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A General Finite Markov Chain Model for Degrading Systems Analysis

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Abstract: *Analytical model of degrading systems based on Finite Markov Chains is defined. This model is analyzed within the finite time horizon for recoverable, partially recoverable, and non-recoverable degrading systems. The set of critical states is identified. This set forms some base for the implementation of a bounded probabilistic analysis of the investigated degrading system.*
Key Word*: degrading systems, finite time horizon, Finite Markov Chains, bounded probabilistic analysis.*

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I. Introduction

It is well-known that any real system is inherently a degrading one, since its functioning is deteriorated by aging and accumulated wear. This is the reason why developing an effective maintenance policy (MP) for the analyzed degrading system (DS) is one of the actual problems.

The framework for the MP study has been laid in [1]. A significant contribution to this area was the study of MPs within the finite time horizon [2], which is one of the essential conditions for any real DS. A fairly comprehensive overview of approaches for developing MPs is presented in [3]. Wide use of Cyber-Physical Systems significantly increased the relevance of the MPs study [4, 5]. For these systems, in addition to fully recoverable DSs, there is an urgent need to investigate also partially recoverable as well as non-recoverable DSs. The latter ones have many applications in various fields, for example in the health care sector [6-8].

An essential component for the development of any MP is modeling the temporal variability of DS deterioration. As a rule, it is based on the analysis of the changes in the parameters of the analyzed DS. To simulate these changes various mathematical models are used, both deterministic and stochastic (see [9], for example). Due to the inherent limitations of deterministic models, stochastic models are known to be more adequate. Among the latter, the Finite Markov Chain (FMC) [10] should be especially noted for the following three reasons.

Firstly, any FMC is a fairly simple model.

Secondly, state transitions in the FMC can naturally be interpreted as the measurements of the system parameters after a fixed period of time (it is used for analysis of a sufficiently wide class of DSs).

Thirdly, symbolic simulation makes it possible to implement probability variation at non-intersecting time intervals using a sequence of analyzed FMCs.

These arguments are confirmed, for example, by the results obtained in [11-13]. Besides, any FMC is a convenient model for performing bounded probabilistic analysis [14, 15] of the investigated DS and for generation probabilistic counterexamples [16, 17], i.e. the sets of finite paths with a critical probability mass.

Although FMC is used to analyze DS in a significant number of studies, the models considered in them are built for specific problems under investigation. An analytical model based on the FMC and intended to analyze fully recoverable and non-recoverable DS has been proposed in [18].

In the given paper, this model is refined and generalized to simulate not only fully recoverable and non-recoverable DSs, but also partially recoverable ones.

The rest of the paper is organized as follows. In Section 2 proposed analytical model is defined. In Section 3 exact analysis of the proposed model is considered. In Section 4 bounded analysis of the proposed model is considered. Section 5 is some discussion of obtained results. Section 6 contains concluding remarks.

II. Proposed FMC model for DS analysis

It is well-known that any FMC C_n with the set of states $S_n = \{s_1, \cdots, s_n\}$ $(n \geq 2)$ can be presented by the stochastic matrix

$$P_{C_n} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix},$$

i.e. $0 \leq p_{ij} \leq 1$ $(i, j = 1, \cdots, n)$ and $\sum_{j=1}^{n} p_{ij} = 1$ $(j = 1, \cdots, n)$. We remind that the element p_{ij} $(i, j = 1, \cdots, n)$ is the probability of the transition from the state s_i to the state s_j in one step. The elements of the stochastic matrix $P_{C_n}^m = \underbrace{P_{C_n} \cdots P_{C_n}}_{m \text{ times}}$ $(m = 1, 2, \cdots)$ are the probabilities of transitions between states in m steps, and for any initial distribution of the probabilities of the states $\mathbf{v}_0 = (v_1^{(0)}, \cdots, v_n^{(0)})$ (where $0 \leq v_i^{(0)} \leq 1$ $(i = 1, \cdots, n)$ and $\sum_{i=1}^{n} v_i^{(0)} = 1$) the vector $\mathbf{u}_m = \mathbf{v}_0 P_{C_n}^m$ $(m = 1, 2, \cdots)$ is the distribution of the probabilities of the states after m steps.

To deal with the FMC C_n as the model of the analyzed DS S_n with n stages of functionality, the following four assumptions are accepted:

Assumption 1. One step of state transition corresponds to two successive measurements of the parameters of the analyzed DS S_n.

Assumption 2. The state s_1 represents the analyzed DS S_n in the fully functional stage.

Assumption 3. The state s_n represents the analyzed DS S_n in the inoperable stage.

Assumption 4. The states $s_2, \cdots, s_{n-1}$ represent the analyzed DS S_n in all possible stages of partial functioning.

Besides, to represent explicitly degradation as well as recovery of the analyzed DS S_n in one step it is assumed that the following two restrictions on the structure of the FMC C_n hold:

Restriction 1. For a given positive integer k $(2 \leq k \leq n)$ some partition $\pi = \{B_1, \cdots, B_k\}$ of the set S_n is fixed such that $B_1 = \{s_{i_1}\}$, $B_j = \{s_{i_{j-1}+1}, \cdots, s_{i_j}\}$ $(j = 2, \cdots, k-1)$, and $B_k = \{s_{i_{k-1}+1}\}$, where $i_1 = 1$, $i_{k-1} = n-1$ and $i_{j-1} < i_j$ for all $j = 2, \cdots, k-1$.

Restriction 2. Elements of the stochastic matrix P_{C_n} satisfy to the following six conditions:

Condition 1. For all $i = 1, \cdots, n-1$ hold the inequalities $0 < p_{ii} < 1$, and $p_{nn} = 1$.

Condition 2. For all $j = 2, \cdots, k-1$ the equality $p_{rh} = 0$ holds for all states $s_r, s_h \in B_j$ $(r \neq h)$.

Condition 3. For any state $s_r \in B_j$ $(j = 1, \cdots, k-1)$ there exists some subset $S_n^{dsc}(r)$ $\left(\varnothing \neq S_n^{dsc}(r) \subseteq \bigcup_{m=j+1}^{k} B_m \right)$ such that

$$p_{rh} \begin{cases} > 0, \text{ for all } s_h \in S_n^{dsc}(r) \\ = 0, \text{ for all } s_h \in \left(\bigcup_{m=j+1}^{k} B_m \right) \setminus S_n^{dsc}(r) \end{cases}.$$

Condition 4. For all $j = 1, \cdots, k-1$ holds the equality $\bigcup_{s_r \in B_j} (S_n^{dsc}(r) \cap B_{j+1}) = B_{j+1}$.

Condition 5. For any state $s_r \in B_j$ $(j = 2, \cdots, k-1)$ there exists some subset $S_n^{anc}(r)$ $(S_n^{anc}(r) \subseteq \bigcup_{m=1}^{j-1} B_m)$ such that

$$p_{rh} \begin{cases} > 0, \text{ for all } s_h \in S_n^{anc}(r) \\ = 0, \text{ for all } s_h \in \left(\bigcup_{m=1}^{j-1} B_m \right) \setminus S_n^{anc}(r) \end{cases}.$$

Condition 6. For all $j = 2,\cdots,k-1$, if $p_{rn} = 0$ for all states $s_r \in B_j$, then $p_{hn} = 0$ for all states $s_h \in B_{j-1}$.

Remark 1. When the investigated DS S_n is a technical system, and deteriorating in the functionality is carried out due to the appearance of faults in it, it is usually assumed that in one step either one new fault can appear, or one of the existing faults can be eliminated. So, in this case, Conditions 3-5 take the following form:

Condition 3A. For any state $s_r \in B_j$ $(j = 1,\cdots,k-1)$ there exists some subset $S_n^{dsc}(r)$ $(\varnothing \neq S_n^{dsc}(r) \subseteq B_{j+1})$ such that

$$p_{rh}\begin{cases} > 0, \text{ for all } s_h \in S_n^{dsc}(r) \\ = 0, \text{ for all } s_h \in B_{j+1} \setminus S_n^{dsc}(r) \end{cases}.$$

Condition 4A. For all $j = 1,\cdots,k-1$ holds the equality $\bigcup_{s_r \in B_j} S_n^{dsc}(r) = B_{j+1}$.

Condition 5A. For any state $s_r \in B_j$ $(j = 2,\cdots,k-1)$ there exists some subset $S_n^{anc}(r)$ $(S_n^{anc}(r) \subseteq B_{j-1})$ such that

$$p_{rh}\begin{cases} > 0, \text{ for all } s_h \in S_n^{anc}(r) \\ = 0, \text{ for all } s_h \in B_{j-1} \setminus S_n^{anc}(r) \end{cases}.$$

On the base of Assumptions 1-4 and Restrictions 1 and 2, we introduce the following two definitions.

Definition 1. *For an FMC C_n:*

1. The critical set of states in the weak sense S_n^{ws-cr} consists of all states $s_r \in \bigcup_{j=1}^{k-1} B_j$ such that $p_{rn} > 0$.

2. The critical set of states in the strong sense S_n^{ss-cr} consists of all states $s_r \in \bigcup_{j=1}^{k-1} B_j$ such that $S_n^{dsc}(r) = \{s_n\}$

.

Remark 2. Definition 1 directly implies that the relations $\varnothing \neq S_n^{ss-cr} \subseteq S_n^{ws-cr} \subset S_n$ are true:

Definition 2. *An FMC C_n is a model of:*

1. A recoverable DS S_n, if for all $j = 2,\cdots,k-1$ and for any state $s_r \in B_j$ holds the disequality $S_n^{anc}(r) \neq \varnothing$.

2. A non-recoverable DS S_n, if for all $j = 2,\cdots,k-1$ and for any state $s_r \in B_j$ holds the equality $S_n^{anc}(r) = \varnothing$.

Example 1. 1. Let us consider a network consisting of three pairwise connected computers C_1, C_2 and C_3. Deteriorating in the functionality of this network is carried out due to the appearance of faults in the computers, and recovery consists of eliminating these faults. Thus, we are dealing with the DS $S_8^{(1)}$. Due to Remark 1, we get that the symbolic mathematical model for this DS is the following FMC $C_8^{(1)}$

$$P_{C_8^{(1)}} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 & 0 & 0 & 0 \\ p_{21} & p_{22} & 0 & 0 & p_{25} & p_{26} & 0 & 0 \\ p_{31} & 0 & p_{33} & 0 & p_{35} & 0 & p_{37} & 0 \\ p_{41} & 0 & 0 & p_{44} & 0 & p_{46} & p_{47} & 0 \\ 0 & p_{52} & p_{53} & 0 & p_{55} & 0 & 0 & p_{58} \\ 0 & p_{62} & 0 & p_{64} & 0 & p_{66} & 0 & p_{68} \\ 0 & 0 & p_{73} & p_{74} & 0 & 0 & p_{77} & p_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where:

$p_{1i} > 0$ $(i = 1,\cdots,4)$ and $p_{11} + p_{12} + p_{13} + p_{14} = 1$;

$p_{2i} > 0$ $(i = 1,2,5,6)$ and $p_{21} + p_{22} + p_{25} + p_{26} = 1$;

$p_{3i} > 0$ $(i = 1,3,5,7)$ and $p_{31} + p_{33} + p_{35} + p_{37} = 1$;

$p_{4i} > 0 \quad (i = 1,4,6,7)$ and $p_{41} + p_{44} + p_{46} + p_{47} = 1$;

$p_{5i} > 0 \quad (i = 2,3,5,8)$ and $p_{52} + p_{53} + p_{55} + p_{58} = 1$;

$p_{6i} > 0 \quad (i = 2,4,6,8)$ and $p_{62} + p_{64} + p_{66} + p_{68} = 1$;

$p_{7i} > 0 \quad (i = 3,4,7,8)$ and $p_{73} + p_{74} + p_{77} + p_{78} = 1$.

In the FMC $C_8^{(1)}$:

1. The state s_1 represents the analyzed DS $S_8^{(1)}$ in the fully functional stage.
2. The state s_2 represents the analyzed DS $S_8^{(1)}$ in the stage when the computer C_1 is faulty and the computers C_2 and C_3 are fault-free.
3. The state s_3 represents the analyzed DS $S_8^{(1)}$ in the stage when the computer C_2 is faulty and the computers C_1 and C_3 are fault-free.
4. The state s_4 represents the analyzed DS $S_8^{(1)}$ in the stage when the computer C_3 is faulty and the computers C_1 and C_2 are fault-free.
5. The state s_5 represents the analyzed DS $S_8^{(1)}$ in the stage when the computers C_1 and C_2 are faulty and the computer C_3 is fault-free.
6. The state s_6 represents the analyzed DS $S_8^{(1)}$ in the stage when the computers C_1 and C_3 are faulty and the computer C_2 is fault-free.
7. The state s_7 represents the analyzed DS $S_8^{(1)}$ in the stage when the computers C_2 and C_3 are faulty and the computer C_1 is fault-free.
8. The state s_8 represents the analyzed DS $S_8^{(1)}$ in the inoperable stage, i.e. when all three computers C_1, C_2 and C_3 are faulty.

For the FMC $C_8^{(1)}$ we get $\pi = \{B_1, B_2, B_3, B_4\}$, where $B_1 = \{s_1\}$, $B_2 = \{s_2, s_3, s_4\}$, $B_3 = \{s_5, s_6, s_7\}$, and $B_4 = \{s_8\}$.

Due to Definition 1, $S_8^{ss-cr} = S_8^{ws-cr} = B_3 = \{s_5, s_6, s_7\}$.

Moreover, since $S_8^{anc}(r) \neq \varnothing$ for all states $s_r \in B_2 \cup B_3$, then due to Definition 2, FMC $C_8^{(1)}$ is a model of the recoverable DS $S_8^{(1)}$.

2. Let us consider some chronic disease progression containing two stages for the given Patient. The deterioration of the Patient's health is caused by the onset of the disease, staying in the first stage of the disease, the transition to the second stage of the disease, and finally, death. Thus, we are dealing with the DS $S_4^{(1)}$. The symbolic mathematical model for this DS is the following FMC $C_4^{(1)}$

$$P_{C_4^{(1)}} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ 0 & p_{22} & p_{23} & p_{24} \\ 0 & 0 & p_{33} & p_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where:

$p_{1i} > 0 \quad (i = 1,\cdots,4)$ and $p_{11} + p_{12} + p_{13} + p_{14} = 1$;

$p_{2i} > 0 \quad (i = 2,3,4)$ and $p_{22} + p_{23} + p_{24} = 1$;

$p_{3i} > 0 \quad (i = 2,3,4)$ and $p_{33} + p_{34} = 1$.

In the FMC $C_8^{(1)}$:

1. The state s_1 represents the analyzed DS $S_4^{(1)}$, when the Patient is in the healthy stage.
2. The state s_2 represents the analyzed DS $S_4^{(1)}$, when the Patient is staying in the first stage of the disease.
3. The state s_3 represents the analyzed DS $S_4^{(1)}$, when the Patient is staying in the second stage of the disease.
4. The state s_4 represents the analyzed DS $S_4^{(1)}$, when the Patient is dead.

For the FMC $C_4^{(1)}$ we get $\pi = \{B_1, B_2, B_3, B_4\}$, where $B_1 = \{s_1\}$, $B_2 = \{s_2\}$, $B_3 = \{s_3\}$, and $B_4 = \{s_4\}$.

Due to Definition 1, $S_4^{ws-cr} = B_1 \cup B_2 \cup B_3$, and $S_4^{ss-cr} = B_3$.

Moreover, since $S_4^{anc}(r) = \varnothing$ for all states $s_r \in B_2 \cup B_3$, then due to Definition 2, the FMC $C_4^{(1)}$ is a model of the non-recoverable DS $S_4^{(1)}$.

III. Exact analysis of the proposed model

The occurrences of the behaviors associated with the decreasing in the functionality of the analyzed DS S_n can be estimated via computation for the FMC C_n the probability $\mathrm{P}(s_1, S_n^{trgt})$ to reach this or the other target set S_n^{trgt} $(\varnothing \neq S_n^{trgt} \subseteq S_n \setminus \{s_1\})$ of states starting in the state s_1 as follows (see [17], for example).

Let $\Pi_{s_1, S_n^{trgt}}(m)$ $(m = 1, 2, \cdots)$ be the set of all strings $w = s_{i_0} s_{i_1} \cdots s_{i_m} \in S_n^+$ such that $s_{i_0} = s_1$, $s_{i_j} \notin S_n^{trgt}$ $(j = 1, \cdots, m-1)$, and $s_{i_m} \in S_n^{trgt}$, where $p_{i_j i_{j+1}} > 0$ $(j = 0, 1, \cdots, m-1)$.

Due to [10, 14], with any string $w = s_{i_0} s_{i_1} \cdots s_{i_m} \in \Pi_{s_1, S_n^{trgt}}(m)$ $(m = 1, 2, \cdots)$ can be associated the probability

$$\mathrm{P}(w) = \prod_{j=0}^{m-1} p_{i_j i_{j+1}} . \tag{1}$$

Therefore,

$$\mathrm{P}(s_1, S_n^{trgt}) = \sum_{m=1}^{\infty} \sum_{w \in \Pi_{s_1, S_n^{trgt}}(m)} \mathrm{P}(w) . \tag{2}$$

It is well-known that for real DSs the study of their behavior within the finite time horizon is of practical importance. It follows from (2) that the probability $\mathrm{P}(s_1, S_n^{trgt}; \leq l)$ to reach the set S_n^{trgt} for no more than l of state transitions (where l is a given fixed positive integer) can be computed as follows:

$$\mathrm{P}(s_1, S_n^{trgt}; \leq l) = \sum_{m=1}^{l} \sum_{w \in \Pi_{s_1, S_n^{trgt}}(m)} \mathrm{P}(w) . \tag{3}$$

Example 2. 1. Let us consider the DS $S_8^{(1)}$ (see Example 1.1). Setting $S_8^{trgt} = S_8^{ws-cr} = B_3 = \{s_5, s_6, s_7\}$, $l = 3$, and applying (1), we get the values shown in Table 1.1 (it should be noted that $\Pi_{s_1, S_8^{trgt}}(1) = \varnothing$).

Table 1.1

	w	$\mathrm{P}(w)$		w	$\mathrm{P}(w)$
$\Pi_{s_1, S_8^{trgt}}(2)$	$s_1 s_2 s_5$	$p_{12} p_{25}$	$\Pi_{s_1, S_8^{trgt}}(3)$	$s_1 s_2^2 s_6$	$p_{12} p_{22} p_{26}$
	$s_1 s_2 s_6$	$p_{12} p_{26}$		$s_1^2 s_3 s_5$	$p_{11} p_{13} p_{35}$
	$s_1 s_3 s_5$	$p_{13} p_{35}$		$s_1^2 s_3 s_7$	$p_{11} p_{13} p_{37}$
	$s_1 s_3 s_7$	$p_{13} p_{37}$		$s_1 s_3^2 s_5$	$p_{13} p_{33} p_{35}$
	$s_1 s_4 s_6$	$p_{14} p_{46}$		$s_1 s_3^2 s_7$	$p_{13} p_{33} p_{37}$
	$s_1 s_4 s_7$	$p_{14} p_{47}$		$s_1^2 s_4 s_6$	$p_{11} p_{14} p_{46}$
$\Pi_{s_1, S_8^{trgt}}(3)$	$s_1^2 s_2 s_5$	$p_{11} p_{12} p_{25}$		$s_1^2 s_4 s_7$	$p_{11} p_{14} p_{47}$
	$s_1^2 s_2 s_6$	$p_{11} p_{12} p_{26}$		$s_1 s_4^2 s_6$	$p_{14} p_{44} p_{46}$
	$s_1 s_2^2 s_5$	$p_{12} p_{22} p_{25}$		$s_1 s_4^2 s_7$	$p_{14} p_{44} p_{47}$

From Table 1.1 we get

$$\mathrm{P}(s_1, S_8^{trgt}; \leq 3) = (1 + p_{11} + p_{22}) p_{12} (p_{25} + p_{26}) + (1 + p_{11} + p_{33}) p_{13} (p_{35} + p_{37}) + (1 + p_{11} + p_{44}) p_{14} (p_{46} + p_{47}) .$$

2. Let us consider the DS $S_4^{(1)}$ (see Example 1.2). Setting $S_4^{trgt} = \{s_4\}$, and $l = 3$, and applying (1), we get the values shown in Table 2.1.

Table 2.1

	w	$\mathrm{P}(w)$		w	$\mathrm{P}(w)$
$\Pi_{s_1,S_4^{trgt}}(1)$	$s_1 s_4$	p_{14}	$\Pi_{s_1,S_4^{trgt}}(3)$	$s_1^2 s_2 s_4$	$p_{11} p_{12} p_{24}$
$\Pi_{s_1,S_4^{trgt}}(2)$	$s_1^2 s_4$	$p_{11} p_{14}$		$s_1^2 s_3 s_4$	$p_{11} p_{13} p_{34}$
	$s_1 s_2 s_4$	$p_{12} p_{24}$		$s_1 s_2^2 s_4$	$p_{12} p_{22} p_{24}$
	$s_1 s_3 s_4$	$p_{13} p_{34}$		$s_1 s_2 s_3 s_4$	$p_{12} p_{23} p_{34}$
$\Pi_{s_1,S_4^{trgt}}(3)$	$s_1^3 s_4$	$p_{11}^2 p_{14}$		$s_1 s_3^2 s_4$	$p_{13} p_{33} p_{34}$

From Table 2.1 we get

$$\mathrm{P}(s_1, S_4^{trgt}; \leq 3) = (1 + p_{11}(1 + p_{11}))p_{14} + (1 + p_{11} + p_{22})p_{12}p_{24} + (p_{13}(1 + p_{33} + p_{11}) + p_{12}p_{23})p_{34} .$$

IV. Bounded analysis of the proposed model

For any real DS S_n the cardinality of the sets $\Pi_{s_1,S_n^{trgt}}(m)$ $(m = 1, 2, \cdots, l)$ grows rather quickly with an increase in the integer m. By this reason, instead of constructing the entire set $\bigcup_{m=1}^{l} \Pi_{s_1,S_n^{trgt}}(m)$ in an explicit form, bounded reachability properties [14-17] can be analyzed as follows.

Let λ $(0 < \lambda < 1)$ be a given number, and $P(s_1, S_n^{trgt}; \leq l, \leq \lambda)$ be the property that for the analyzed DS S_n the probability to reach the set S_n^{trgt} starting in the state s_1 by at most l steps is not greater then λ.

It follows from (3) that the analyzed DS S_n satisfies to the property $P(s_1, S_n^{trgt}; \leq l, \leq \lambda)$ if and only if the inequality $\sum_{m=1}^{l} \sum_{w \in \Pi_{s_1,S_n^{trgt}}(m)} \mathrm{P}(w) \leq \lambda$ holds. Therefore, the property $P(s_1, S_n^{trgt}; \leq l, \leq \lambda)$ fails for the analyzed DS S_n if and only if for some subset $\mathrm{S} \subseteq \bigcup_{m=1}^{l} \Pi_{s_1,S_n^{trgt}}(m)$ the inequality $\mathbf{P}(\mathrm{S}) = \sum_{w \in \mathrm{S}} \mathbf{P}(w) > \lambda$ holds. This subset S is called a counterexample.

It is evident that an attempt to construct a counterexample can reduce computations for checking the property $P(s_1, S_n^{trgt}; \leq l, \leq \lambda)$. For this purpose, we can use the following Algorithm 1 proposed in [17].

Algorithm 1 (Checking the property $P(s_1, S_n^{trgt}; \leq l, \leq \lambda)$).

Step 1. $m := 1$, $\mathrm{S} := \varnothing$, $\mathbf{P}(\mathrm{S}) := 0$.

Step 2. If $\Pi_{s_1,S_n^{trgt}}(m) \neq \varnothing$, then go to Step 3, else go to Step 6.

Step 3. $w :=$ The first element of the set $\Pi_{s_1,S_n^{trgt}}(m)$, $\mathrm{S} := \mathrm{S} \cup \{w\}$, $\Pi_{s_1,S_n^{trgt}}(m) := \Pi_{s_1,S_n^{trgt}}(m) \setminus \{w\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(w)$.

Step 4. If $\mathbf{P}(\mathrm{S}) > \lambda$, then go to Step 5, else go to Step 2.

Step 5. Print: "For the DS S_n the property $P(s_1, S_n^{trgt}; \leq l, \leq \lambda)$ is false", print the counterexample S in the explicit form, and HALT.

Step 6. $m := m + 1$.

Step 7. If $m \leq l$, then go to Step 2, else go to Step 8.

Step 8. Print: "For the DS S_n the property $P(s_1, S_n^{trgt}; \leq l, \leq \lambda)$ is true", and HALT.

Remark 3. In [15-17], when checking the property $P(s_1, S_n^{trgt}; \leq l, \leq \lambda)$ it has been assumed that the elements of any non-empty set of strings $\Pi_{s_1,S_n^{trgt}}(m)$ $(m = 1, 2, \cdots, l)$ are enumerated according to non-increase of their probabilities. It should be noted that proof of Theorem 2 in [17] on the correctness and the soundness of Algorithm 1 does not use this assumption. Since providing such enumeration of elements of the sets $\Pi_{s_1,S_n^{trgt}}(m)$ $(m = 1, 2, \cdots, l)$ is a hard problem, we discard this assumption. This does not affect either the correctness or the soundness of the Algorithm 1, but can only lead to the fact that the constructed counterexample is not minimal in cardinality. The advantage of the discard of this assumption is that there is no need to construct the entire set $\Pi_{s_1, S_n^{trgt}}(m)$ $(m = 1, 2, \cdots, l)$ explicitly in advance. Instead, it is sufficient to generate its elements as needed (this can be done by presentation state transitions of the FMC C_n via rooted ranked tree with the root labeled by the state s_1).

Example 3. 1. Let us consider the DS $S_8^{(1)}$ (see Example 1.1) for the following numerical values of the transition probabilities between the states

$$P_{C_8^{(1)}} = \begin{bmatrix} 0.85 & 0.06 & 0.04 & 0.05 & 0 & 0 & 0 & 0 \\ 0.30 & 0.50 & 0 & 0 & 0.12 & 0.08 & 0 & 0 \\ 0.30 & 0 & 0.60 & 0 & 0.05 & 0 & 0.05 & 0 \\ 0.40 & 0 & 0 & 0.40 & 0 & 0.10 & 0.10 & 0 \\ 0 & 0.30 & 0.10 & 0 & 0.40 & 0 & 0 & 0.20 \\ 0 & 0.20 & 0 & 0.20 & 0 & 0.50 & 0 & 0.10 \\ 0 & 0 & 0.20 & 0.20 & 0 & 0 & 0.40 & 0.20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Let $S_8^{trgt} = S_8^{ws-cr} = B_3 = \{s_5, s_6, s_7\}$, and $l = 3$.

Substituting these numerical values into Table 1.1 (see Example 2.1), we get the following Table 1.2.

Table 1.2

	w	$\mathrm{P}(w)$		w	$\mathrm{P}(w)$
$\Pi_{s_1, S_8^{trgt}}(2)$	$s_1 s_2 s_5$	0.00720	$\Pi_{s_1, S_8^{trgt}}(3)$	$s_1 s_2^2 s_6$	0.00240
	$s_1 s_2 s_6$	0.00480		$s_1^2 s_3 s_5$	0.00170
	$s_1 s_3 s_5$	0.00200		$s_1^2 s_3 s_7$	0.00170
	$s_1 s_3 s_7$	0.00200		$s_1 s_3^2 s_5$	0.00120
	$s_1 s_4 s_6$	0.00500		$s_1 s_3^2 s_7$	0.00010
	$s_1 s_4 s_7$	0.00500		$s_1^2 s_4 s_6$	0.00425
$\Pi_{s_1, S_8^{trgt}}(3)$	$s_1^2 s_2 s_5$	0.00612		$s_1^2 s_4 s_7$	0.00425
	$s_1^2 s_2 s_6$	0.00408		$s_1 s_4^2 s_6$	0.00240
	$s_1 s_2^2 s_5$	0.00360		$s_1 s_4^2 s_7$	0.00200

Applying Algorithm 1 to check the property $P(s_1, S_8^{trgt}; \le 3, \le 0.04)$, we get:

Step 1. $m := 1$, $\mathrm{S} := \varnothing$, $\mathbf{P}(\mathrm{S}) := 0$.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(1) = \varnothing$, we go to Step 6.

Step 6. $m := 2$.

Step 7. Since $2 \le 3$, we go to Step 2.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_2 s_5$ $\mathrm{S} := \{s_1 s_2 s_5\}$, $\Pi_{s_1, S_8^{trgt}}(2) := \Pi_{s_1, S_8^{trgt}}(2) \setminus \{s_1 s_2 s_5\}$, $\mathbf{P}(\mathrm{S}) := 0 + \mathbf{P}(s_1 s_2 s_5) = 0.00720$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.00720 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_2 s_6$ $\mathrm{S} := \mathrm{S} \cup \{s_1 s_2 s_6\}$, $\Pi_{s_1, S_8^{trgt}}(2) := \Pi_{s_1, S_8^{trgt}}(2) \setminus \{s_1 s_2 s_6\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_2 s_6) = 0.01200$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.01200 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_3 s_5$, $\mathrm{S} := \mathrm{S} \cup \{s_1 s_3 s_5\}$, $\Pi_{s_1, S_8^{trgt}}(2) := \Pi_{s_1, S_8^{trgt}}(2) \setminus \{s_1 s_3 s_5\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_3 s_5) = 0.01400$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.01400 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, s_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_3 s_7$, $\mathrm{S} := \mathrm{S} \cup \{s_1 s_3 s_7\}$, $\Pi_{s_1, s_8^{trgt}}(2) := \Pi_{s_1, s_8^{trgt}}(2) \setminus \{s_1 s_3 s_7\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_3 s_7) = 0.01600$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.01600 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, s_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_4 s_6$, $\mathrm{S} := \mathrm{S} \cup \{s_1 s_4 s_6\}$, $\Pi_{s_1, s_8^{trgt}}(2) := \Pi_{s_1, s_8^{trgt}}(2) \setminus \{s_1 s_4 s_6\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_4 s_6) = 0.02100$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.02100 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, s_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_4 s_7$, $\mathrm{S} := \mathrm{S} \cup \{s_1 s_4 s_7\}$, $\Pi_{s_1, s_8^{trgt}}(2) := \Pi_{s_1, s_8^{trgt}}(2) \setminus \{s_1 s_4 s_7\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_4 s_7) = 0.02600$

.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.02100 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, s_8^{trgt}}(2) = \varnothing$, we go to Step 6.

Step 6. $m := 3$.

Step 7. Since $3 \le 3$, we go to Step 2.

Step 2. Since $\Pi_{s_1, s_8^{trgt}}(3) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1^2 s_2 s_5$, $\mathrm{S} := \mathrm{S} \cup \{s_1^2 s_2 s_5\}$, $\Pi_{s_1, s_8^{trgt}}(2) := \Pi_{s_1, s_8^{trgt}}(2) \setminus \{s_1^2 s_2 s_5\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1^2 s_2 s_5) = 0.03212$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.03212 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, s_8^{trgt}}(3) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1^2 s_2 s_6$, $\mathrm{S} := \mathrm{S} \cup \{s_1^2 s_2 s_6\}$, $\Pi_{s_1, s_8^{trgt}}(2) := \Pi_{s_1, s_8^{trgt}}(2) \setminus \{s_1^2 s_2 s_6\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1^2 s_2 s_6) = 0.03620$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.03620 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, s_8^{trgt}}(3) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_2^2 s_5$, $\mathrm{S} := \mathrm{S} \cup \{s_1 s_2^2 s_5\}$, $\Pi_{s_1, s_8^{trgt}}(2) := \Pi_{s_1, s_8^{trgt}}(2) \setminus \{s_1 s_2^2 s_5\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_2^2 s_5) = 0.03980$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.03980 \le 0.04$, we go to Step 2.

Step 2. Since $\Pi_{s_1, s_8^{trgt}}(3) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_2^2 s_6$, $\mathrm{S} := \mathrm{S} \cup \{s_1 s_2^2 s_6\}$, $\Pi_{s_1, s_8^{trgt}}(2) := \Pi_{s_1, s_8^{trgt}}(2) \setminus \{s_1 s_2^2 s_6\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_2^2 s_6) = 0.04220$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.04220 > 0.04$, we go to Step 5.

Step 5. Print:

"For the DS $S_8^{(1)}$ the property $P(s_1, S_8^{trgt}; \le 3, \le 0.4)$ is false",

Print:

$$\mathrm{S} = \{s_1 s_2 s_5, s_1 s_2 s_6, s_1 s_3 s_5, s_1 s_2 s_7, s_1 s_4 s_6, s_1 s_4 s_7, s_1^2 s_2 s_5, s_1^2 s_2 s_6, s_1 s_2^2 s_5, s_1 s_2^2 s_6\},$$

and HALT.

It should be noted that in the process of applying Algorithm 1 to check the property $P(s_1, S_8^{trgt}; \le 3, \le 0.4)$, the last eight rows of Table 1.2 were not used at all. This situation is typical when a counterexample exists.

2. Let us consider the DS $S_4^{(1)}$ (see Example 1.2) for the following numerical values of the transition probabilities between the states

$$P_{C_4^{(1)}} = \begin{bmatrix} 0.70 & 0.20 & 0.08 & 0.02 \\ 0 & 0.65 & 0.25 & 0.10 \\ 0 & 0 & 0.60 & 0.40 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let $S_4^{trgt} = \{s_4\}$, and $l = 3$.

Substituting these numerical values into Table 2.1 (see Example 2.1), we get the following Table 2.2.

Table 2.2

	w	$P(w)$		w	$P(w)$
$\Pi_{s_1,S_4^{trgt}}(1)$	$s_1 s_4$	0.0200	$\Pi_{s_1,S_4^{trgt}}(3)$	$s_1^2 s_2 s_4$	0.0140
$\Pi_{s_1,S_4^{trgt}}(2)$	$s_1^2 s_4$	0.0140		$s_1^2 s_3 s_4$	0.0224
	$s_1 s_2 s_4$	0.0200		$s_1 s_2^2 s_4$	0.0130
	$s_1 s_3 s_4$	0.0320		$s_1 s_2 s_3 s_4$	0.0200
$\Pi_{s_1,S_4^{trgt}}(3)$	$s_1^3 s_4$	0.0098		$s_1 s_3^2 s_4$	0.0192

Applying Algorithm 1 to check the property $P(s_1, S_4^{trgt}; \le 3, \le 0.2)$, we get:

Step 1. $m := 1$, $\mathrm{S} := \varnothing$, $\mathbf{P}(\mathrm{S}) := 0$.

Step 2. Since $\Pi_{s_1,S_n^{trgt}}(1) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_4$ $\mathrm{S} := \{s_1 s_4\}$, $\Pi_{s_1,S_8^{trgt}}(2) := \Pi_{s_1,S_8^{trgt}}(2) \setminus \{s_1 s_4\}$, $\mathbf{P}(\mathrm{S}) := 0 + \mathbf{P}(s_1 s_4) = 0.0200$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.0200 \le 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1,S_8^{trgt}}(1) = \varnothing$, we go to Step 6.

Step 6. $m := 2$.

Step 7. Since $2 \le 3$, we go to Step 2.

Step 2. Since $\Pi_{s_1,S_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1^2 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1^2 s_4\}$, $\Pi_{s_1,S_8^{trgt}}(2) := \Pi_{s_1,S_8^{trgt}}(2) \setminus \{s_1^2 s_4\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1^2 s_4) = 0.0340$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.0340 \le 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1,S_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_2 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1 s_2 s_4\}$, $\Pi_{s_1,S_8^{trgt}}(2) := \Pi_{s_1,S_8^{trgt}}(2) \setminus \{s_1 s_2 s_4\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_2 s_4) = 0.0540$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.0540 \le 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1,S_8^{trgt}}(2) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_3 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1 s_3 s_4\}$, $\Pi_{s_1,S_8^{trgt}}(2) := \Pi_{s_1,S_8^{trgt}}(2) \setminus \{s_1 s_3 s_4\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_3 s_4) = 0.0860$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.0860 \le 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1,S_8^{trgt}}(2) = \varnothing$, we go to Step 6.

Step 6. $m := 3$.

Step 7. Since $3 \le 3$, we go to Step 2.

Step 2. Since $\Pi_{s_1,S_8^{trgt}}(3) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1^3 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1^3 s_4\}$, $\Pi_{s_1,S_8^{trgt}}(2) := \Pi_{s_1,S_8^{trgt}}(2) \setminus \{s_1^3 s_4\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1^3 s_4) = 0.0958$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.0958 \le 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1,S_8^{trgt}}(3) \ne \varnothing$, we go to Step 3.

Step 3. $w := s_1^2 s_2 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1^2 s_2 s_4\}$, $\Pi_{s_1,S_8^{trgt}}(2) := \Pi_{s_1,S_8^{trgt}}(2) \setminus \{s_1^2 s_2 s_4\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1^2 s_2 s_4) = 0.1098$

.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.1098 \le 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(3) \neq \varnothing$, we go to Step 3.

Step 3. $w := s_1^2 s_3 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1^2 s_3 s_4\}$, $\Pi_{s_1, S_8^{trgt}}(2) := \Pi_{s_1, S_8^{trgt}}(2) \setminus \{s_1^2 s_3 s_4\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1^2 s_3 s_4) = 0.1322$

.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.1322 \leq 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(3) \neq \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_2^2 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1 s_2^2 s_4\}$, $\Pi_{s_1, S_8^{trgt}}(2) := \Pi_{s_1, S_8^{trgt}}(2) \setminus \{s_1 s_2^2 s_4\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_2^2 s_4) = 0.1452$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.1452 \leq 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(3) \neq \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_2 s_3 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1 s_2 s_3 s_4\}$, $\Pi_{s_1, S_8^{trgt}}(3) := \Pi_{s_1, S_8^{trgt}}(3) \setminus \{s_1 s_2 s_3 s_4\}$,

$\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_2 s_3 s_4) = 0.1652$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.1652 \leq 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(3) \neq \varnothing$, we go to Step 3.

Step 3. $w := s_1 s_3^2 s_4$ $\mathrm{S} := \mathrm{S} \cup \{s_1 s_3^2 s_4\}$, $\Pi_{s_1, S_8^{trgt}}(3) := \Pi_{s_1, S_8^{trgt}}(3) \setminus \{s_1 s_3^2 s_4\}$, $\mathbf{P}(\mathrm{S}) := \mathbf{P}(\mathrm{S}) + \mathbf{P}(s_1 s_3^2 s_4) = 0.1844$.

Step 4. Since $\mathbf{P}(\mathrm{S}) = 0.1844 \leq 0.2$, we go to Step 2.

Step 2. Since $\Pi_{s_1, S_8^{trgt}}(3) = \varnothing$, we go to Step 6.

Step 6. $m := 4$.

Step 7. Since $4 \not\leq 3$, we go to Step 8.

Step 8. Print:

"For the DS $S_4^{(1)}$ the property $P(s_1, S_4^{trgt}; \leq 3, \leq 0.2)$ is true",

and HALT.

It should be noted that in the process of applying Algorithm 1 to check the property $P(s_1, S_4^{trgt}; 3, 0.2)$ all rows of Table 2.2 were used. This situation always occurs when a counterexample does not exist.

V. Discussion

The main aim of the given paper was to develop an analytical model based on FMC and intended for analysis of recoverable, partially recoverable, and non-recoverable DSs within the finite time horizon, for which the measurements of the system parameters are carried out after a fixed period of time. The use of FMC assumes that the state transition probabilities are constant. Hence it follows that the finite time horizon must be divided into disjoint intervals, at each of which this assumption can be accepted. As a result, the analyzed DS will be represented by a sequence of FMCs, each of which corresponds to a certain interval. The methods of exact and bounded analysis of the investigated DS on an interval via FMC are considered in this paper.

Definition 2 identifies two types of DS, namely recoverable and non-recoverable ones. These two types of DSs correspond to the extreme cases between which there is a whole spectrum of DSs. This spectrum of DSs can be defined as follows.

Definition 3. *An FMC* C_n *is a model of a* m *-recoverable* $(2 \leq m \leq k-1)$ *DS* S_n *, if for all* $j = m, \cdots, k-1$ *and for any state* $s_r \in B_j$ *holds the disequality* $S_n^{anc}(r) \neq \varnothing$.

Definition 3 implies that a 2 -recoverable DS is a recoverable DS in the sense of Definition 2. Obviously, the methods of exact and bounded analysis considered in this paper can be applied to any m -recoverable $(2 \leq m \leq k-1)$ DS S_n .

It is noted in Remark 3 that generation of elements of the set $\Pi_{s_1, S_n^{trgt}}(m)$ $(m = 1, 2, \cdots, l)$ can be done by presentation state transitions of the FMC C_n via rooted ranked tree with the root labeled by the state s_1 . Obviously, the following approach can be applied to compute the probability $\mathrm{P}(s_1, S_n^{trgt}; \leq l)$.

Let $\mathrm{P}(s_1, S_n^{trgt}; = m)$ $(m = 1, \cdots, l)$ be the probability to reach the set S_n^{trgt} exactly for m state transitions. Then

$$\mathrm{P}(s_1, S_n^{trgt}; \leq l) = \sum_{m=1}^{l} \mathrm{P}(s_1, S_n^{trgt}; = m) . \tag{4}$$

Setting $\mathbf{u}_m = (u_1^{(m)}, \cdots, u_n^{(m)}) = \mathbf{v}_0 P_{c_n}^m$, where $\mathbf{v}_0 = (1, \underbrace{0, \cdots, 0}_{n-1\ times})$, we get

$$\mathrm{P}(s_1, S_n^{trgt}; = m) = \sum_{i \in \{j | s_j \in S_n^{trgt}\}} u_i^{(m)} . \quad (5)$$

Applying (4) and (5), we can compute any set $\Pi_{s_1, S_n^{trgt}}(m) \quad (m = 2, \cdots, l)$ by using recurrence relation $\mathbf{u}_m = \mathbf{u}_{m-1} P_{c_n}$.

V. Conclusion

In the given paper an analytical model based on FMC that presents the transitions between the functionality stages of the analyzed DS is proposed. The advantage of this model is in the possibility to carry out symbolic simulation of the transitions between the functionality stages of the analyzed DS by using these or the other suitable software tools. Besides, the critical sets of states in the weak as well as in the strong sense can be defined. The problem of exact and bounded analysis for the target set of states reachability within a given finite time horizon is solved. The essential characteristic of the proposed model is that it can be used for the analysis of the effect of the variations of transition probabilities on the probability of the target set of states' reachability. Possible area of future research is software implementation of Algorithm 1. Another area is the development of methods for choosing the probabilities of the FMC states transitions to minimize the probability of the target set of states' reachability.

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