General Proof of Goldbach’s Conjecture

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Abstract: The general proof of Goldbach’s conjecture in number theory is drawn in this paper by applying a specific bounding condition from Bertrand’s postulate or Chebyshev’s theorem and general concept of number theory.

Keywords: Bertrand’s postulate & Chebyshev’s theorem, Goldbach’s conjecture, prime number, numbers series, number theory.

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I. Introduction

It is known that the original form of Goldbach’s conjecture in number theory is: Every even integer greater than 2 can be expressed as the sum of two primes and a specific form of Goldbach’s conjecture in number theory is: Every even integer greater than 4 can be expressed as the sum of two odd primes. These even numbers (>4) are called Goldbach’s numbers. If n be an integer, where n>2; then 2n is an even integer, where 2n>4. So the mathematical formulation of above conjecture is 2n=p1+p2; where p1 & p2 are two odd prime numbers and 2n>4. Now two lowest odd prime numbers are 3 & 5. So if p1>p2, then the lowest value of p1+p2=8 and if p1>p2, then p1=5 & p2=3. Hence 2n=8. Here 2n is an even integer, where 2n>6 as well as n is an integer, where n≥3. Thus Goldbach’s conjecture can be written as a new form with certain consideration that: Every even integer greater than 6 can be expressed as the sum of two odd primes, when the primes are not equal to each other. However every even integer (2n) is twice of an integer (n) as well as every even integer (2n) is the sum of two integers located at equal distance along with both sides from an integer which is its half (n) in the numbers series. Again according to Goldbach’s conjecture p1+p2=2n; where n≥3 and when p1>p2. Therefore p1 & p2 are two integers located at equal distance along with both sides from the integer (n) which is the half of even integer (2n). If n>2, then p1-n=n-p2; where p1>p2. It is a specific form of Goldbach’s conjecture. Thus if it will be proved that there exist at least two primes located at equal distance along with both sides from an integer greater than 3 in numbers series; then the specific and the above considered form of Goldbach’s conjecture will be automatically proved.

II. Explanation of Proof

Bertrand’s postulate (Chebyshev’s theorem) states that: There exists at least a prime number (p1) in between n and 2n-2 for any integer n>3; where 2n is twice of n. Such that n,p1<2n-2. Now 2 is only the even prime number and every even integer is the twice of a number in number series. Thus a is always an even integer to maintain the conditions are transferred into 2(n+a/2)=p1+p2=3n-2; where x & r are the integers and x & r both are odd. Now n+2 (x=3n-2) or x+r=2n-4 or x+r=2(n-2)=(n-2)+(n-2). The relation x+r=(n-2)+(n-2) shows the general value of x+r in all situations of x & r (i.e. for all possible values of x & r) with respect to n-2 and 2(n-2) is the twice of n-2. That means for all possible values of x & r with respect to n-2, the general value of x+r can be described as x+r=(n-2)+(n-2)); where a is an integer≥0 & a=0, 1, 2, 3, …, (n-2). Thus x=0 and x+r=(n-2) and vice versa. Now if x=(n-2)+a & x=(n-2)-a, then the relation n+2=x+p1+p2=3n-2-(r) shows that n+2+(n-2)+p1+p2=3n-2-a or p1+p2=a. Again on the other hand if x=(n-2)-a & x=(n-2)+a, then the relation n+2=x+p1+p2=3n-2-r shows that n+2+(n-2)+p1+p2=3n-2-a or p1+p2=2a. Here 2n+a-a are integers as p1+p2 or 2n & a are integers. Now p1 & p2 are odd primes. So p1+p2 is an even integer. So 2n+a-2n-a must be an even integer. Hence 2n is always an even integer for any value (even or odd) of n. Thus a is always an even integer to maintain the situation (as even-even or even-even). Therefore a=1, 3, 5, 7, …, (n-1) when n is even & a=1, 3, 5, 7, …, (n-2) when n is odd not valid; rather a=0, 2, 4, 6, …, (n-2) when n is even & a=0, 2, 4, 6, …, (n-2) when n is odd valid. Hence p1+p2=2a=2(n-2) & p1+p2=2a=2(n-2) or p1+p2=2a. On the other hand p1+p2=2n-2(a)=3n-2(b) & n-2=a. Suppose a=2b, b be an integer; where b=0, 1, 2, 4, …, (n-2)/2 for n is even & b=0, 1, 2, 4, …, [(n-2)/2]-1/2 for n is odd. Hence p1+p2=(b+1)+(n+b) and on the other hand p1+p2=(n-b)+(b-1). Again n=4, 5, 6, …, n (i.e. n≥4) and b=0, 1, 2, 4, …, (n-2)/2 for n is even & b=0, 1, 2, 4, …, (n-2)/2 for n is odd (i.e. b=0). So for a specific even or odd value of n (i.e. any fixed even or odd value of n and its corresponding b values for an even or odd value of n); it can be concluded that n+b is an integer which is shown in the following way: For n=4 & b=0, 1, 2, 4, …, (n-2)/2; so n=b=4, 5; for n=5 & b=0, 1, 2, 4, …, [(n-2)/2] 1/2, so n=b=5, 6; for n=6 & b=0, 1, 2, 4, …, (n-2)/2, so n=b=6, 7, 8; for n=7 & b=0, 1, 2, 4, …, [(n-2)/2] 1/2, so n=b=7, 8, 9; … …; for n=n & b=0, 1, 2, 4, …, (n-2)/2, so n=b=n+1, n+2, n+3, …, [(n-2)/2] for n is even or for n=n & b=0, 1, 2, 4, …, [(n-2)/2] 1/2, so n=b=n, n+1, n+2, n+3, …, [(n-2)/2] for n is odd. That means for n=4, 5, 6, …, n & b=0, 1, 2, 4, …
... (n-2)/2 for n is even & b=0, 1, 2, 4, ..., \{n-2\}/2 for n is odd, the general situations of n+b values with respect to n & b are: n+b=n, n+1, n+2, n+3, ..., \{n+(n-2)\}/2 for n is even & n+b=n, n+1, n+2, n+3, ..., \{n+(n-2)\}/2 for n is odd. On the other hand it can be said that n is an integer which is shown in the following way:

For n=4 & b=0, 1, 2, 4, ..., \{n-2\}/2, so n-b=4, 3, 2, 0, 1, \{n-2\}/2, so n-b=5, 4, 6, 7, 8, ..., \{n-2\}/2. For n=6 & b=0, 1, 2, 4, ..., \{n-2\}/2, so n-b=6, 5, 4, for \n=7 & b=0, 1, 2, 4, ..., \{n-2\}/2, so n-b=7, 6, 5, ..., \{n-2\}/2 for n=8 & b=0, 1, 2, 4, ..., \{n-2\}/2, so n-b=8, 7, 6, ..., \{n-2\}/2. For n=10 & b=0, 1, 2, 4, ..., \{n-2\}/2, so n-b=10, 9, 8, 7, ..., \{n-2\}/2.

The above means for n=4, 5, 6, 7, ..., \{n-2\}/2 for n=8 & b=0, 1, 2, 4, ..., \{n-2\}/2 for n is even & n=10, 9, 8, 7, ..., \{n-2\}/2 for n is odd, the general situations of n+b values with respect to n & b are: n+b=n, \n+1, n+2, n+3, ..., \{n+(n-2)\}/2 for n is even & n+b, \n+1, n+2, n+3, ..., \{n+(n-2)\}/2 for n is odd. Now let it be considered that for even & odd all situations of n, the values of n+b=m; where m is an integer. As n=4 & b=0; so n+b=4. Thus considering all values of n, in this case m=n-b, \n+1, n+2, n+3, ..., \{n+(n-2)\}/2 for n is even & m\n=b+n, \n+1, n+2, n+3, ..., \{n+(n-2)\}/2 for n is odd; where m=b=0, \n+1, \n+2, \n+3, ..., \{n+(n-2)\}/2 for n is even or m=b=n=0, \n+1, \n+2, \n+3, ..., \{n+(n-2)\}/2 for n is odd. Hence from both cases p1+p2=p+m or p1+p2=2m. The relation p1+p2=m shows the general value of p1+p2 in all situations of p1 & p2 (i.e. for all possible values of p1 & p2) with respect to m and 2m is the twice of m. Therefore for all possible values of p1 & p2 situation happens that the highest number is \{m+(p1+p2)\} and the lowest odd two primes are \{m-1\}, \{m+1\} which are integers. So the relation \{p1+p2=m\} is the first term of numbers series m=n+b or m=b respectively; so considering each term of both numbers series p1<sp2 (as p1<p2). Thus it can always be written that p1<sp+m=p<sp2. Now the above explanation shows that m is an integer & m\n=b3 only the exception is m=b3 for n=4 & b=1, so according to Bertrand’s postulate (Chebyshev’s theorem), it is stated that: There exists at least a prime number (p1<sp-m) in between n and 2m-2 for any integer m\n=3; where 2m is twice of m as well as from the general conception, it is obtained that: There exists at least a prime number (p1<sp-m) in between 2 and m for any integer m\n=3 simultaneously. Such that m<n<2m & 2m<n<m+2m & 2m<n+2m. That is why it can be drawn from the above fact (the conditions m<n<2m & 2m<n+2m are simultaneously exist in this situation) that there exist at least two primes (p1 & p2) located at equal distance along with both sides from an integer (m>3) in numbers series. That means from p1<sp+m & p2<sp-m; it is written that p1=m & p2=m-s respectively. Thus p1<sp-m=p1<sp2=2m; where m\n=3, p1<sp2 & p2<sp2. Again on the other hand, every even integer (2m) is twice of an integer (m) as well as every even integer (2m) is the sum of two integers located at equal distance between m which is its half (m) in the numbers series. So every even integer (2m+n) is the sum of two primes (p1 & p2) as p1<sp2 are located at equal distance s (as p1<sp+m & p2<sp-m) along with both sides from the integer m; where p2<sp2. Therefore p2<sp2=2m; where m\n=3, p1<sp2 & p2<sp2. It is nothing but the specific situation of Goldbach’s conjecture. However when s=0, then from p1<sp+m & p2<sp-m; it can be obtained that p1=m & p2=m. It is only possible when m is itself a prime. Here the situation holds the condition p1<sp2 in this respect. Again when s=m, then p1<sp2 & p2<sp2. Now 2m is always even for any value of n and both p1 & p2 are neither even (although 2 is an exception, but it does not hold the conditions of discussed proof) nor zero according to consideration of above proof. So it can be obtained from above explanation that s can accept at least a value of s=1, 3, 5, 7, ..., (m-3) for m is even & s=2, 4, 6, 8, ..., (m-3) for m is odd to maintain all the situations of this proof to hold the condition m\n=3, p1<sp2 & p2<sp2. In case of m=n=b3 (for n=4 & b=1) discussed above, there is only possibility to assume that p1=3 & p2=5 are only valid; because there exists no number in between 2 & 3 (i.e. in between 2 & m) and in between 3 & 4 (i.e. in between m & 2m-2) in numbers series. Surprisingly it is itself a prime number, so its twice is expressed as 6=3+3; where m\n=3, 2n=6, p1<sp3 & p2<sp3. Thus the specific form of Goldbach’s conjecture (Every even integer greater than 4 can be expressed as the sum of two odd primes) is proved in the general way.

III. Summary

It is written that p1<sp2=2n+a+2n+2b=2(n+b)=2n or p1<sp2=2n-a+2n-2b=(n-b)=2m; where a=2b, m\n=3, p1<sp2 & p2<sp2. Now if b=0, then from both cases m=n; so it can be written that p1<sp2=2n or p1<sp2=2n-n; where n, p1<sp2 & p2<sp2. Therefore p1<sp2=2m. It is a specific form of Goldbach’s conjecture. The relation p1<sp2=m+n shows the general value of p1<sp2 in all situations of p1 & p2 (i.e. for all possible values of p1 & p2) with respect to n and 2n is the twice of n. That means for all possible values of p1 & p2 with respect to n, the general value of p1<sp2 can be described as p1<sp2=n(d)+(n-d); where d is an integer<0 & d=0, 1, 2, 3, ..., n. Hence p1<sp2=2m & p2<sp2=d and vice versa. As p1<sp2=d, so p1<sp2=d & p2<sp2=d. From that above situation the lowest value of p1<sp2=8 as the lowest two odd primes are p1=5 & p2=3 (as p1<sp2). So the relation p1<sp2=2n is always valid for p1<sp2<8 in the above conditions n<3, p3<sp2 & p4<sp2. From the above discussion it is obtained that p1<sp2=2n or p1<sp2=2n. Thus 2n+2=8 or 2n+2=8. Again as b=0, so a=0. Therefore in both cases 2n+2=8 or 2n=4. It is the required condition of the specific form of Goldbach’s conjecture. Now if d=0, then p1<sp2=d & p2<sp2=d. It is only possible when n is itself a prime and here the situation holds the condition p1<sp2. However the described proof of Goldbach’s conjecture is valid for the condition \n=2 and it is also written above that the case for number 3 is a specific situation of above proof; but 2 is itself the only even prime number, so its twice is expressed as 4=2+2; where n=2, 4, 6, 8, ..., (m-3) for m is even & \n=2, 4, 6, 8, ..., (m-3) for m is odd as well as original form of Goldbach’s conjecture (Every even integer greater than 2 can be expressed.

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as the sum of two primes) and specific form of Goldbach’s conjecture (Every even integer greater than 4 can be expressed as the sum of two odd primes) in number theory both are true side by side.

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References
