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Vector Identities

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Abstract- *Vector identities are produced in non-customary way. The results are shown in different forms such as to stick on tips. The method adopted exhibits the new operators appearing with commutation brackets. Nabla with order of dot, cross and their combination appears to have new type of operators.*
Keywords- *vector identities, operator in commutation bracket, combination of dot, cross with nabla.*

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I. Manuscript

The radius vector is increasing constantly with unit spatial speed (space gradient), hence we should expect its gradient as the unit radius vector. The radius vector is straightforward, it never bends or rotates, and hence we must hope for its vanishing curl. However, the separation between nearby radii vectors continuously increasing, giving indication of non-zero divergence.

(1) $\nabla r = \hat{r}$ $\qquad$ $\nabla r = \hat{r}\frac{\partial}{\partial r}(r) = \hat{r}$

Thus, we are having two types of vectors, direct ones like radius vectors, and others obtained from gradients. Direct ones are termed as contra-valiant vectors and others as co-valiant vectors.
The gradient is an intrinsic property of a scalar field quite independent of any coordinate system.

(2) $\nabla \frac{1}{r} = \hat{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\right) = -\frac{\hat{r}}{r^2}$

(3) $\nabla \cdot \vec{r} = \sum_x \frac{\partial}{\partial x} x = 1 + 1 + 1 = 3$

A scalar is an invariant; hence, divergence remains the same in all coordinate systems.

(4) $\nabla \times \vec{r} = \nabla \times (r\hat{r}) = \nabla r \times \hat{r} + r\nabla \times \hat{r} = \hat{r} \times \hat{r} + r\nabla \times (\nabla r) = r(\nabla \times \nabla)r = 0$

$$\nabla \times \nabla = 0$$

(5) $d\vec{r} \cdot \nabla = d$

$$d\Phi = \frac{\partial \Phi}{\partial x}dx + \frac{\partial \Phi}{\partial y}dy + \frac{\partial \Phi}{\partial z}dz$$
$$= dx\frac{\partial \Phi}{\partial x} + dy\frac{\partial \Phi}{\partial y} + dz\frac{\partial \Phi}{\partial z} = d\vec{r} \cdot \nabla\Phi$$
$$\Rightarrow [d]\Phi = [d\vec{r} \cdot \nabla]\Phi$$

(6) $\text{divgrad } \Phi = \nabla^2\Phi$ $\qquad$ $dg = \nabla^2$

$$\text{divgrad } \Phi = \sum_x \hat{x}\frac{\partial}{\partial x} \cdot \sum_x \hat{x}\frac{\partial \Phi}{\partial x} = \sum_x \frac{\partial^2 \Phi}{\partial x^2} = \nabla^2\Phi$$
$$\nabla \cdot \nabla = \nabla^2$$

(7) $\operatorname{cur\,lg\,rad}\Phi = 0$ $\quad cg = 0$

$$\mathbf{C}_{\text{url}}\ \ \mathbf{G}_{\text{radient}}\ \ \mathbf{S}_{\text{calar}} = \mathbf{0}$$

(8) divergence curl $\vec{V} = 0$ $\quad dc = 0$

$$\text{div curl}\,\vec{V} = \sum\nolimits_x \hat{x}\frac{\partial}{\partial x}\cdot\sum\nolimits_x \hat{x}\left[\frac{\partial V_z}{\partial y}-\frac{\partial V_y}{\partial z}\right] = \sum\nolimits_x \frac{\partial}{\partial x}\left[\frac{\partial V_z}{\partial y}-\frac{\partial V_y}{\partial z}\right] = 0$$

The above can also be obtained in a simple manner if we treat nabla or del as a vector. In that case we find the box product of three vectors.

$$\nabla\cdot\nabla\times\vec{V} = \{\nabla\times\nabla\}\cdot\vec{V} = \left[\nabla\ \ \nabla\ \ \vec{V}\right] = 0$$

Since the box product vanishes for any two identical elements. Here we reach at two conclusions: (l) the cross product of nabla with itself identically vanishes, and (2) the nabla can be incorporated in vector products.

$$\mathbf{D}_{\text{ivergence}}\ \ \mathbf{C}_{\text{url}}\ \ \mathbf{V}_{\text{ector}} = \mathbf{0}$$

(9) $cc = [g,d]\ \neq 0$

Hence the vector operators possess the property of violation of commutability. Therefore non-commutative quantities can be represented by such operators. This fact is well utilized by quantum physicists.

$$\nabla\times(\nabla\times\vec{V}) = \nabla\{\nabla\cdot\vec{V}\} - \{\nabla\cdot\nabla\}\vec{V} = (gd-dg)V$$

$$cc = gd - dg = [\mathrm{g}\ \ \mathrm{d}]_-\ \neq 0$$

(10) <u>Divergence of cross product of two vectors</u>

$$D(u\otimes v) = (Du)\otimes v + u\otimes(Dv) = (D_u u)\otimes v + u\otimes(D_v v)$$
$$= [(D_u + D_v)u]\otimes v + u\otimes[(D_u+D_v)v] = [(D_u+D_v)](u\otimes v)$$

The above identity is valid only if we put the condition:

$$D_u v\ = 0 =\ D_v u$$

$$\nabla\cdot(\vec{a}\times\vec{b}) = (\nabla_a+\nabla_b)\cdot(\vec{a}\times\vec{b}) = \nabla_a\cdot(\vec{a}\times\vec{b}) + \nabla_b\cdot(\vec{a}\times\vec{b})$$
$$= (\nabla_a\times\vec{a})\cdot\vec{b} - \nabla_b\cdot(\vec{b}\times\vec{a}) = (\nabla_a\times\vec{a})\cdot\vec{b} - (\nabla_b\times\vec{b})\cdot\vec{a}$$
$$= \vec{b}\cdot(\nabla_a\times\vec{a}) - \vec{a}\cdot(\nabla_b\times\vec{b})$$
$$= \vec{b}\cdot[(\nabla_a+\nabla_b)\times\vec{a}] - \vec{a}\cdot[(\nabla_a+\nabla_b)\times\vec{b}]$$
$$= \vec{b}\cdot(\nabla\times\vec{a}) - \vec{a}\cdot(\nabla\times\vec{b})$$
$$= -\begin{vmatrix}\vec{a}\cdot & \vec{b}\cdot\\ \nabla\times\vec{a} & \nabla\times\vec{b}\end{vmatrix}$$

$$\vec{O} =\ \cdot\nabla\times$$
$$\nabla\cdot(\vec{a}\times\vec{b}) = -\vec{O}[\vec{a}\ \ \vec{b}]_- =\ -[\vec{a}\ \ \vec{O}\ \ \vec{b}]_-$$
$$\nabla\cdot(\vec{a}\times\vec{b}) =\ -[\vec{a}\ \ \ \cdot\nabla\times\ \ \vec{b}]_-$$

(11) $\nabla\cdot[(\nabla\Phi_1)\times(\nabla\Phi_2)]$
$= \nabla\Phi_2\cdot(\nabla\times\nabla\Phi_1)-\nabla\Phi_1\cdot(\nabla\times\nabla\Phi_2)=0;\ \nabla\times\nabla=0$

(12)

$$
\begin{aligned}
\nabla\times(\vec{a}\times\vec{b}) &= (\nabla_a+\nabla_b)\times(\vec{a}\times\vec{b})=\nabla_a\times(\vec{a}\times\vec{b})+\nabla_b\times(\vec{a}\times\vec{b})\\
&=(\vec{b}\cdot\nabla_a)\vec{a}-(\nabla_a\cdot\vec{a})\vec{b}-(\vec{a}\cdot\nabla_b)\vec{b}+(\nabla_b\cdot\vec{b})\vec{a}\\
&=(\vec{b}\cdot\nabla)\vec{a}-(\nabla\cdot\vec{a})\vec{b}-(\vec{a}\cdot\nabla)\vec{b}+(\nabla\cdot\vec{b})\vec{a}\\
&=(\vec{b}\cdot\nabla)\vec{a}-(\vec{a}\cdot\nabla)\vec{b}+\vec{a}(\nabla\cdot\vec{b})-\vec{b}(\nabla\cdot\vec{a})\\
&=-\begin{vmatrix}\vec{a} & \vec{b}\\ \nabla\cdot\vec{a} & \nabla\cdot\vec{b}\end{vmatrix}-\begin{vmatrix}\vec{a}\cdot\nabla & \vec{b}\cdot\nabla\\ \vec{a} & \vec{b}\end{vmatrix}
\end{aligned}
$$

$$\nabla\times(\vec{a}\times\vec{b})=[\vec{a}\quad\nabla\cdot\quad\vec{b}]_- - [\vec{a}\quad\cdot\nabla\quad\vec{b}]_-$$

(13) $\nabla(\vec{a}\cdot\vec{b})=?$

$\because\ \vec{a}\times(\nabla_b\times\vec{b})=\nabla_b(\vec{a}\cdot\vec{b})-(\vec{a}\cdot\nabla_b)\vec{b}\Rightarrow\vec{a}\times(\nabla\times\vec{b})=\nabla_b(\vec{a}\cdot\vec{b})-(\vec{a}\cdot\nabla)\vec{b}$

$\because\ \vec{b}\times(\nabla_a\times\vec{a})=\nabla_a(\vec{a}\cdot\vec{b})-(\vec{b}\cdot\nabla_a)\vec{a}\Rightarrow\vec{b}\times(\nabla\times\vec{a})=\nabla_a(\vec{a}\cdot\vec{b})-(\vec{b}\cdot\nabla)\vec{a}$

$\therefore\ \vec{a}\times(\nabla\times\vec{b})+\vec{b}\times(\nabla\times\vec{a})=(\nabla_a+\nabla_b)(\vec{a}\cdot\vec{b})-(\vec{a}\cdot\nabla)\vec{b}-(\vec{b}\cdot\nabla)\vec{a}$

$\Rightarrow\nabla(\vec{a}\cdot\vec{b})=\vec{a}\times(\nabla\times\vec{b})+\vec{b}\times(\nabla\times\vec{a})+(\vec{a}\cdot\nabla)\vec{b}+(\vec{b}\cdot\nabla)\vec{a}$

$\vec{Q}=\ \times\nabla\times$; $\overrightarrow{Q'}=\ \cdot\nabla$; $\nabla(\vec{a}\cdot\vec{b})=[\vec{a}\quad\vec{Q}\quad\vec{b}]_+ + [\vec{a}\quad\overrightarrow{Q'}\quad\vec{b}]_+$

$$\nabla(\vec{a}\cdot\vec{b})=[\vec{a}\quad\times\nabla\times\quad\vec{b}]_+ + [\vec{a}\quad\cdot\nabla\quad\vec{b}]_+$$

II. Conclusion

The commutator bracket is modified in the following way.

$[A\ \ ,\ \ B]_\pm = AB\pm BA$

$[A\ \ O\ \ B]_+ = AOB\pm BOA$

The new class of operators $\vec{O}=\ \cdot\nabla\times$ $\quad\vec{Q}=\ \times\nabla\times$ $\quad\overrightarrow{Q'}=\ \cdot\nabla$

are quite useful in expressing vector identities in concise form.

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