\alpha_A}(x*y))\}$

$= \min\{\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \geq \min\{\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

(iii) Now $\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \min(\alpha', \mu^{N}_{\alpha_A}(y*z))$

$\leq \min(\alpha', \max\{\mu^{N}_{\alpha_A}(x*z), \mu^{N}_{\alpha_A}(x*y)\})$

$= \max\{\min(\alpha', \mu^{N}_{\alpha_A}(x*z)), \min(\alpha', \mu^{N}_{\alpha_A}(x*y))\}$

$= \max\{\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \leq \max\{\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

(iv) Now $\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \min(\beta, \nu^{P}_{\alpha_A}(0))$

$\leq \min(\beta, \nu^{P}_{\alpha_A}(x))$

$= \nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \leq \nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Now $\nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \max(\beta', \nu^{N}_{\alpha_A}(0))$

$\geq \max(\beta', \nu^{N}_{\alpha_A}(x))$

$= \nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \geq \nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

(v) Now $\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \min(\beta, \nu^{P}_{\alpha_A}(y*z))$

$\leq \min(\beta, \max\{\nu^{P}_{\alpha_A}(x*z), \nu^{P}_{\alpha_A}(x*y)\})$

$= \max\{\min(\beta, \nu^{P}_{\alpha_A}(x*z)), \min(\beta, \nu^{P}_{\alpha_A}(x*y))\}$

$= \max\{\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \leq \max\{\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

(vi) Now $\nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \max(\beta', \nu^{N}_{\alpha_A}(y*z))$

$\geq \max(\beta', \min\{\nu^{N}_{\alpha_A}(x*z), \nu^{N}_{\alpha_A}(x*y)\})$

$= \min\{\max(\beta', \nu^{N}_{\alpha_A}(x*z)), \max(\beta', \nu^{N}_{\alpha_A}(x*y))\}$

$= \min\{\nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \geq \min\{\nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $P_{\alpha,\alpha',\beta,\beta'}(A)$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 2

If A and B are bipolar intuitionistic fuzzy α-ideal of X, then $P_{\alpha,\alpha',\beta,\beta'}(A \cap B) = P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)$ is also a bipolar intuitionistic fuzzy α-ideal of X, and for every $\alpha, \beta \in [0, 1]$, $\alpha', \beta' \in [-1, 0]$ and $\alpha + \beta \leq 1$, $\alpha' + \beta' \geq -1$.

Proof: Let A and B are bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z ∈ A ∩ B then 0, x, y, z ∈ A and 0, x, y, z ∈ B.

(i) Now $\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \max(\alpha, \mu^{P}_{\alpha_{A\cap B}}(0))$

$= \max(\alpha, \min\{\mu^{P}_{\alpha_A}(0), \mu^{P}_{\alpha_B}(0)\})$

$\geq \max(\alpha, \min\{\mu^{P}_{\alpha_A}(x), \mu^{P}_{\alpha_B}(x)\})$

$= \min\{\max(\alpha, \mu^{P}_{\alpha_A}(x)), \max(\alpha, \mu^{P}_{\alpha_B}(x))\}$

$= \min\{\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x), \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$

$= \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Therefore $\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) \geq \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Now $\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \min(\alpha', \mu^{N}_{\alpha_{A\cap B}}(0))$

$= \min(\alpha', \max\{\mu^{N}_{\alpha_A}(0), \mu^{N}_{\alpha_B}(0)\})$

$\leq \min(\alpha', \max\{\mu^{N}_{\alpha_A}(x), \mu^{N}_{\alpha_B}(x)\})$

$= \max\{\min(\alpha', \mu^{N}_{\alpha_A}(x)), \min(\alpha', \mu^{N}_{\alpha_B}(x))\}$

$= \max\{\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x), \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$

$= \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Therefore $\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) \leq \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

(ii) Now $\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) = \max(\alpha, \mu^{P}_{\alpha_{A\cap B}}(y*z))$

$= \max(\alpha, \min\{\mu^{P}_{\alpha_A}(y*z), \mu^{P}_{\alpha_B}(y*z)\})$

$\geq \max(\alpha, \min\{\min\{\mu^{P}_{\alpha_A}(x*z), \mu^{P}_{\alpha_A}(x*y)\}, \min\{\mu^{P}_{\alpha_B}(x*z), \mu^{P}_{\alpha_B}(x*y)\}\})$

$= \max(\alpha, \min\{\min\{\mu^{P}_{\alpha_A}(x*z), \mu^{P}_{\alpha_B}(x*z)\}, \min\{\mu^{P}_{\alpha_A}(x*y), \mu^{P}_{\alpha_B}(x*y)\}\})$

$= \min\{\max(\alpha, \min\{\mu^{P}_{\alpha_A}(x*z), \mu^{P}_{\alpha_B}(x*z)\}), \max(\alpha, \min\{\mu^{P}_{\alpha_A}(x*y), \mu^{P}_{\alpha_B}(x*y)\})\}$

$= \min\{\min\{\max(\alpha, \mu^{P}_{\alpha_A}(x*z)), \max(\alpha, \mu^{P}_{\alpha_B}(x*z))\},$
$\min\{\max(\alpha, \mu^{P}_{\alpha_A}(x*y)), \max(\alpha, \mu^{P}_{\alpha_B}(x*y))\}\}$

$= \min\{\min\{\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)\},$
$\min\{\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y), \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}\}$

$= \min\{\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$

Therefore

$\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) \geq \min\{\mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \mu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$

(iii) Now $\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) = \min(\alpha', \mu^{N}_{\alpha_{A\cap B}}(y*z))$

$= \min(\alpha', \max\{\mu^{N}_{\alpha_A}(y*z), \mu^{N}_{\alpha_B}(y*z)\})$

$\leq \min(\alpha', \max\{\max\{\mu^{N}_{\alpha_A}(x*z), \mu^{N}_{\alpha_A}(x*y)\}, \max\{\mu^{N}_{\alpha_B}(x*z), \mu^{N}_{\alpha_B}(x*y)\}\})$

$= \min(\alpha', \max\{\max\{\mu^{N}_{\alpha_A}(x*z), \mu^{N}_{\alpha_B}(x*z)\}, \max\{\mu^{N}_{\alpha_A}(x*y), \mu^{N}_{\alpha_B}(x*y)\}\})$

$= \max\{\min(\alpha', \max\{\mu^{N}_{\alpha_A}(x*z), \mu^{N}_{\alpha_B}(x*z)\}), \min(\alpha', \max\{\mu^{N}_{\alpha_A}(x*y), \mu^{N}_{\alpha_B}(x*y)\})\}$

$= \max\{\max\{\min(\alpha', \mu^{N}_{\alpha_A}(x*z)), \min(\alpha', \mu^{N}_{\alpha_B}(x*z))\},$
$\max\{\min(\alpha', \mu^{N}_{\alpha_A}(x*y)), \min(\alpha', \mu^{N}_{\alpha_B}(x*y))\}\}$

$= \max\{\max\{\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)\},$
$\max\{\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y), \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}\}$

$= \max\{\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$

Therefore

$\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) \leq \max\{\mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \mu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$

(iv) Now $\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \min(\beta, \nu^{P}_{\alpha_{A\cap B}}(0))$

$= \min(\beta, \max(\nu^{P}_{\alpha_A}(0), \nu^{P}_{\alpha_B}(0)))$

$\leq \min(\beta, \max(\nu^{P}_{\alpha_A}(x), \nu^{P}_{\alpha_B}(x)))$

$= \max\{\min(\beta, \nu^{P}_{\alpha_A}(x)), \min(\beta, \nu^{P}_{\alpha_B}(x))\}$

$= \max\{\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x), \nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$

$= \nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Therefore $\nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) \leq \nu^{P}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Now $\nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \max(\beta', \nu^{N}_{\alpha_{A\cap B}}(0))$

$= \max(\beta', \min(\nu^{N}_{\alpha_A}(0), \nu^{N}_{\alpha_B}(0)))$

$\geq \max(\beta', \min(\nu^{N}_{\alpha_A}(x), \nu^{N}_{\alpha_B}(x)))$

$= \min\{\max(\beta', \nu^{N}_{\alpha_A}(x)), \max(\beta', \nu^{N}_{\alpha_B}(x))\}$

$= \min\{\nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x), \nu^{N}_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$

$= \nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Therefore $\nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(0) \geq \nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

(v) Now $\nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) = \min(\beta, \nu^P_{\alpha_{A\cap B}}(y * z))$

$= \min(\beta, \max\{\nu^P_{\alpha_A}(y * z), \nu^P_{\alpha_B}(y * z)\})$

$\leq \min(\beta, \max\{\max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}, \max\{\nu^P_{\alpha_B}(x * z), \nu^P_{\alpha_B}(x * y)\}\})$

$= \min(\beta, \max\{\max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_B}(x * z)\}, \max\{\nu^P_{\alpha_A}(x * y), \nu^P_{\alpha_B}(x * y)\}\})$

$= \max\{\min(\beta, \max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_B}(x * z)\}), \min(\beta, \max\{\nu^P_{\alpha_A}(x * y), \nu^P_{\alpha_B}(x * y)\})\}$

$= \max\{\max\{\min(\beta, \nu^P_{\alpha_A}(x * z)), \min(\beta, \nu^P_{\alpha_B}(x * z))\},$
$\max\{\min(\beta, \nu^P_{\alpha_A}(x * y)), \min(\beta, \nu^P_{\alpha_B}(x * y))\}\}$

$= \max\{\max\{\nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x * z), \nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z)\},$
$\max\{\nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x * y), \nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}\}$

$= \max\{\nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z), \nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}$

Therefore

$\nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) \leq \max\{\nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)\cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z), \nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}$

(vi) Now $\nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) = \max(\beta', \nu^N_{\alpha_{A\cap B}}(y * z))$

$= \max(\beta', \min(\nu^N_{\alpha_A}(y * z), \nu^N_{\alpha_B}(y * z)))$

$\geq \max(\beta', \min(\min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}, \min\{\nu^N_{\alpha_B}(x * z), \nu^N_{\alpha_B}(x * y)\}))$

$= \max(\beta', \min(\min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_B}(x * z)\}, \min\{\nu^N_{\alpha_A}(x * y), \nu^N_{\alpha_B}(x * y)\}))$

$= \min\{\max(\beta', \min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_B}(x * z)\}), \max(\beta', \min\{\nu^N_{\alpha_A}(x * y), \nu^N_{\alpha_B}(x * y)\})\}$

$= \min\{\min\{\max(\beta', \nu^N_{\alpha_A}(x * z)), \max(\beta', \nu^N_{\alpha_B}(x * z))\},$
$\min\{\max(\beta', \nu^N_{\alpha_A}(x * y)), \max(\beta', \nu^N_{\alpha_B}(x * y))\}\}$

$= \min\{\min\{\nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x * z), \nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z)\},$
$\min\{\nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A)}}(x * y), \nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}\}$

$= \min\{\nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z), \nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}$

Therefore

$\nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) \geq \min\{\nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z), \nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}$

Therefore $P_{\alpha,\alpha',\beta,\beta'}(A \cap B) = P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 3

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $Q_{\alpha,\alpha',\beta,\beta'}(A)$ is a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z ∈ A.

(i) Now $\mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \min(\alpha, \mu^P_{\alpha_A}(0))$

$\geq \min(\alpha, \mu^P_{\alpha_A}(x))$

$= \mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \geq \mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Now $\mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \max(\alpha', \mu^N_{\alpha_A}(0))$

$\leq \max(\alpha', \mu^N_{\alpha_A}(x))$

$= \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \leq \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

(ii) Now $\mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(y * z) = \min(\alpha, \mu^P_{\alpha_A}(y * z))$

$\geq \min(\alpha, \min\{\mu^P_{\alpha_A}(x * z), \mu^P_{\alpha_A}(x * y)\})$

$= \min\{\min(\alpha, \mu^P_{\alpha_A}(x * z)), \min(\alpha, \mu^P_{\alpha_A}(x * y))\}$

$= \min \{ \mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \geq \min \{ \mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

(iii) Now $\mu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \max(\alpha', \mu^{N}_{\alpha_A}(y*z))$

$\leq \max(\alpha', \max\{ \mu^{N}_{\alpha_A}(x*z),\ \mu^{N}_{\alpha_A}(x*y)\})$

$= \max\{ \max(\alpha',\ \mu^{N}_{\alpha_A}(x*z)),\ \max(\alpha', \mu^{N}_{\alpha_A}(x*y))\}$

$= \max\{ \mu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \mu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\mu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \leq \max\{ \mu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \mu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

(iv) Now $\nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \max(\beta,\ \nu^{P}_{\alpha_A}(0))$

$\leq \max(\beta,\ \nu^{P}_{\alpha_A}(x))$

$= \nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \leq \nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Now $\nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \min(\beta', \nu^{N}_{\alpha_A}(0))$

$\geq \min(\beta', \nu^{N}_{\alpha_A}(x))$

$= \nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \geq \nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

(v) Now $\nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \max(\beta,\ \nu^{P}_{\alpha_A}(y*z))$

$\leq \max(\beta,\ \max\{\nu^{P}_{\alpha_A}(x*z),\ \nu^{P}_{\alpha_A}(x*y)\})$

$= \max\{ \max(\beta,\ \nu^{P}_{\alpha_A}(x*z)),\ \max(\beta,\ \nu^{P}_{\alpha_A}(x*y))\}$

$= \max\{\nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \leq \max\{\nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \nu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

(vi) Now $\nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \min(\beta', \nu^{N}_{\alpha_A}(y*z))$

$\geq \min(\beta',\ \min\{\nu^{N}_{\alpha_A}(x*z),\ \nu^{N}_{\alpha_A}(x*y)\})$

$= \min\{ \min(\beta',\ \nu^{N}_{\alpha_A}(x*z)),\ \min(\beta',\ \nu^{N}_{\alpha_A}(x*y))\}$

$= \min\{\nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \geq \min\{\nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \nu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $Q_{\alpha,\alpha',\beta,\beta'}(A)$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 4

If A and B are bipolar intuitionistic fuzzy α-ideal of X, then $Q_{\alpha,\alpha',\beta,\beta'}(A \cap B) = Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)$ is also a bipolar intuitionistic fuzzy α-ideal of X, and for every $\alpha, \beta \in [0, 1]$, $\alpha', \beta' \in [-1, 0]$ and $\alpha + \beta \leq 1$, $\alpha' + \beta' \geq -1$.

Proof: Let A and B are bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z ∈ A ∩ B then 0, x, y, z ∈ A and 0, x, y, z ∈ B.

(i) Now $\mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \min(\alpha, \mu^{P}_{\alpha_{A\cap B}}(0))$

$= \min(\alpha,\ \min\{\mu^{P}_{\alpha_A}(0), \mu^{P}_{\alpha_B}(0)\})$

$\geq \min(\alpha,\ \min\{\mu^{P}_{\alpha_A}(x), \mu^{P}_{\alpha_B}(x)\})$

$= \min\{\min(\alpha, \mu^{P}_{\alpha_A}(x)), \min(\alpha, \mu^{P}_{\alpha_B}(x))\}$

$= \min\{ \mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x),\ \mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$

$= \mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Therefore $\mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) \geq \mu^{P}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Now $\mu^{N}_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \max(\alpha', \mu^{N}_{\alpha_{A\cap B}}(0))$

$= \max(\alpha', \max\{ \mu^{N}_{\alpha_A}(0),\ \mu^{N}_{\alpha_B}(0)\})$

$\leq \max(\alpha', \max\{ \mu^{N}_{\alpha_A}(x),\ \mu^{N}_{\alpha_B}(x)\})$

$$= \max \{ \max (\alpha', \mu^N_{\alpha_A}(x)),\ \max (\alpha', \mu^N_{\alpha_B}(x))\}$$
$$= \max \{\mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x),\ \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$$
$$= \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$$

Therefore $\mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(0) \le \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

(ii) Now $\mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) = \min (\alpha, \mu^P_{\alpha_{A\cap B}}(y * z))$

$$= \min (\alpha,\ \min \{ \mu^P_{\alpha_A}(y * z),\ \mu^P_{\alpha_B}(y * z)\})$$
$$\ge \min (\alpha,\ \min \{ \min\{ \mu^P_{\alpha_A}(x * z),\ \mu^P_{\alpha_A}(x * y)\},\ \min\{\mu^P_{\alpha_B}(x * z),\ \mu^P_{\alpha_B}(x * y)\}\})$$
$$= \min (\alpha, \min \{ \min\{ \mu^P_{\alpha_A}(x * z),\ \mu^P_{\alpha_B}(x * z)\},\ \min\{\mu^P_{\alpha_A}(x * y),\ \mu^P_{\alpha_B}(x * y)\}\})$$
$$= \min \{ \min (\alpha, \min\{ \mu^P_{\alpha_A}(x * z),\ \mu^P_{\alpha_B}(x * z)\}),\ \min (\alpha,\ \min\{\mu^P_{\alpha_A}(x * y),\ \mu^P_{\alpha_B}(x * y)\})\}$$
$$= \min \{ \min \{ \min (\alpha, \mu^P_{\alpha_A}(x * z)),\ \min (\alpha, \mu^P_{\alpha_B}(x * z))\},\ \min\{ \min(\alpha,\mu^P_{\alpha_A}(x * y)),\ \min (\alpha,\ \mu^P_{\alpha_B}(x * y))\}\}$$
$$= \min \{ \min \{\mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x * z),\ \mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z)\},\ \min \{\mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x * y),\ \mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}\}$$
$$= \min \{ \mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z),\ \mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}$$

Therefore

$$\mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) \ge \min \{\mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z), \mu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}$$

(iii) Now $\mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) = \max (\alpha', \mu^N_{\alpha_{A\cap B}}(y * z))$

$$= \max (\alpha', \max \{ \mu^N_{\alpha_A}(y * z),\ \mu^N_{\alpha_B}(y * z)\})$$
$$\le \max (\alpha', \max \{ \max\{ \mu^N_{\alpha_A}(x * z),\ \mu^N_{\alpha_A}(x * y)\},\ \max\{\mu^N_{\alpha_B}(x * z),\ \mu^N_{\alpha_B}(x * y)\}\})$$
$$= \max (\alpha', \max \{ \max\{ \mu^N_{\alpha_A}(x * z),\ \mu^N_{\alpha_B}(x * z)\},\ \max\{\mu^N_{\alpha_A}(x * y),\ \mu^N_{\alpha_B}(x * y)\}\})$$
$$= \max \{ \max (\alpha',\ \max \{ \mu^N_{\alpha_A}(x * z),\ \mu^N_{\alpha_B}(x * z)\}),\ \max (\alpha',\ \max\{\mu^N_{\alpha_A}(x * y),\ \mu^N_{\alpha_B}(x * y)\})\}$$
$$= \max \{ \max \{ \max (\alpha', \mu^N_{\alpha_A}(x * z)), \max (\alpha', \mu^N_{\alpha_B}(x * z))\},\ \max \{ \max (\alpha',\mu^N_{\alpha_A}(x * y)), \max (\alpha',\mu^N_{\alpha_B}(x * y))\}\}$$
$$= \max \{ \max \{\mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x * z),\ \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z)\},\ \max \{\mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x * y),\ \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}\}$$
$$= \max\{ \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z),\ \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}$$

Therefore

$$\mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) \le \max\{ \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * z), \mu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x * y)\}$$

(iv) Now $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(0) = \max (\beta,\ \nu^P_{\alpha_{A\cap B}}(0))$

$$= \max (\beta, \max (\nu^P_{\alpha_A}(0), \nu^P_{\alpha_B}(0)))$$
$$\le \max (\beta, \max (\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)))$$
$$= \max \{ \max (\beta, \nu^P_{\alpha_A}(x)),\ \max (\beta,\ \nu^P_{\alpha_B}(x))\}$$
$$= \max \{ \nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x), \nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$$
$$= \nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$$

Therefore $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(0) \le \nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Now $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(0) = \min (\beta', \nu^N_{\alpha_{A\cap B}}(0))$

$$= \min (\beta', \min (\nu^N_{\alpha_A}(0), \nu^N_{\alpha_B}(0)))$$
$$\ge \min (\beta', \min (\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x)))$$
$$= \min \{ \min (\beta', \nu^N_{\alpha_A}(x)),\ \min (\beta', \nu^N_{\alpha_B}(x))\}$$
$$= \min \{\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x), \nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$$
$$= \nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$$

Therefore $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(0) \ge \nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

(v) Now $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A \cap B)}}(y * z) = \max (\beta,\ \nu^P_{\alpha_{A\cap B}}(y * z))$

$$= \max (\beta, \max (\nu^P_{\alpha_A}(y * z), \nu^P_{\alpha_B}(y * z)))$$

$\leq$ max (β, max { max {$\nu^P_{\alpha_A}(x*z)$, $\nu^P_{\alpha_A}(x*y)$}, max{$\nu^P_{\alpha_B}(x*z)$, $\nu^P_{\alpha_B}(x*y)$}})

= max (β, max { max {$\nu^P_{\alpha_A}(x*z)$, $\nu^P_{\alpha_B}(x*z)$}, max{$\nu^P_{\alpha_A}(x*y)$, $\nu^P_{\alpha_B}(x*y)$}})

= max { max (β, max {$\nu^P_{\alpha_A}(x*z)$, $\nu^P_{\alpha_B}(x*z)$}), max (β , max{$\nu^P_{\alpha_A}(x*y)$, $\nu^P_{\alpha_B}(x*y)$})}

= max { max { max (β, $\nu^P_{\alpha_A}(x*z)$), max (β, $\nu^P_{\alpha_B}(x*z)$)},
max { max (β, $\nu^P_{\alpha_A}(x*y)$), max (β, $\nu^P_{\alpha_B}(x*y)$)}}

= max { max { $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z)$, $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)$ },
max {$\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)$, $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)$}}

= max { $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)$, $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)$}

Therefore
$\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) \leq$ max {$\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)$, $\nu^P_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)$}

(vi) Now $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z)$ = min (β', $\nu^N_{\alpha_{A\cap B}}(y*z)$)

= min (β', min ($\nu^N_{\alpha_A}(y*z)$, $\nu^N_{\alpha_B}(y*z)$))

$\geq$ min (β', min (min {$\nu^N_{\alpha_A}(x*z)$, $\nu^N_{\alpha_A}(x*y)$}, min { $\nu^N_{\alpha_B}(x*z)$, $\nu^N_{\alpha_B}(x*y)$}))

= min (β', min (min {$\nu^N_{\alpha_A}(x*z)$, $\nu^N_{\alpha_B}(x*z)$}, min {$\nu^N_{\alpha_A}(x*y)$, $\nu^N_{\alpha_B}(x*y)$}))

= min { min (β', min {$\nu^N_{\alpha_A}(x*z)$, $\nu^N_{\alpha_B}(x*z)$}), min (β', min {$\nu^N_{\alpha_A}(x*y)$, $\nu^N_{\alpha_B}(x*y)$})}

= min { min { min (β', $\nu^N_{\alpha_A}(x*z)$), min (β', $\nu^N_{\alpha_B}(x*z)$)},
min{ min (β', $\nu^N_{\alpha_A}(x*y)$), min (β', $\nu^N_{\alpha_B}(x*y)$)}}

= min {min {$\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z)$), $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)$},
min {$\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)$, $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)$}}

= min { $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)$, $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)$}

Therefore
$\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) \geq$ min { $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)$, $\nu^N_{\alpha_{Q_{\alpha,\alpha',\beta,\beta'}(A)\cap Q_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)$}

Therefore $Q_{\alpha,\alpha',\beta,\beta'}$(A ∩ B) = $Q_{\alpha,\alpha',\beta,\beta'}$(A) ∩ $Q_{\alpha,\alpha',\beta,\beta'}$(B) is also a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 5

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $G_{\alpha,\alpha',\beta,\beta'}$(A) is also a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.
Consider 0, x, y, z ∈ A.

(i) Now $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \alpha\, \mu^P_{\alpha_A}(0)$

$\geq \alpha\, \mu^P_{\alpha_A}(x)$

$= \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \geq \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Now $\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \alpha'\, \mu^N_{\alpha_A}(0)$

$\leq \alpha'\, \mu^N_{\alpha_A}(x)$

$= \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \leq \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

(ii) Now $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \alpha\, \mu^P_{\alpha_A}(y*z)$

$\geq \alpha$ min { $\mu^P_{\alpha_A}(x*z)$, $\mu^P_{\alpha_A}(x*y)$}

= min { $\alpha\, \mu^P_{\alpha_A}(x*z)$, $\alpha\, \mu^P_{\alpha_A}(x*y)$}

= min {$\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z)$, $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)$}

Therefore $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \geq$ min {$\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z)$, $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)$}

(iii) Now $\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \alpha'\, \mu^N_{\alpha_A}(y*z)$

$\leq \alpha'$ max { $\mu^N_{\alpha_A}(x*z)$, $\mu^N_{\alpha_A}(x*y)$}

$= \max\{ \alpha' \mu^N_{\alpha_A}(x*z),\ \alpha' \mu^N_{\alpha_A}(x*y)\}$

$= \max\{ \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z),\ \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \le \max\{ \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \beta\, \nu^P_{\alpha_A}(0)$

$\le \beta\, \nu^P_{\alpha_A}(x)$

$= \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \le \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Now $\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(0) = \beta'\, \nu^N_{\alpha_A}(0)$

$\ge \beta'\, \nu^N_{\alpha_A}(x)$

$= \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

Therefore $\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(0) \ge \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x)$

(v) Now $\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \beta\, \nu^P_{\alpha_A}(y*z)$

$\le \beta \max\{\nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_A}(x*y)\}$

$= \max\{ \beta\, \nu^P_{\alpha_A}(x*z),\ \beta\, \nu^P_{\alpha_A}(x*y)\}$

$= \max\{\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \le \max\{\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) = \beta'\, \nu^N_{\alpha_A}(y*z)$

$\ge \beta' \min\{\nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\}$

$= \min\{ \beta'\nu^N_{\alpha_A}(x*z),\ \beta'\nu^N_{\alpha_A}(x*y)\}$

$= \min\{\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(y*z) \ge \min\{\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y)\}$

Therefore $G_{\alpha,\alpha',\beta,\beta'}(A)$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 6

If A and B are bipolar intuitionistic fuzzy α-ideal of X, then $G_{\alpha,\alpha',\beta,\beta'}(A \cap B) = G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)$ is also a bipolar intuitionistic fuzzy α-ideal of X, and for every $\alpha, \beta \in [0, 1]$, $\alpha', \beta' \in [-1, 0]$ and $\alpha + \beta \le 1$, $\alpha' + \beta' \ge -1$.

Proof: Let A and B are bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z ∈ A ∩ B then 0, x, y, z ∈ A and 0, x, y, z ∈ B.

(i) Now $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \alpha\, \mu^P_{\alpha_{A\cap B}}(0)$

$= \alpha \min\{\mu^P_{\alpha_A}(0), \mu^P_{\alpha_B}(0)\}$

$\ge \alpha \min\{\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)\}$

$= \min\{\alpha\mu^P_{\alpha_A}(x),\ \alpha\mu^P_{\alpha_B}(x)\}$

$= \min\{ \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x),\ \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$

$= \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Therefore $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) \ge \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Now $\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \alpha'\, \mu^N_{\alpha_{A\cap B}}(0)$

$= \alpha' \max\{ \mu^N_{\alpha_A}(0),\ \mu^N_{\alpha_B}(0)\}$

$\le \alpha' \max\{ \mu^N_{\alpha_A}(x),\ \mu^N_{\alpha_B}(x)\}$

$= \max\{\alpha'\mu^N_{\alpha_A}(x),\ \alpha'\mu^N_{\alpha_B}(x)\}$

$= \max\{\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x),\ \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$

$= \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Therefore $\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) \le \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

(ii) Now $\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) = \alpha\, \mu^P_{\alpha_{A\cap B}}(y*z)$

$$= \alpha \min\{\mu^P_{\alpha_A}(y*z), \mu^P_{\alpha_B}(y*z)\}$$
$$\geq \alpha \min\{ \min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_A}(x*y)\}, \min\{\mu^P_{\alpha_B}(x*z), \mu^P_{\alpha_B}(x*y)\}\}$$
$$= \alpha \min\{ \min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_B}(x*z)\}, \min\{\mu^P_{\alpha_A}(x*y), \mu^P_{\alpha_B}(x*y)\}\}$$
$$= \min\{ \min\{\alpha\mu^P_{\alpha_A}(x*z), \alpha\mu^P_{\alpha_B}(x*z)\}, \min\{\alpha\mu^P_{\alpha_A}(x*y), \alpha\mu^P_{\alpha_B}(x*y)\}\}$$
$$= \min\{ \min\{\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)\},$$
$$\min\{\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y), \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}\}$$
$$= \min\{ \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$$

Therefore

$$\mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) \geq \min\{ \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$$

(iii) Now $\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) = \alpha'\, \mu^N_{\alpha_{A\cap B}}(y*z)$

$$= \alpha' \max\{\mu^N_{\alpha_A}(y*z), \mu^N_{\alpha_B}(y*z)\}$$
$$\leq \alpha' \max\{ \max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*y)\}, \max\{\mu^N_{\alpha_B}(x*z), \mu^N_{\alpha_B}(x*y)\}\}$$
$$= \alpha' \max\{ \max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_B}(x*z)\}, \max\{\mu^N_{\alpha_A}(x*y), \mu^N_{\alpha_B}(x*y)\}\}$$
$$= \max\{ \max\{\alpha'\mu^N_{\alpha_A}(x*z), \alpha'\mu^N_{\alpha_B}(x*z)\}, \max\{\alpha'\mu^N_{\alpha_A}(x*y), \alpha'\mu^N_{\alpha_B}(x*y)\}\}$$
$$= \max\{ \max\{\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)\},$$
$$\max\{\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y), \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}\}$$
$$= \max\{ \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$$

Therefore

$$\mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) \leq \max\{ \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$$

(iv) Now $\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \beta\, \nu^P_{\alpha_{A\cap B}}(0)$

$$= \beta \max\{\nu^P_{\alpha_A}(0), \nu^P_{\alpha_B}(0)\}$$
$$\leq \beta \max\{\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)\}$$
$$= \max\{ \beta\,\nu^P_{\alpha_A}(x), \beta\,\nu^P_{\alpha_B}(x)\}$$
$$= \max\{\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x), \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$$
$$= \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$$

Therefore $\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) \leq \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

Now $\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) = \beta'\, \nu^N_{\alpha_{A\cap B}}(0)$

$$= \beta' \min\{\nu^N_{\alpha_A}(0), \nu^N_{\alpha_B}(0)\}$$
$$\geq \beta' \min\{\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x)\}$$
$$= \min\{ \beta'\nu^N_{\alpha_A}(x), \beta'\nu^N_{\alpha_B}(x)\}$$
$$= \min\{\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x), \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)\}$$
$$= \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$$

Therefore $\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(0) \geq \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x)$

(v) Now $\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) = \beta\, \nu^P_{\alpha_{A\cap B}}(y*z)$

$$= \beta \max\{\nu^P_{\alpha_A}(y*z), \nu^P_{\alpha_B}(y*z)\}$$
$$\leq \beta \max\{ \max\{\nu^P_{\alpha_A}(x*z), \nu^P_{\alpha_A}(x*y)\}, \max\{\nu^P_{\alpha_B}(x*z), \nu^P_{\alpha_B}(x*y)\}\}$$
$$= \beta \max\{ \max\{\nu^P_{\alpha_A}(x*z), \nu^P_{\alpha_B}(x*z)\}, \max\{\nu^P_{\alpha_A}(x*y), \nu^P_{\alpha_B}(x*y)\}\}$$
$$= \max\{ \max\{\beta\nu^P_{\alpha_A}(x*z), \beta\nu^P_{\alpha_B}(x*z)\}, \max\{\beta\nu^P_{\alpha_A}(x*y), \beta\nu^P_{\alpha_B}(x*y)\}\}$$
$$= \max\{ \max\{\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)\},$$
$$\max\{\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y), \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}\}$$
$$= \max\{ \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$$

Therefore

$$\nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) \leq \max\{ \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$$

(vi) Now $\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) = \beta'\, \nu^N_{\alpha_{A\cap B}}(y*z)$

$$= \beta' \min\{\nu^N_{\alpha_A}(y*z), \nu^N_{\alpha_B}(y*z)\}$$
$$\geq \beta' \min\{\min\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_A}(x*y)\}, \min\{\nu^N_{\alpha_B}(x*z), \nu^N_{\alpha_B}(x*y)\}\}$$
$$= \beta' \min\{\min\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_B}(x*z)\}, \min\{\nu^N_{\alpha_A}(x*y), \nu^N_{\alpha_B}(x*y)\}\}$$
$$= \min\{\min\{\beta'\nu^N_{\alpha_A}(x*z), \beta'\nu^N_{\alpha_B}(x*z)\}, \min\{\beta'\nu^N_{\alpha_A}(x*y), \beta'\nu^N_{\alpha_B}(x*y)\}\}$$
$$= \min\{\min\{\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*z), \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z)\}, \min\{\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)}}(x*y), \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}\}$$
$$= \min\{\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$$

Therefore

$$\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A\cap B)}}(y*z) \geq \min\{\nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*z), \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(A)\cap G_{\alpha,\alpha',\beta,\beta'}(B)}}(x*y)\}$$

Therefore $G_{\alpha,\alpha',\beta,\beta'}(A\cap B) = G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)$ is also a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 7

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = Q_{\beta,\beta',\alpha,\alpha'}(A)$ is also a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z ∈ A.

(i) Now $\mu^P_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) = \nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(0)$

$$= \min\{\beta, \nu^P_{\alpha_{\bar{A}}}(0)\}$$
$$= \min\{\beta, \mu^P_{\alpha_A}(0)\}$$
$$\geq \min\{\beta, \mu^P_{\alpha_A}(x)\}$$
$$= \mu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$$

Therefore $\mu^P_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) \geq \mu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Now $\mu^N_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) = \nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(0)$

$$= \max\{\beta', \nu^N_{\alpha_{\bar{A}}}(0)\}$$
$$= \max\{\beta', \mu^N_{\alpha_A}(0)\}$$
$$\leq \max\{\beta', \mu^N_{\alpha_A}(x)\}$$
$$= \mu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$$

Therefore $\mu^N_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) \leq \mu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

(ii) Now $\mu^P_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) = \nu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(y*z)$

$$= \min\{\beta, \nu^P_{\alpha_{\bar{A}}}(y*z)\}$$
$$= \min\{\beta, \mu^P_{\alpha_A}(y*z)\}$$
$$\geq \min\{\beta, \min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_A}(x*y)\}\}$$
$$= \min\{\min\{\beta, \mu^P_{\alpha_A}(x*z)\}, \min\{\beta, \mu^P_{\alpha_A}(x*y)\}\}$$
$$= \min\{\mu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \mu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$$

Therefore $\mu^P_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) \geq \min\{\mu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \mu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) = \nu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(y*z)$

$$= \max\{\beta', \nu^N_{\alpha_{\bar{A}}}(y*z)\}$$
$$= \max\{\beta', \mu^N_{\alpha_A}(y*z)\}$$
$$\leq \max\{\beta', \max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*y)\}\}$$
$$= \max\{\max\{\beta', \mu^N_{\alpha_A}(x*z)\}, \max\{\beta', \mu^N_{\alpha_A}(x*y)\}\}$$
$$= \max\{\mu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \mu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$$

Therefore $\mu^N_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) \leq \max\{\mu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \mu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) = \mu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(0)$

$= \max\{\alpha,\ \mu^P_{\alpha_{\bar{A}}}(0)\}$

$= \max\{\alpha, \nu^P_{\alpha_A}(0)\}$

$\leq \max\{\alpha, \nu^P_{\alpha_A}(x)\}$

$= \nu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Therefore $\nu^P_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) \leq \nu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Now $\nu^N_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) = \mu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(0)$

$= \min\{\alpha', \mu^N_{\alpha_{\bar{A}}}(0)\}$

$= \min\{\alpha', \nu^N_{\alpha_A}(0)\}$

$\geq \min\{\alpha', \nu^N_{\alpha_A}(x)\}$

$= \nu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Therefore $\nu^N_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) \geq \nu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

(v) Now $\nu^P_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) = \mu^P_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(y*z)$

$= \max\{\alpha,\ \mu^P_{\alpha_{\bar{A}}}(y*z)\}$

$= \max\{\alpha, \nu^P_{\alpha_A}(y*z)\}$

$\leq \max\{\alpha, \max\{\nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_A}(x*y)\}\}$

$= \max\{\max\{\alpha, \nu^P_{\alpha_A}(x*z)\},\ \max\{\alpha,\ \nu^P_{\alpha_A}(x*y)\}\}$

$= \max\{\nu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \nu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

Therefore $\nu^P_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) \leq \max\{\nu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \nu^P_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) = \mu^N_{\alpha_{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(y*z)$

$= \min\{\alpha', \mu^N_{\alpha_{\bar{A}}}(y*z)\}$

$= \min\{\alpha', \nu^N_{\alpha_A}(y*z)\}$

$\geq \min\{\alpha', \min\{\nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\}\}$

$= \min\{\min\{\alpha', \nu^N_{\alpha_A}(x*z)\},\ \min\{\alpha',\ \nu^N_{\alpha_A}(x*y)\}\}$

$= \min\{\nu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \nu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

Therefore $\nu^N_{\alpha_{\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) \geq \min\{\nu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \nu^N_{\alpha_{Q_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

Therefore $\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = Q_{\beta,\beta',\alpha,\alpha'}(A)$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 8

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = G_{\beta,\beta',\alpha,\alpha'}(A)$ is also a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z ∈ A.

(i) Now $\mu^P_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) = \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(0)$

$= \beta\ \nu^P_{\alpha_{\bar{A}}}(0)$

$= \beta\ \mu^P_{\alpha_A}(0)$

$\geq \beta\ \mu^P_{\alpha_A}(x)$

$= \mu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Therefore $\mu^P_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) \geq \mu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Now $\mu^N_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) = \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(0)$

$= \beta'\ \nu^N_{\alpha_{\bar{A}}}(0)$

$= \beta'\ \mu^N_{\alpha_A}(0)$

$\leq \beta'\ \mu^N_{\alpha_A}(x)$

$= \mu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Therefore $\mu^N_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) \leq \mu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

(ii) Now $\mu^P_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) = \nu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(y*z)$

$= \beta\, \nu^P_{\alpha_{\bar{A}}}(y*z)$

$= \beta\, \mu^P_{\alpha_A}(y*z)$

$\geq \beta \min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_A}(x*y)\}$

$= \min\{\beta\, \mu^P_{\alpha_A}(x*z), \beta\, \mu^P_{\alpha_A}(x*y)\}$

$= \min\{\mu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \mu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

Therefore $\mu^P_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) \geq \min\{\mu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \mu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) = \nu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(y*z)$

$= \beta'\, \nu^N_{\alpha_{\bar{A}}}(y*z)$

$= \beta'\, \mu^N_{\alpha_A}(y*z)$

$\leq \beta' \max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*y)\}$

$= \max\{\beta'\, \mu^N_{\alpha_A}(x*z), \beta'\, \mu^N_{\alpha_A}(x*y)\}$

$= \max\{\mu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \mu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

Therefore $\mu^N_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) \leq \max\{\mu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \mu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) = \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(0)$

$= \alpha\, \mu^P_{\alpha_{\bar{A}}}(0)$

$= \alpha\, \nu^P_{\alpha_A}(0)$

$\leq \alpha\, \nu^P_{\alpha_A}(x)$

$= \nu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Therefore $\nu^P_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) \leq \nu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Now $\nu^N_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) = \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(0)$

$= \alpha'\, \mu^N_{\alpha_{\bar{A}}}(0)$

$= \alpha'\, \nu^N_{\alpha_A}(0)$

$\geq \alpha'\, \nu^N_{\alpha_A}(x)$

$= \nu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

Therefore $\nu^N_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(0) \geq \nu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x)$

(v) Now $\nu^P_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) = \mu^P_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(y*z)$

$= \alpha\, \mu^P_{\alpha_{\bar{A}}}(y*z)$

$= \alpha\, \nu^P_{\alpha_A}(y*z)$

$\leq \alpha \max\{\nu^P_{\alpha_A}(x*z), \nu^P_{\alpha_A}(x*y)\}$

$= \max\{\alpha\, \nu^P_{\alpha_A}(x*z), \alpha\, \nu^P_{\alpha_A}(x*y)\}$

$= \max\{\nu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \nu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

Therefore $\nu^P_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) \leq \max\{\nu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \nu^P_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) = \mu^N_{\alpha_{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}(y*z)$

$= \alpha'\, \mu^N_{\alpha_{\bar{A}}}(y*z)$

$= \alpha'\, \nu^N_{\alpha_A}(y*z)$

$\geq \alpha' \min\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_A}(x*y)\}$

$= \min\{\alpha'\, \nu^N_{\alpha_A}(x*z), \alpha'\, \nu^N_{\alpha_A}(x*y)\}$

$= \min\{\nu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \nu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

Therefore $\nu^N_{\alpha_{\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})}}}(y*z) \geq \min\{\nu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*z), \nu^N_{\alpha_{G_{\beta,\beta',\alpha,\alpha'}(A)}}(x*y)\}$

Therefore $\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = G_{\beta,\beta',\alpha,\alpha'}(A)$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 9
If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then $P_{\alpha,\alpha',\beta,\beta'}(A)$ is a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem: 10
If A and B are bipolar intuitionistic anti fuzzy α-ideal of X, then
$P_{\alpha,\alpha',\beta,\beta'}(A \cap B) = P_{\alpha,\alpha',\beta,\beta'}(A) \cap P_{\alpha,\alpha',\beta,\beta'}(B)$ is also a bipolar intuitionistic anti fuzzy α-ideal of X, and for every $\alpha, \beta \in [0, 1]$, $\alpha', \beta' \in [-1, 0]$ and $\alpha + \beta \leq 1$, $\alpha' + \beta' \geq -1$.

Theorem: 11
If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then $Q_{\alpha,\alpha',\beta,\beta'}(A)$ is a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem: 12
If A and B are bipolar intuitionistic anti fuzzy α-ideal of X, then
$Q_{\alpha,\alpha',\beta,\beta'}(A \cap B) = Q_{\alpha,\alpha',\beta,\beta'}(A) \cap Q_{\alpha,\alpha',\beta,\beta'}(B)$ is also a bipolar intuitionistic anti fuzzy α-ideal of X, and for every $\alpha, \beta \in [0, 1]$, $\alpha', \beta' \in [-1, 0]$ and $\alpha + \beta \leq 1$, $\alpha' + \beta' \geq -1$.

Theorem: 13
If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then $G_{\alpha,\alpha',\beta,\beta'}(A)$ is also a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem: 14
If A and B are bipolar intuitionistic anti fuzzy α-ideal of X, then
$G_{\alpha,\alpha',\beta,\beta'}(A \cap B) = G_{\alpha,\alpha',\beta,\beta'}(A) \cap G_{\alpha,\alpha',\beta,\beta'}(B)$ is also a bipolar intuitionistic anti fuzzy α-ideal of X, and for every $\alpha, \beta \in [0, 1]$, $\alpha', \beta' \in [-1, 0]$ and $\alpha + \beta \leq 1$, $\alpha' + \beta' \geq -1$.

Theorem: 15
If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then $\overline{P_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = Q_{\beta,\beta',\alpha,\alpha'}(A)$ is also a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem: 16
If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then $\overline{G_{\alpha,\alpha',\beta,\beta'}(\bar{A})} = G_{\beta,\beta',\alpha,\alpha'}(A)$ is also a bipolar intuitionistic anti fuzzy α-ideal of X.

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