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Complementary Circuit Domination in Graphs

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Abstract: *A set D of a graph $G=(V,E)$ is a dominating set if every vertex in $V\setminus D$ is adjacent to some vertex in D. The domination number of G is the minimum cardinality of a dominating set. In this paper, complementary circuit domination number is defined and some of its properties are studied.*
Keywords: *Domination number, complementary circuit domination number*

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I. Introduction

Graphs considered in this paper are finite, undirected, nontrivial, connected simple graphs.For a graph G, let $V(G)$ and $E(G)$ denote the vertex set and the edge set respectively. For $v\in V(G)$, the set of all vertices adjacent to v is called the neighborhood of v and is denoted by $N(v)$. $N[v]=N(v)\bigcup\{v\}$ is called the closed neighborhood of v. Any undefined terms in this paper may be found in Harary [2].

Ore introduced the concept of domination in graphs. A set $D\subseteq V(G)$ is called a dominating set of G, if every vertex in $V(G)\setminus D$ is adjacent to some vertex in D. D is said to be a minimal dominating set if no proper subset of D is a dominating set. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set. We call a set of vertices a $\gamma-$ set if it is a dominating set with cardinality $\gamma(G)$. A dominating set D of G is a connected dominating set if the induced subgraph $\langle D\rangle$ is connected. The connected domination number $\gamma_c(G)$ is the minimum cardinality of a connected dominating set. A survey of results on domination can be found in [5].

Recently S. Muthammai , M. Bhanumathi and P. Vidhya [3] introduced the concept of complementary tree domination. A dominating set D is called a complementary tree dominating set if the induced sub-graph $\langle V\setminus D\rangle$ is a tree. The minimum cardinality of a complementary tree dominating set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}(G)$.

This paper is concerned with complementary circuit domination, which is of great practical importance. In real life there are many situations in which every point on a circuit or a cycle is dominated by minimum number of points outside the set. Especially when the under lying graph structure is complete or a wheel graph, all the points on the circuit or cycle can be dominated by a single point. Complementary circuit domination gives a way for this. This paper is organized as follows. Section II the complementary circuit domination is defined and some of its properties are discussed. Section III deals with the of complimentary circuit domination number $\gamma_{ccd}(G)$ of some graphs and in the last section bounds of complimentary circuit domination number are obtained.

II. Complementary Circuit Domination

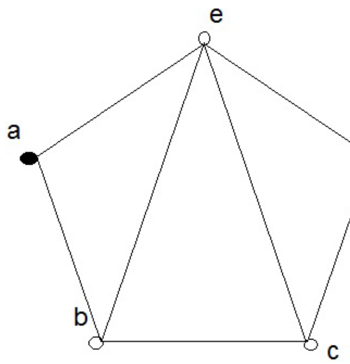
A set $D\subseteq V(G)$ of $G=(V,E)$ is called a dominating set if every vertex in $V\setminus D$ is adjacent to some vertex in D. The minimum cardinality of all dominating sets is called the domination number $\gamma(G)$ of G. In this section a complementary circuit dominating set is defined by imposing a condition on the dominating set.

Definition 2.1. A dominating set D is called a complementary circuit dominating set if the induced subgraph $\langle V \setminus D \rangle$ is a circuit. The minimum cardinality of a complementary circuit dominating set is called the complementary circuit domination number of G and is denoted by $\gamma_{ccd}(G)$

When $\langle V \setminus D \rangle$ is a cycle C_n for $n \geq 3$, D reduces to a complementary cycle dominating set. Let us denote ccd − set for a complementary circuit dominating set and γ_{ccd} − set for a minimum cardinality ccd − set.

Since every ccd − set is a dominating set, Theorem 2.2 follows directly from the definition of complementary circuit dominating set.

Theorem 2.2. For any connected graph G , $\gamma(G) \leq \gamma_{ccd}(G)$
.
Following example gives a graph for which $\gamma(G) < \gamma_{ccd}(G)$



In the above graph, $\{a.d\}$ is a ccd − set and $\{e\}$ is a γ -set of G. Hence $\gamma(G) < \gamma_{ccd}(G)$

Observation:
1. A graph contains a complementary circuit dominating set only if G is not a cycle and G contains a circuit.
2. Pendant vertices belongs to every complementary circuit dominating set.

Theorem 2.3. If D is a ccd − set of a graph G then D contains no cut vertex of G .

Proof. : Suppose if possible, $v \in D$ is a cut vertex. Then $\langle V(G) \setminus \{v\} \rangle$ is disconnected, a contradiction to the assumption that $\langle V(G) \setminus \{v\} \rangle$ is a circuit. Therefore v is not a cut vertex for any $v \in D$.

Theorem 2.4. Every minimal ccd −set of a graph G is properly contained in a ctd −set of G .

Proof. : Let D be a minimal ccd − set of a graph G . Then $\langle V \setminus D \rangle$ is a circuit.
Case I: $\langle V \setminus D \rangle$ is a cycle C_n for $n \geq 3$. Then for any edge $e \in E(C_n)$, $\langle V \setminus D \rangle \setminus \{e\}$, we get a spanning tree T of $\langle V \setminus D \rangle$. In this case T is simply a path P_n . By adjoining one of the pendant vertices u of T to its dominating vertex in D , we get a ctd −set $D' = D \cup \{u\}$ of G properly containing D.
Case II: $\langle V \setminus D \rangle$ is a circuit other than a cycle. Then for a finite number of edges $e_1, e_2, ..., e_n$ of $\langle V \setminus D \rangle$, $\langle V \setminus D \rangle \setminus \{e_1, e_2, ..., e_n\}$ is a spanning tree of $\langle V \setminus D \rangle$. As in case I, by adjoining one of the pendant

vertices u of T to its neighboring vertex in D, we get a ctd−set $D^{'} = D \bigcup \{u\}$ of G properly containing D

Corollary2.5 and Corollary 2.6 follows directly from Theorem 2.4

Corollary 2.5. Every minimal ccd − set can be extended to a ctd − set

Corollary 2.6 Every minimal ccd − set is a proper subset of a ctd − set

Remark: Every minimal ccd − set is properly contained in a ctd − set. But this does not imply that every minimal ctd − set contains a ccd − set.

III. The value of complementary circuit domination number for some particular graphs.

Theorem 3.1. For a graph G, $\gamma_{ccd}(G) = 1$ if and only if $G \cong G_1 + K_1$ for some graph G_1

Proof. : Let $\gamma_{ccd}(G) = 1$. Then there exists a complementary circuit dominating set D of G with $|D| = 1$ such that $\langle V \setminus D \rangle$ is a circuit. Since each vertex in $V \setminus D$ is adjacent to the single vertex in D,
$G \cong G_1 + K_1$ where $G_1 = \langle V \setminus D \rangle$
Conversely if $G \cong G_1 + K_1$, where G_1 is a circuit, then the set $\{v\}$ is a complementary circuit dominating set of G where $\{v\} = V(K_1)$

The following results are direct conquences of Theorem 3.1

Corollary 3.2. For any complete graph K_n with $n \geq 4$ vertices, $\gamma_{ccd}(K_n) = 1$

Corollary 3.3. For any wheel graph W_{n+1} with $n \geq 3$, $\gamma_{ccd}(W_{n+1}) = 1$

IV. Bounds of Complementary Domination Number

In this section we consider connected graphs G containing a ccd − set and with p vertices where $p \geq 4$.

Theorem 4.1. Let G be a graph, then $\gamma_{ccd}(G) \leq p - 3$

Proof. : Let D be a γ_{ccd}−set of G. Then $p = |V| = |D| + |V \setminus D|$ Since $\langle V \setminus D \rangle$ is a circuit, $|V \setminus D|$ must be atleast 3. Therefore $|D| \leq p - 3$ ie., $\gamma_{ccd}(G) \leq p - 3$.

Theorem 4.2. $\gamma_{ccd}(G) = p - 3$ if and only if $\langle V \setminus D \rangle = K_3$ for any γ_{ccd} − set D of G

Proof. : If $\langle V \setminus D \rangle = K_3$, then $|V \setminus D| = 3$ and $|D| = p - 3$. Therefore $\gamma_{ccd}(G) = p - 3$.
Conversely if D is a ccd − set and $\gamma_{ccd}(G) = p - 3$, then the circuit $\langle V \setminus D \rangle$ contains 3 vertices and the only possibility is $\langle V \setminus D \rangle = K_3$.

Corollary 4.3. $\left\lceil p \big/ (\Delta + 1) \right\rceil \leq \gamma_{ccd}(G) \leq p - 3$

Proof. : Since $\left\lceil \frac{p}{(\Delta+1)} \right\rceil \le \gamma(G) \le \gamma_{ccd}(G)$, first part of the inequality holds. Hence by the previous theorem we have $\left\lceil \frac{p}{(\Delta+1)} \right\rceil \le \gamma_{ccd}(G) \le p-3$

Theorem 4.4. $\gamma_{ccd}(G) + \Delta(G) \le 2p-4$

Proof. : Since $\Delta(G) \le p-1$, for any connected graph with p vertices, we have,
$\gamma_{ccd}(G) + \Delta(G) \le 2p-4$

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