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Some Biconditional Cordial Graphs

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Abstract : *For a graph $G = (V, E)$, a binary vertex labeling function $f : V(G) \rightarrow \{0,1\}$ of G is called a biconditional cordial labeling if an induced edge labeling function $f^* : E(G) \rightarrow \{0,1\}$ defined by $f^*(uv) = \begin{cases} 1; f(u) = f(v) \\ 0; f(u) \neq f(v) \end{cases}$, $\forall\, uv \in E(G)$ satisfies the two conditions: $(i)\ |v_f(0) - v_f(1)| \leq 1$ and $(ii) |e_f(0) -$ $ef1 \leq 1$, where vfi and efi denotes respectively the number of vertices and edges of G having label i. A graph is called biconditional cordial graph if it admits biconditional cordial labeling. In this manuscript we have proved that the crown graph, armed crown, helms, closed helms, gears and flower graphs are biconditional cordial graphs.*

Key Word*: Crown graph; Flower graph; Biconditional cordial labeling; Biconditional cordial graph.*

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I. Introduction

We begin with simple, finite, connected and undirected graph $G = (V, E)$ with order p and size q which we also denote as $G = (p, q)$. The terms not defined here are used in the sense of Harary [4]. For an extensive survey on graph labeling and bibliographic references we refer to Gallian [2]. Most of the graph labeling techniques trace their origin to graceful labeling introduced independently by Rosa [11] and Golomb [3]. A S. K. Vaidya, C. M. Barasara, Edge Product Cordial Labeling of Graphs, Journal of Mathematical and Computational Science, 2(5), (2012), 1436-1450version of graceful labeling were introduced by Cahit [1] and called it cordial labeling. In this article Cahit has presented results on cordial labeling of tree, complete bipartite graph, friendship graph, fan and wheel graph. Some more results on graph cordiality can be found in [6, 10]. The concept of edge product cordial labeling was first introduced by Vaidya and Barasara [12]. They studied edge product cordial property for cycle C_n for odd n, trees with order greater than 2, crown graph $C_n \odot K_1$, armed crown, helms, closed helms, web, gear graph G_n for odd n, flowers, shell graph S_n for odd n. Murali et al.[8] introduced the concept of biconditional cordial labelling and proved that path and cycles are biconditional cordial graphs. Murli et al. [7] proved that ladder graph and $K_2 + nK_1$ graph admits biconditional cordial labeling. After that Nedumaran et al.[9] proved that k copies of double star admits biconditional cordial labeling. Further, Kalaimathi and Balamurugan [5] proved that complete bipartite graph, book graph with triangular pages and web graphs are biconditional cordial graphs. Here in this article we discuss the biconditional cordial labeling of crown graph $C_n \odot K_1$, armed crown, helms, closed helms, gears and flower graphs.

Following is a brief summary of definitions which are useful for the present investigations.

Definition 1 : For a graph $G = (V, E)$, a binary vertex labeling function $f : V(G) \rightarrow \{0,1\}$ induces an edge labeling function $f^* : E(G) \rightarrow \{0,1\}$ defined as $f^*(uv) = \begin{cases} 1; f(u) = f(v) \\ 0; f(u) \neq f(v) \end{cases}$, $\forall\, uv \in E(G)$. Then f is called a bi-conditional cordial labeling of graph G if $|v_f(0) - v_f(1)| \leq 1$and$|e_f(0) - e_f(1)| \leq 1$. Where $v_f(i)$ and $e_f(i)$ be the number of vertices and edges of G having labels i respectively.

Definition 2 : The crown $C_n \odot K_1$ is obtained by joining a pendant edge to each vertex of C_n

Definition 3: The armed crown is a graph in which path P_2 is attached at each vertex of cycle C_n by an edge. It is denoted by AC_n where n is the number of vertices in cycle C_n.

Definition 4 : The wheel W_n is defined to be the join $K_1 + C_n$. The vertex corresponding to K_1 is known as apex vertex, the vertices corresponding to cycle are known as rim vertices while the edges corresponding to cycle are known as rim edges. Edges joining apex and vertices of cycle are spoke edges.

Definition 5 : The helm H_n is the graph obtained from wheel W_n by joining a pendant edge to each rim vertex of wheel W_n.

Definition 6 : The closed helm CH_n is the graph obtained from a helm H_n by joining each pendant vertex of helm H_n to form a cycle.

Definition 7 : The flower Fl_n is the graph obtained from a helm H_n by joining each pendant vertex of helm to the apex vertex of the helm.
Definition 8 : Let $e = uv$ be an edge of graph G and w is not a vertex of G. The edge e is subdivided when it is replaced by the edges $e' = uw$ and $e'' = wv$.
Definition 9 : The gear graph G_n is obtained from the wheel W_n subdividing each of its rim edge.

II. Main Results

Theorem 1 *:* The crown $C_n \odot K_1$ is a bi-conditional cordial graph.
Proof *:* Let $v_1, v_2, \dots, v_n$ be the vertices of C_n and $u_1, u_2, \dots, u_n$ be the pendant vertices of crown $C_n \odot K_1$.
Now, we define $f : V(C_n \odot K_1) \to \{0,\ 1\}$ as

$$f(v_i) = 0; \text{ for all } i \text{ and } f(u_i) = 1; \text{ for all } i.$$

Thus we have

$$v_f(0) = \text{n} = v_f(1) \text{ and } e_f(0) = \text{n} = e_f(1)$$

$$\Rightarrow |v_f(0) - v_f(1)| \leq 1 \text{ and } |e_f(0) - e_f(1)| \leq 1.$$

Hence the crown $C_n \odot K_1$ is a bi-conditional cordial graph.

Illustration 1 : $C_4 \odot K_1$ and its biconditional cordial labeling is shown in Figure 1.

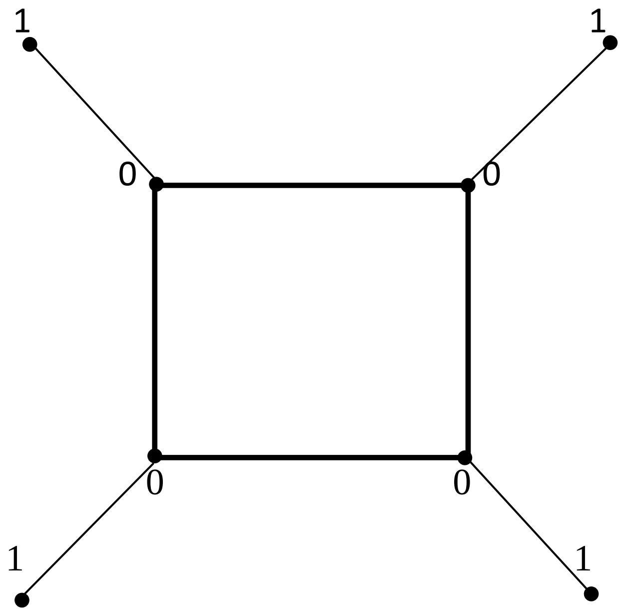


Figure 1: $C_4 \odot K_1$ and its biconditional cordial labeling

Theorem 2 : The armed crown AC_n is a bi-conditional cordial graph.
Proof : Let $v_1, v_2, \dots, v_n$ be the vertices of C_n. Let $u_i,\ w_i$ be the vertices of path ${P_2}^i$. To construct armed crown AC_n join vertex v_i of cycle $C_n$to vertex u_i of path with an edge.
Case 1 : When n is an even.
Now, we define $f : V(AC_n) \to \{0,\ 1\}$ as

$$f(v_i) = 0; \quad \text{for all } i,$$
$$f(u_i) = 1; \quad \text{for all } i,$$
$$f(w_{2i-1}) = 1; \ 1 \leq i \leq \frac{n}{2},$$
$$f(w_{2i}) = 0; \quad 1 \leq i \leq \frac{n}{2},$$

Thus we have

$$v_f(0) = n + \frac{n}{2} = \frac{3n}{2},$$
$$v_f(1) = n + \frac{n}{2} = \frac{3n}{2},$$
$$e_f(0) = n + \frac{n}{2} = \frac{3n}{2} \ ,$$
$$e_f(1) = n + \frac{n}{2} = \frac{3n}{2}$$

$\Rightarrow |v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Case 2 : When n is an odd.
Now, we define $f : V(AC_n) \to \{0,\ 1\}$ as

$f(v_i) = 0;$ for all i,
$f(u_i) = 1;$ for all i,
$f(w_{2i-1}) = 1;\ 1 \le i \le \lceil \frac{n}{2} \rceil$,
$f(w_{2i}) = 0;\ 1 \le i \le \lfloor \frac{n}{2} \rfloor$,

Thus we have

$v_f(1) = n + \lceil \frac{n}{2} \rceil = \frac{3n+1}{2}$,
$v_f(0) = n + \lfloor \frac{n}{2} \rfloor = \frac{3n-1}{2}$,
$e_f(1) = n + \lceil \frac{n}{2} \rceil = \frac{3n+1}{2}$,
$e_f(0) = n + \lfloor \frac{n}{2} \rfloor = \frac{3n-1}{2}$

$\Rightarrow |v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.
Hence the armed crown AC_n is a bi-conditional cordial graph.

Illustration 2 : AC_4 and AC_5 with its biconditional cordial labeling is shown in Figure 2.

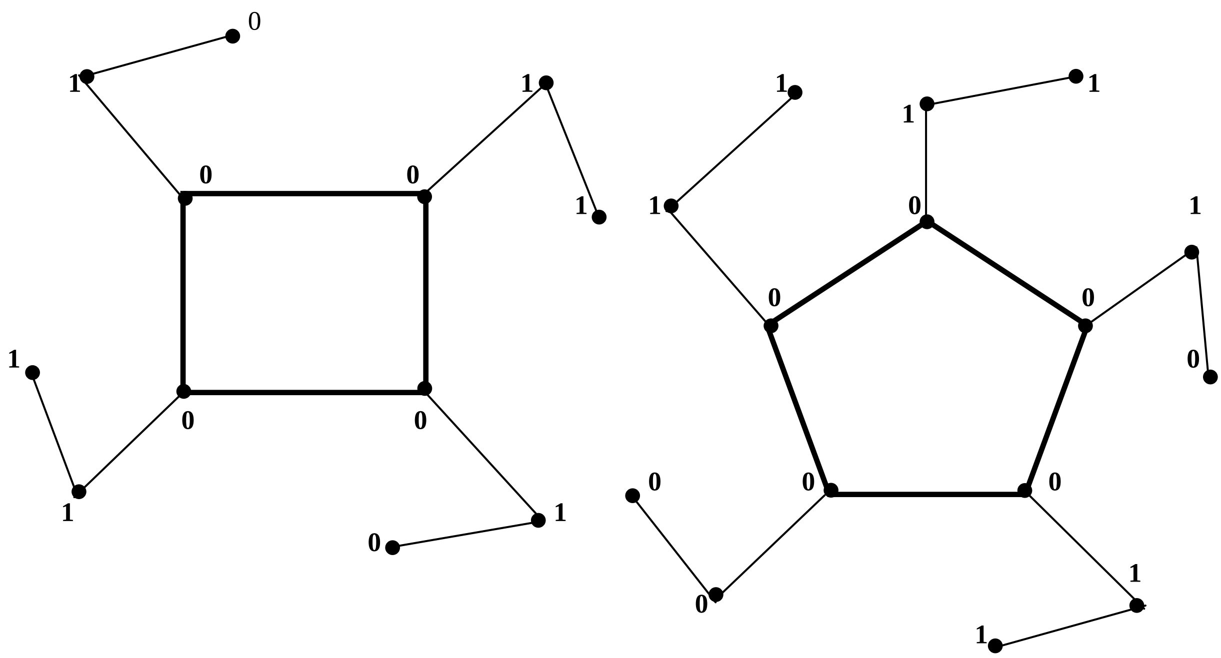


Figure 2 : AC_4 and AC_5 with its biconditional cordial labeling

Theorem 3 : The helm H_n is a bi-conditional cordial graph.
Proof : Let $v_1, v_2, \ldots, v_n$ be the rim vertices and $u_1, u_2, \ldots, u_n$ be the pendant vertices of helm H_n. Let v be an apex vertex.
Case 1 : When n is an even.
Now, we define $f : V(H_n) \to \{0,\ 1\}$ as

$f(v_{2i-1}) = 1;\quad 1 \le i \le \frac{n}{2},\qquad f(v_{2i}) = 0;\quad 1 \le i \le \frac{n}{2}$,
$f(u_{2i-1}) = 1;\quad 1 \le i \le \frac{n}{2},\qquad f(u_{2i}) = 0;\quad 1 \le i \le \frac{n}{2}$,
$f(v) = 0$

Thus we have

$v_f(0) = \frac{n}{2} + \frac{n}{2} + 1 = n+1$,

$v_f(1) = \frac{n}{2} + \frac{n}{2}$ = n,
$e_f(0) = n + \frac{n}{2} = \frac{3n}{2}$,
$e_f(1) = n + \frac{n}{2} = \frac{3n}{2}$
$\Rightarrow \left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$.

Case 2 : When n is an odd.

Now, we define $f : V(H_n) \rightarrow \{0,\ 1\}$ as

$f(v_{2i-1}) = 1;\ \ 1 \leq i \leq \lceil \frac{n}{2} \rceil$,
$f(v_{2i}) = 0;\ \ \ \ 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$,
$f(u_{2i-1}) = 1;\ \ 1 \leq i \leq \lceil \frac{n}{2} \rceil$,
$f(u_{2i}) = 0;\ \ \ \ 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$
$f(v) = 0$

Thus we have

$v_f(0) = \lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{2} \rfloor + 1$ = n,
$v_f(1) = \lceil \frac{n}{2} \rceil + \lceil \frac{n}{2} \rceil$ = n+1,
$e_f(0) = (n-1) + \lceil \frac{n}{2} \rceil = \frac{3n-1}{2}$,
$e_f(1) = 1 + n + \lfloor \frac{n}{2} \rfloor = \frac{3n+1}{2}$
$\Rightarrow \left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$.

Hence the helm H_n is a bi-conditional cordial graph.

Illustration 3 : H_6 with its biconditional cordial labeling is shown in Figure 3.

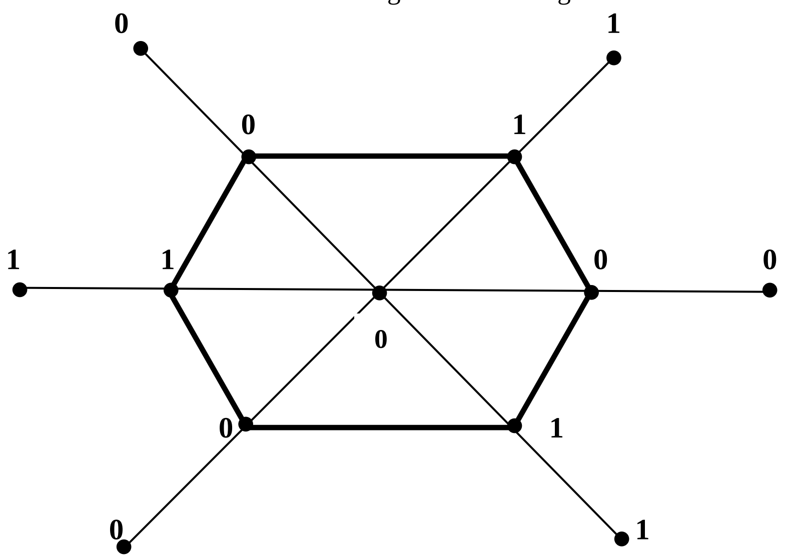


Figure 3 : H_6 with its biconditional cordial labeling

Theorem 4 : The closed helm CH_n is a bi-conditional cordial graph.

Proof : Let $v_1, v_2, \ldots, v_n$ be the rim vertices and $u_1, u_2, \ldots, u_n$ be the vertices of outer cycle. Let v be an apex vertex.

Now, we define $f : V(CH_n) \rightarrow \{0,\ 1\}$ as

$f(v_i) = 1;$ for all i,
$f(u_i) = 0;$ for all i,
$f(v) = 0$

Thus we have

$v_f(0) = n + 1$,
$v_f(1)$ = n,
$e_f(0) = n + n = 2n$,
$e_f(1) = n + n = 2n$

$\Rightarrow \left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$.
Hence the closed helm is a bi-conditional cordial graph.

Illustration 4 : CH_4 and CH_5 with its biconditional cordial labeling is shown in Figure 4.

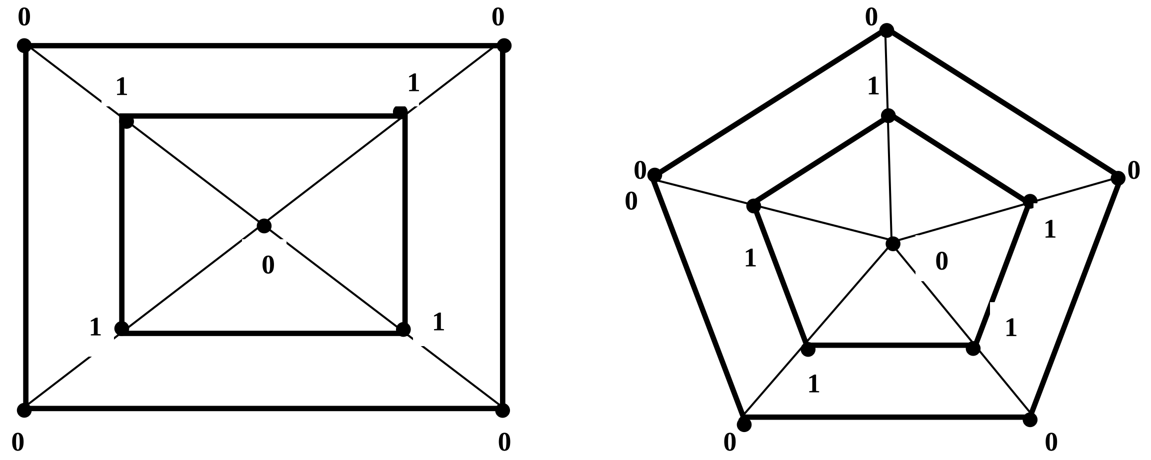


Figure 4: CH_4 and CH_5 with its biconditional cordial labeling

Theorem 5 : The gear graph G_n is a bi-conditional cordial graph.
Proof : Let $v, v_1, v_2, \ldots, v_{2n}$ be the vertices of gear graph G_n.
Now, we define $f: V(G_n) \rightarrow \{0, 1\}$ as

$$f(v_i) = \begin{cases} 1; \ i \cong 1,2(mod4) \\ 0; \ i \cong 0,3(mod4) \end{cases}, f(v) = 0$$

Case 1 : When n is an even.
Thus we have

$v_f(0) = n + 1, \qquad v_f(1) = \text{n}$

$e_f(0) = n + \frac{n}{2} = \frac{3n}{2} = e_f(1)$

$\Rightarrow \left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$.

Case 2 : When n is an odd.
Thus we have

$v_f(0) = \frac{2n-2}{2} + 1 = \text{n},$

$v_f(1) = \frac{2n-2}{2} + 2 = n + 1,$

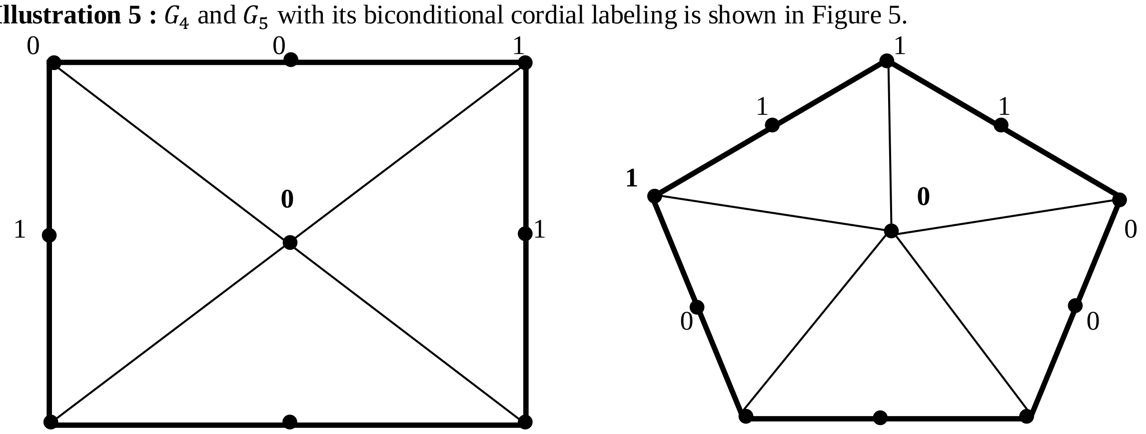
$e_f(0) = \frac{2n-2}{2} + \lceil \frac{n}{2} \rceil = \frac{3n-1}{2},$

$e_f(1) = \frac{2n-2}{2} + 2 + \lfloor \frac{n}{2} \rfloor = \frac{3n+1}{2}$

$\Rightarrow \left|v_f(0) - v_f(1)\right| \leq 1$ and $\left|e_f(0) - e_f(1)\right| \leq 1$.

Hence the gear graph G_n is a bi-conditional cordial graph.

Illustration 5 : G_4 and G_5 with its biconditional cordial labeling is shown in Figure 5.



1 0 0 0 1 1

Figure 5 : G_4 and G_5 with and its biconditional cordial labeling

Theorem 6 : The flower graph Fl_n is a bi-conditional cordial graph.
Proof : Let v be an apex vertex of helm. Let $v_1, v_2, \ldots, v_n$ be the rim vertices of helm H_n and $u_1, u_2, \ldots, u_n$ be the pendant vertices of helm H_n. To construct flower graph Fl_n join the pendant vertex u_i of helm H_n to an apex vertex of helm H_n .
Now, we define $f : V(Fl_n) \rightarrow \{0,\ 1\}$ as

$f(v_i) = 1;$ for all i,
$f(u_i) = 0;$ for all i,
$f(v) = 0$

Thus we have

$v_f(0) = n + 1,$
$v_f(1) = \text{n},$
$e_f(0) = n + n = 2n,$
$e_f(1) = n + n = 2n$

$\Rightarrow |v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1.$

Hence the flower graph Fl_n is a bi-conditional cordial graph.

Illustration 6: Fl_4 and its biconditional cordial labeling is shown in Figure 6.

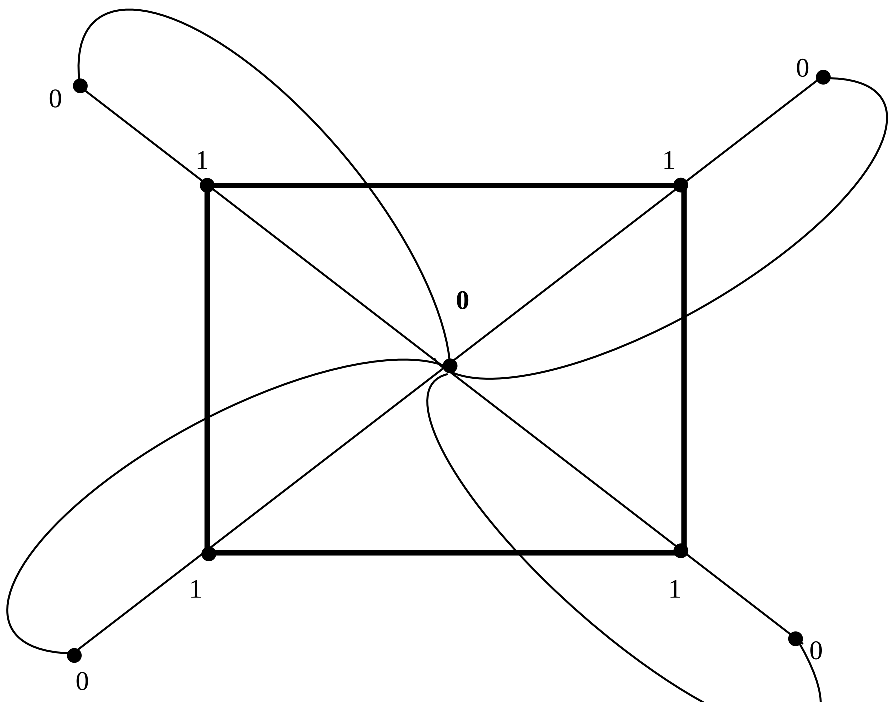


Figure 6: Fl_4 and its biconditional cordial labeling

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