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Fermat Quadratic Equation and Prime Numbers

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Abstract: *The article provides a general solution of Fermat quadratic equation, analyzes the behavior of different classes of the basic solutions of the equation. The author shows the possibility to use the solutions for generating prime numbers and proving Landau's hypothesis and Goldbach's binary hypothesis. Numerical calculations are made to explain the general relationships.*

Keywords and phrases: *Theory of numbers, natural numbers, prime numbers, Goldbach's binary hypothesis, Landau's hypothesis*

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I. Introduction. Construction of basic solutions

The problem of solving the quadratic Fermat equation has a long history and many papers are devoted to it. The purpose of this article is to give a general solution of the quadratic equation in a new way and explore its connection with the problems of prime numbers. Quadratic Fermat's equation is obtained from the general equation, when $p = 2$ and has the form

$$x^2 + y^2 = z^2, \tag{1}$$

where x, y, z are natural numbers. Put $y = x + m$; $z = x + n$; $n > m$. We introduce the new parameters δ_1 and δ_2, we put $n - m = \delta_1 x/3$, $n^2 - m^2 = \delta_2 x^2/3$, i.e. $n + m = \delta_2 x/\delta_1$, then equation (1) is transformed to the form

$$2\delta_1 + \delta_2 = 3, \tag{2}$$

where δ_1, δ_2 are a positive rational numbers; x is a free variable; its value is chosen the smallest, so that x, y and z are natural numbers. The triplet (x, y, z), which is the solution of the quadratic equation (1), is given by the relations

$$x = 2s_1 s_2, \tag{3a}$$

$$y = x \cdot [1 + 1/2(\delta_2/\delta_1 - \delta_1/3)], \tag{3b}$$

$$z = x \cdot [1 + 1/2(\delta_2/\delta_1 + \delta_1/3)], \tag{3c}$$

where s_1 is the denominator of the number δ_2/δ_1, and s_2 is the denominator of the number $\delta_1/3$; numerator and denominator of δ_2/δ_1, and also, the numerator and denominator of $\delta_1/3$ are relatively prime numbers. In this case, the triplet will be the basic solution (see below). The purpose of this article is to study the general solution of equation (1) and its connection with prime numbers. In the general case, to obtain solutions of the equation put $\delta_1 = 1/k$, $\delta_2 = (3k - 2)/k$, to satisfy (2), where k is an arbitrary positive rational number greater than 2/3. In this regard, there are a number of interesting questions. We will show how to obtain all the basic solutions of the quadratic equation, i.e. such that x, y, z have no common divisor. Note that, if x, y, z all even, then the solution is not basic. We distinguish three cases.

1. Set of solutions generated by natural values of k, i.e., $k = 1, 2, 3$, etc. Each value of k corresponds to a basic solution. These solutions will be called solutions of the first kind (type).
2. Set of solutions generated by rational values of k, when k is a proper fraction in the range $2/3 < k < 1$. Put $k = n/(n+1)$, where $n = 3, 4, 5$, etc., then $k = 3/4, 4/5, 5/6, 6/7$, etc. Each value of k corresponds to a basic solution. These solutions we call solutions of the second type. In general, they can be represented in the form $k = q/(n+1)$, where $2/3(n+1) < q \le n$. In some cases, x and y may change places, for example (3, 4, 5) → (4, 3, 5). Such solutions are considered equivalent.
3. Set of solutions generated by rational values of k, when k is an improper fraction in the interval $1 < k < \infty$. It is convenient to distinguish two cases depending on whether the denominator is even or odd. If the denominator is an even number, we put $k = l_1/2l$, where $l, l_1 = 1, 2, 3$, etc., $l_1 \ge 2l$, l_1 and $2l$ – mutually prime numbers. Fixing

the denominator and changing the numerator, we obtain a sequence of values of k, each of which corresponds to a basic solution. For example, for $l = 1$ we have a sequence with denominator 2, i.e., $k = 3/2, 5/2, 7/2$, etc. For $l = 2$ we have the sequence with a denominator of 4, i.e. $k = 5/4, 7/4, 9/4$, etc. If the denominator is an odd number, put $k = l_1/(2l + 1)$, where again l, $l_1 = 1, 2, 3$, etc., $l_1 \geq (2l + 1)$, l_1 and $(2l + 1)$ – mutually prime numbers. For example, for $l = 1$ we have a sequence with denominator 3, i.e., $k = 4/3, 5/3, 7/3, 8/3, 11/3$ etc. For $l = 2$ we get $k = 6/5, 7/5, 8/5, 9/5, 11/5$ etc. These solutions we call solutions of the third type. The above procedure gives all the solutions of the quadratic equation.

II. Use of basic solutions for obtaining primes

As a second problem, consider obtaining the basic solutions of the quadratic equation such that y, z both odd, $z – y = n – m = 2$, as this task is related to the problem of generating prime numbers. It is easy to find that the solutions have the form $x = 6k$, $y = 9k^2 – 1$, $z = 9k^2 + 1$. Since y, z are both odd numbers, then k must be an even number, i.e. $k = 2, 4, 6, 10$, etc. It is clear that x and y are always composite numbers and z may be a prime number if k ends in 0, 2 or 8, thus z ends in 1 or 7, respectively. In the first case, k takes the values 10, 20, 30, 40, 50, etc., and z forms a sequence with the first member 901 and a variable period equal to $180(k + 5)$. The expression for z in this case can be conveniently represented in the form $z = 9k^2 + 1 = 900t^2 + 1$, where $t = 1, 2, 3$, etc. Note that the integer k can take values of powers of 10, thus z can be represented as $9{\cdot}10^{2r} + 1$, where $r = 1, 2, 3$, etc. It is easy to verify that among these numbers z there are the prime numbers, for example, 8101, 14401, 22501, 32401, 44101, 57601, 72901, 90001, 176401(the check was carried out in the range up to 200000). In the second case, k takes values 2, 12, 22, 32, 42, 52, etc., and z forms a sequence with the first member 37 and a variable period equal to $180(k + 5)$. The expression for z in this case can be conveniently represented in the form $z = 9k^2 + 1 = 36(1 + 5t)^2 + 1$, where $t = 0, 1, 2, 3$, etc. Note that the integer k can take values of powers of 2, thus z can be represented as $36{\cdot}2^{8t} + 1$, where $t = 0, 1, 2, 3$, etc. It is easy to verify that among the numbers z there are the prime numbers, for example, 37, 1297, 4357, 15877, 24337, 93637, 156817 (the check was carried out in the range up to 200000). In the third case, k takes the values 8, 18, 28, 38, 48, etc. The number z in this case can be conveniently represented in the form $z = 9k^2 + 1 = 36(4 + 5t)^2 + 1$, where $t = 0, 1, 2, 3$, etc. Among these numbers there are also prime, for example, 577, 2917, 7057, 41617, 69697, 197137 etc. (the test was carried out in the range up to 200000). Since the period for ends (but of course not for numbers) is equal to 5, it is convenient to represent numbers $9k^2 + 1$ in the form of pentad (fives). We have (1pr, 2ex, 1pr, 1c), (1pr, 2ex, 1pr, 1c), (1pr, 2ex, 1pr, 1pr), (1c, 2ex, 1c, 1pr), (1pr, 2ex, 1c, 1pr), (1pr, 2ex, 1c, 1pr), (1c, 2ex, 1pr, 1pr), (1c, 2ex, 1c, 1pr), (1c, 2ex, 1pr, 1pr), (1c, 2ex, 1c, 1pr), (1pr, 2ex, 1c, 1c), (1c, 2ex, 1c, 1c), (1c, 2ex, 1c, 1c), (1pr, 2ex, 1c, 1pr), etc., where 1pr means one prime number, 2ex – two exceptions, i.e., the numbers ending in 5, 1c – one composite number. The notation in the first pentad corresponds to the numbers for $k = 2$ (37), $k = 4$ (145), $k = 6$ (325), $k = 8$ (577), $k = 10$ (901), etc. with step for k equal to 2. The ends of numbers in each pentad change as follows: 7, 5, 5, 7, 1. Regularities in the alternation of the numbers are repeated regularly enough, so the sequence of numbers $z = 9k^2 + 1$ that simultaneously belong to the solution of the quadratic equation can be used to obtain prime numbers. Let us now consider the numbers $y = 9k^2 – 1 = (3k – 1){\cdot}(3k + 1) = y_1{\cdot}y_2$. Since k is an even number and takes the values 2, 4, 6, 10, etc., both cofactor are odd and differ by 2. The numbers $(3k – 1)$ form an arithmetic progression $5 + 6l$ with period 6, where $l = 0, 1, 2, 3$, etc., and the numbers $(3k + 1)$ – an arithmetic progression $7 + 6l$ with the same period. The numbers y_1, y_2 can be both prime, only one of them prime or both composite, and the difference between them is equal to 2. These cases are repeated regularly. Analysis of the tables of prime numbers, carried out in [1], shows that the difference between adjacent primes, as a rule, does not exceed 30, while the difference in 32, 34 and 36 units or more is rare (for example, the difference in 32, 34 and 36 units are found only once among the first 10000 primes), therefore these sequences can be used to obtain primes. Increasing the step of k, we obtain the samples in which both the number (or one of them) are in many cases prime. The numbers $(3k - 1)$ and $(3k + 1)$ take all endings characteristic of primes, and in different combinations, i.e. in a certain sense complement each other. In particular, their ends are respectively 5 and 7 when $l = 0$ ($k = 2$), 1 and 3, when $l = 1$ ($k = 4$), 7 and 9 when $l = 2$ ($k = 6$), 3 and 5, when $l = 3$ ($k = 8$), 9 and 1 when $l = 4$ ($k = 10$), and then they are repeated with period 5. The pairs (5, 7) and (3, 5) should be excluded. For convenience of analysis, we designate $n = 6k = (3k – 1) + (3k + 1)$, where $3k = n/2$ is the center of representation of number n. The pair of endings (1, 3) corresponds to a sequence $k_{13} = 4 + 10l$ ($l = 0, 1, 2, 3$, etc.), the pair of endings (7, 9) corresponds to a sequence $k_{79} = 6 + 10l$ ($l = 0, 1, 2, 3$, etc.), the pair of endings (9, 1) corresponds to a sequence $k_{91} = 10 + 10l$ ($l = 0, 1, 2, 3$, etc.). Thus, the sequences $(3k – 1)$ and $(3k + 1)$ when changing k allow us to obtain the prime numbers and, moreover, pairs of primes that differ by 2. Pairs of prime numbers appear regularly with increasing l. We can choose a step by l so as to get a pair of prime numbers. It of course depends on the order of magnitude of the number n. As the Fermat's quadratic equation has infinitely many solutions of this type, and any opportunity we cannot prefer to another, it can be argued that there are infinitely many primes that differ by 2, i.e. the conjecture of Landau is valid. We have considered the case when y and z both odd, $z – y = n – m = 2$, then x is an even number. The case when x and y are odd and z is even impossible, as follows from the previous analysis. Consider the case when x and z are odd and y is even. Using the previous reasoning, we obtain the basic solution of the form $x = 3k$, $y = (9k^2 – 1)/2$, $z = (9k^2 + 1)/2$, where k = 1, 3, 5, 7, etc. For k =1, we get x = 3, y = 4, z = 5; for k =3 we have x = 9, y = 40, z = 41, etc. Consider the connection of these solutions with prime numbers. The expression for z can be prime number if k ends in 3, 5 or 7; then z ends in 1, 3 and 1, respectively. Using again the above notation in the form of pentad, we write the result in abbreviated form. We have (3pr, 1c, 1ex), (1ex, 3pr, 1ex), (1ex, 1pr, 1c, 1c, 1ex), (1ex, 3c, 1ex), (1ex, 2c, 1pr, 1ex), (1ex, 3pr, 1ex), (1ex, 1c, 2pr, 1ex), (1ex, 1pr, 1c, 1pr, 1ex), (1ex, 2c, 1pr, 1ex), (1ex, 1pr, 2c, 1ex), etc. The notation in the first pentad corresponds to the numbers for k = 1 (5), k = 3 (41), k = 5 (113), k = 7 (221) k = 9 (365), etc. with step of k equal to 2. The ends of each pentad alternate in the following way: 5, 1, 3, 7, 5, etc. Regularity in the alternation of numbers with these ends repeated regularly enough, so the sequence of numbers $z = (9k^2 + 1)/2$ that belong to the solution of the quadratic equation can be used to obtain primes. The expression for y we write in the form $y = (3k – 1)·(3k + 1)/2$. If k ends in 1, 3, 5, 7 or 9, each of the factors alternately after dividing by 2 is odd and can be prime number. Thus for $(3k – 1)/2$, we have to exclude k ending in 7, and for $(3k + 1)/2$ ending in 3, as in this case, the corresponding expression is a multiple of 5. We have for k = 1: $(3k – 1)/2$ = 1, for k = 3: $(3k + 1)/2$ = 5, for k = 5: $(3k – 1)/2$ = 7, for k = 7: $(3k + 1)/2$ = 11, for k = 9: $(3k – 1)/2$ = 13, for k = 11: $(3k + 1)/2$ = 17, for k = 13: $(3k – 1)/2$ = 19 , for k = 15: $(3k + 1)/2$ = 23, for k = 17: $(3k – 1)/2$ = 25 (excluded), etc. In the general case $(3k + 1)/2$ is odd, if $k = 3 + 4t$, and $(3k – 1)/2$ is odd, if $k = 1 + 4t$, where t = 0, 1, 2, 3, etc. The numbers $(3k – 1)/2$ form an arithmetic progression $y_1 = 1 + 6l$ with period 6, where l = 0, 1, 2, 3, etc., and the numbers $(3k + 1)/2$ – progression $y_2 = 5 + 6l$ with the same period. The numbers y_1, y_2 can be both prime, only one of them prime or both composite, and the difference between them is equal to 2 or 4. These situations are repeated regularly. The results are the same obtained above for even k. Thus, the solution of quadratic equation z, as well as $(3k + 1)/2$ and $(3k – 1)/2$ can be used to generate primes.

III. Connection of solutions with Goldbach's problem

In addition, it can be concluded that the justice of the Goldbach's binary conjecture extends to infinite series of numbers. Before giving the proof we summarize the results obtained. If k is even, odd numbers in the sequence $(3k – 1)$ followed by step of 6 units, as well as in the sequence $(3k + 1)$. Against each other numbers in these sequences differ by 2 for the same value of k. If you combine sequences and rank the numbers in ascending order, they alternate with step 2 – 4 – 2 – 4 etc., namely, we have pairs 5 – 7 (k = 2), 11 – 13 (k = 4), 17 – 19 (k = 6), etc.. The first number in a pair corresponds to $(3k – 1)$ and the second $(3k + 1)$. For the sequences $(3k – 1)/2$ and $(3k + 1)/2$ if k is odd the regularities are the same, but the odd numbers correspond to neighboring k (at the same k the second factor is even). If you combine sequences and rank the numbers in ascending order, they alternate with step 4 – 2 – 4 – 2 etc, namely, we have pairs 1(k = 1) – 5(k = 3), 7(k = 5) – 11 (k = 7), 13(k = 9) – 17 (k = 11), 19(k = 13) – 23(k = 15), etc. The first number in a pair corresponds to the $(3k – 1)/2$, and the second $(3k + 1)/2$. Prime numbers appear in regular sequences consisting of the solutions of the quadratic Fermat equation. In these sequences there are no odd numbers divisible by 3, i.e., the number with the smallest repetition period. In addition, odd numbers divisible by 5 have a period of 30 (5·6), and divisible by 7 – period 42 (7·6), etc., i.e., appear seldom enough. Now we prove Goldbach's binary hypothesis by induction on k. If k = 2 it is true, as $x = 6k = 12 = (3·2 – 1) + (3·2 + 1) = 5 + 7$. Suppose it is valid for arbitrary $p = k$, i.e., $x = 6k = (3k – 1) + (3k + 1)$, and both summands in the right part of the equation are prime numbers. Note that they differ by 2, and the distribution center is $3k$. Put $p = k + 2$. We have $x_1 = 6(k + 2) = x + 12 = [(3k – 1) + 6] + [(3k + 1) + 6] = (3k_1 – 1) + (3k_1 + 1)$, where $k_1 = k + 2$. The terms differ by 2 and the distribution center is equal to $3k + 6 = 3k_1 = 3(k + 2)$. If both terms are prime numbers, then the hypothesis is true. Otherwise, successively decrease the first term by 2, 4, 6, etc. and simultaneously increasing the second term by the same value until both numbers are not primes. This happens in finite number of steps, as it follows from the analysis, and that proves the Goldbach conjecture. Here we must make one remark. Even numbers go in step of 6. Representation for the intermediate numbers is obtained by selection of the members of the sequences symmetrically located relative to the center, that sum is a represented even number. Let us consider a special case. Suppose, for example, k = 20, then the number is $6k$ = 120, the center is 60. We have a representation 120 = 59+61 = 55+65 = 53+67 etc., where the first term in each sum belongs to the sequence $(3k – 1)$ or $(3k + 1)$ and the second to the sequence $(3k + 1)$ or $(3k – 1)$ respectively. Let k = 22, then the number is $6k$ = 132, the center is 66. We have a representation 132 = 65 + 67 = 61 + 71 = 59 + 73 = 53 + 79 etc., where the first term in each sum belongs to the sequence $(3k – 1)$ or $(3k + 1)$ and the second to sequence $(3k + 1)$ or $(3k – 1)$ respectively. To get a representation of the intermediate even numbers the terms can belong to one or different sequences. For example, for the number 122 with the center 61 the terms are taken from the sequence $(3k + 1)$, so 122 = 55 + 67 = 49 + 73 = 43 + 79 etc. For the number 124 with the center 62 the terms are taken from the sequence $(3k – 1)$, we have 124 = 53 + 71 = 47 + 77 = 41 + 83 etc. For the number 126 with the center 63 the summands are taken from both sequences with a shift in k, we have 126 = 61 + 65 = 59 + 67 = 55 + 71 = 53

+ 73 etc. It is easy to write general relations for representation of even integers in the interval from $6k$ to $6(k + 2)$. Let $k = u$ (u is an even number), then the represented number is $6u$; $3u$ is distribution center, and we have $6u = (3u – 1) + (3u + 1)$. Let $k = u + 2$, then the represented number is equal to $6(u + 2)$; distribution center is $3(u + 2)$, and we have $6(u + 2) = [3(u + 2) – 1] + [3(u + 2) + 1]$. The difference between the numbers $6u$ and $6(u + 2)$ is equal to 12. We write representations for the intermediate numbers. For the number $6u + 2$ with center $3u + 1$ we have $6u + 2 = [(3u + 1) – 6] + [(3u + 1) + 6] = [(3u + 1) – 12] + [(3u + 1) + 12]$, etc. For the number $6u + 4$ with the center $3u + 2$ we have $6u + 4 = [(3u – 1) – 6] + [(3u – 1) + 12] = [(3u – 1) – 12] + [(3u – 1) + 18]$, etc. For the number $6u + 6$ with the center $3u + 3$ we have $6u + 6 = [(3u – 1)] + [(3u + 1) + 6] = [(3u – 1) – 6] + [(3u + 1) + 12]$, etc. For the number $6u + 8$ with the center $3u + 4$ we obtain $6u + 8 = [(3u + 1)] + [(3u + 1) + 6] = [(3u + 1) – 6] + [(3u + 1) + 12] = [(3u + 1) – 12] + [(3u + 1) + 18]$, etc. Finally, for the number $6u + 10$ with the center $3u + 5$ we have $6u + 10 = [(3u – 1)] + [(3u – 1) + 12] = [(3u – 1) – 6] + [(3u – 1) + 18] = [(3u – 1) – 12] + [(3u – 1) + 24]$, etc. for any k.

IV. Using the solutions of the quadratic equation to obtain large prime numbers

To obtain large primes we use solutions of the second type in the form $k = n/(n +1)$. We choose the values $n +1$, which are divided by 3; so $n +1 = 3l$. In this case, the calculations are simplified. The triplet (x, y, z), which is the solution of the quadratic equation (1), is given by the relations obtained from (3a) − (3c)

$$x_l = 2l \cdot (3l - 1), \tag{4a}$$

$$y_l = x_l \cdot [1 + 1/2(\frac{l-1}{l} - \frac{l}{3l-1})], \tag{4b}$$

$$z_l = x_l \cdot [1 + 1/2(\frac{l-1}{l} + \frac{l}{3l-1})], \tag{4c}$$

where $l = 1, 2, 3,\ldots$. It is easy to verify that the triplet (x, y, z) defined from (4a) − (4c) satisfies equation (1). The values of x, y, z can also be represented in the form of recurrence relations

$$x_{l+1} = x_l + 4 + 12l, \tag{5a}$$

$$y_{l+1} = y_l + 2 + 16l, \tag{5b}$$

$$z_{l+1} = z_l + 4 + 20l. \tag{5c}$$

It is easy to verify that the triplet (x, y, z) defined from (5a) − (5c) satisfies equation (1). We give several triplets (x, y, z) for different l. l =1: (4, 3, 5), l =2: (20, 21, 29), l =3: (48, 55, 73), l =4: (88, 105, 137), l =5: (140, 171, 221), l =6: (204, 253, 325), l =7: (280, 351, 449) etc. All z except z_6 are prime numbers. It can be seen that the end of z runs through all odd values with period 5. Since there are infinitely many prime numbers, as well as the solutions of the quadratic equation (1), among these solutions z there are prime numbers. We give some more values of z, which are prime numbers, for different values of l. We have $z_8 = 593$, $z_9 = 757$, $z_{10} = 941$, $z_{13} = 1613$, $z_{14} = 1877$, $z_{15} = 2161$, $z_{17} = 2789$, $z_{20} = 3881$, $z_{23} = 5153$, $z_{25} = 6101$ etc. The algorithm for calculating the values of z can be used to obtain large prime numbers. To simplify the calculations, we can use the values of z with the end 1, which are obtained for $l = 5 + 5i$ (i=1, 2, 3,…). Among such numbers z_l, there are certainly prime numbers. In particular, for $l = 10^r$, where r is an arbitrarily large natural number, z can be written in a general form: z = 999...9400...01; z contains r nines and $r −1$ zeros. For example, for $r = 1$, $z_{10} = 941$(see above), for $r = 2$, $z_{100} = 99401$ (prime number), for $r = 3$, $z_{1000} = 9994001$, etc., for $r = 57885161$, z = 9...940...01; this number contains 57885161 nines and 57885160 zeros. Solutions of the third type (see above), as calculations show can also be used to obtain prime numbers. For each value of l, sets of triplets are obtained, depending on l_1, in which z can be prime number. This case requires a separate study because of the large number of possibilities; therefore we consider the limiting case for which the general relations can be obtained. Put $k = (n + 1)/n$. To simplify the calculations we choose the values of n, which are divisible by 3, so that $n = 3l$. Formulas (3a) - (3c) are transformed to the form

$$x_l = 2l \cdot (3l + 1), \tag{6a}$$

$$y_l = x_l \cdot [1 + 1/2(\frac{l+1}{l} - \frac{l}{3l+1})], \tag{6b}$$

$$z_l = x_l \cdot [1 + 1/2(\frac{l+1}{l} + \frac{l}{3l+1})], \tag{6c}$$

The values of x, y, z can also be represented in the form of recurrence relations

$$x_{l+1} = x_l + 8 + 12l, \tag{7a}$$

$$y_{l+1} = y_l + 14 + 16l\ , \tag{7b}$$

$$z_{l+1} = z_l + 16 + 20l\ . \tag{7c}$$

where l = 1, 2, 3,…. It is easy to verify that the triplet (x, y, z) defined from (6a) − (6c) or (7a) − (7c) satisfies equation (1). We give several triplets (x, y, z) for different l. l =1: (8, 15, 17), l =2: (28, 45, 53), l =3: (60, 91, 109), l =4: (104, 153, 185), l =5: (160, 231, 281), l =6: (228, 325, 397) etc. All z except z_4 are prime numbers. It can be seen that the end of z runs through all odd values with period 5. The algorithm for calculating the values of z can be used to obtain large prime numbers. We give some more values of z, which are prime numbers, for different values of l. We have z_{10} = 1061, z_{11} = 1277, z_{15} = 2341, z_{16} = 2657, z_{22} = 4973 etc. To simplify the calculations, we can use, as above, the values of z with the end 1, which are obtained for $l = 5 + 5i$ (i=1, 2, 3,…). Among such numbers z_l, there are certainly prime numbers. In particular, for $l = 10^r$, where r is an arbitrarily large natural number, z can be written in a general form: z = 10...060...01; z contains r zeros before the number 6 and r −1 zeros after 6. For example, for r = 1, z_{10} = 1061(see above), for r = 2, z_{100} = 100601, for r = 3, z_{1000} = 10006001, etc., for r = 57885161, z = 10...060...01; this number contains 57885161 zeros before 6 and 57885160 zeros after 6.

V. Conclusion

Thus, this study allows us to determine all solutions of Fermat quadratic equation and shows the possibility of their use for generation of prime numbers and proof of Goldbach's binary hypothesis and conjecture of Landau.

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