Proof of Goldbach’s Conjecture

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Abstract: The mathematical proof of Goldbach’s conjecture in number theory is drawn in this paper by applying a specific bounding condition from Bertrand’s postulate & Chebyshev’s theorem.

Keywords: Bertrand’s postulate & Chebyshev’s theorem, Goldbach’s conjecture, prime number, even & odd number, natural numbers series.

I. Introduction

It is already known that Goldbach’s conjecture in number theory is: Every even integer greater than 2 can be expressed as the sum of two primes. If n be an integer, where n>1; then 2n is an even integer, where 2n>2. Thus the mathematical formulation of above conjecture is 2n=p₁+p₂; where p₁ & p₂ are two prime numbers. Again from the other way the conjecture states that: Every even integer greater than 4 can be expressed as the sum of two odd primes. These even numbers (>4) are called Goldbach’s numbers.

II. Notes of Proof

Bertrand’s postulate (Chebyshev’s theorem) states that:

(i) There exists at least a prime number (p) between n and 2n for any integer n>1. Such that n<p<2n. Let it be considered that n₁ and n₂ are two integers; where n₁ & n₂ both are greater than 1. Now 2n₁ & 2n₂ are the twice of n₁ & n₂ respectively. Suppose p₁ be at least a prime in between n₁ & 2n₁ and p₂ be at least a prime in between n₂ & 2n₂. Hence from the above postulate it is written that n₁<p₁<2n₁ and n₂<p₂<2n₂. So from these relations it can be determined that n₁+n₂<p₁+p₂<2n₁+2n₂ or n₁+n₂<p₁+p₂<2(n₁+n₂). As n₁>1 & n₁>1, so if n₁=constant i.e. any fixed value of n₁=2, 3, 4, … (any integer greater than 1) & n₁=m, where m=2, 3, 4, … (any integer greater than 1); then u=m<p₁+p₂<2(u+m) or m<u<p₁+p₂<2(m+u). After addition of -u, it is obtained that m+u<u<p₁+p₂<u<2(m+u) or m<u<p₁+p₂<u<2m+u. Now the above relation shows that p₁+p₂<2m+u, so there is at least the possibility either p₁+p₂=u+r=2m+u or p₁+p₂=u<2m+u; where r is an integer>0. Hence p₁+p₂=2(m+u)-r. As p₁+p₂=u<2m+u, so r=u+x; where x=0, 1, 2, 3, … (any integer). Again every even number (2n) is the twice of a natural number (n). Thus 2(m+u) is even for any value of m and u. Now to consider Goldbach’s number for even numbers except 4, p₁ & p₂ both are always odd (because of all primes are odd in natural numbers series except 2), as a result p₁+p₂ is always even as (odd+odd)=even. That means r is always even as (even+even)=even. Hence r is even when x=0, 2, 4, 6, … (any even integer) if u is an even & x=1, 3, 5, 7, … (any odd integer) if u is an odd because of (even+even)=even & (odd+odd)=even. Suppose u=2, x=0 & m=2, 3, 4, …; then p₁+p₂=6, 8, 10, … etc (all even integers>4). In this way by choosing the proper values of m, u & r from the above bounding condition it can be determined that every even integer greater than 4 can be expressed as the sum of at least two primes. This is nothing but a specific situation of Goldbach’s conjecture.

However the above proof shows that p₁+p₂≥6 (according to consideration the lowest values of m, u & x are 2, 2 & 0 respectively). Thus 2(m+u)≥6. Hence 2(m+u)-(u+x)≥6 or 2m+u≥6 or 2m+u≥6x. i.e. x<≤(2m+u)-6.

(ii) There exists at least one prime number (p) for integer n>3 with n<p<2n-2. Let it be considered that n₁ and n₂ are two integers; where n₁ & n₂ both are greater than 3 and p₁ & p₂ are the at least prime numbers with n₁<p₁<2n₁-2 and n₂<p₂<2n₂-2 respectively. In the above way it can be drawn that n₁+n₂<p₁+p₂<2(n₁+n₂)-4. Here as n₁>3 & n₂>3, so if n₁=constant i.e. any fixed value of n₁=4, 5, 6, … (any integer greater than 3) & n₂=m, where m=4, 5, 6, … (any integer greater than 3); then u=m<p₁+p₂<2(u+m) or m<u<p₁+p₂<2(m+u)-4. After addition of -u, it is obtained that m<u<p₁+p₂<2m+u-4. Now the above relation shows that p₁+p₂<2m+u-4, so there is at least the possibility either p₁+p₂=u+r=2m+u-4 or p₁+p₂=u<2m+u-4; where r is an integer>0. Hence p₁+p₂=2(m+u)-4-r. As p₁+p₂<u<2m+u-4, so r=u+x; where x=0, 1, 2, 3, … (any integer). Again every even number (2n) is the twice of a natural number (n). Thus 2(m+u) is even for any value of m and u. Hence to consider Goldbach’s number for even numbers except 4, p₁ & p₂ both are always odd (because of all primes are odd in natural numbers series except 2), as a result p₁+p₂ is always even as (odd+odd)=even. That means r is always even as (even+even)=even and 4 is even number. Hence r is even when x=0, 2, 4, 6, … (any even integer) if u is an even & x=1, 3, 5, 7, … (any odd integer) if u is an odd because of (even+even)=even & (odd+odd)=even.

DOI: 10.9790/5728-1603035152 www.iosrjournals.org 51 | Page
(odd+odd)=even. Suppose u=4, x=0 & m=4, 5, 6, …; then p₁+p₂=8, 10, 12, … etc (all even integers>6). In this way by choosing the proper values of m, u & r from the above bounding condition it can be determined that every even integer greater than 6 can be expressed as the sum of at least two primes. Here it is also nothing but a specific situation of Goldbach’s conjecture.

However the above proof shows that p₁+p₂≥8 (according to consideration the lowest values of m, u & x are 4, 4 & 0 respectively). Thus 2(m+u)-4≥8. Hence 2(m+u)-4-(u+x)≥8 or 2m+u-4-x≥8 or 2m+u-12≥x. i.e. x≤(2m+u)-12.

III. Conclusion
Thus Goldbach’s conjecture can be proved from Bertrand’s postulate or Chebyshev’s theorem with applying a special bounding condition for even integers n>4 (Goldbach’s numbers). However the proof cannot be applicable for even number 4. Because 4=2+2; where 2 is only the even prime.

Acknowledgement
I like to thank Sir Larry J. Gerstein, Ex. Professor, California University, Loss Angeles, USA for his valuable comments on Goldbach’s conjecture.

References