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Geometric decomposition in R^n

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Abstract : *the present article discusses advances in the study of geometry, presenting a way of calculating the geometric decomposition of figures in any dimension. In it is presented a theorem that allows to measure how many regular geometric figures it is possible to decompose by means of a bigger figure. However, this work addresses this decomposition in R^n, which allows its application in topology or cohomology. It is hoped that the results obtained here will allow advances in studies that are being developed in this field of studies.*
Keywords : *Geometry, Topology, Cohomology, Hilbert space, Decomposition.*

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I. Introduction

The study of geometry is fascinating and allows the observation, knowledge of forms, behaviors of nature, their impacts on actions and analyzes of varied phenomena [1]. From Euclidean geometry and the definitions presented by Descartes, these forms are used daily to understand the universe as a whole.

The present work contributes and build another step in this ladder of the revelation of the geometric spaces. Aimed at algebra, it is presented here, concepts that will allow to study the basic structure of construction of a geometric form.

Geometry exists in several planes, from point definitions, limited to the one-dimensional plane, straight lines and polygons, which already exist on a flat surface, polyhedral on three-dimensional surfaces, as well as for larger generalizations of so-called Hilbert spaces [2].

This work proposes a technique that allows the knowledge of a certain concept, immutable and symmetrical that covers its study from a space in R^1 to R^n. The main point is a Theorem that allows you to accurately measure the number of regular figures, which are possible to construct within a given regular figure, in whatever dimension they are.

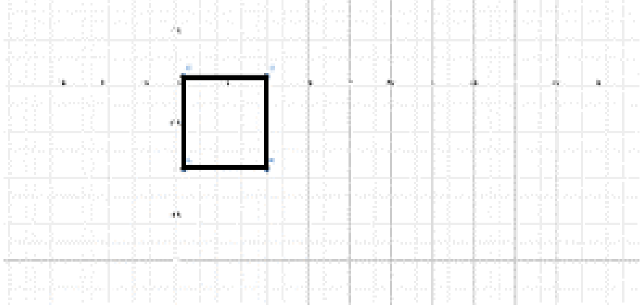
This article will allow advances and contributions within the understanding of Geometry, Topology and Cohomology [3], [4] and [5]. Thus, in the following topics, there are already previous concepts of two articles published by the present author [6] and [7], this application being a utility resulting from the amplitude of their actions.

II. Initial Settings

Initial definitions will be presented here, allowing to understand and enunciate the proposed equation.

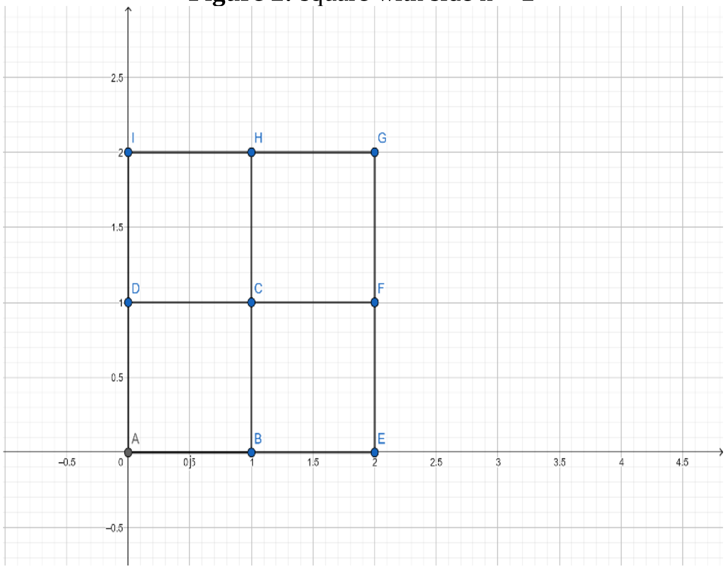
Consider that we have a regular quadrilateral polygon with measure of **n** side, where $n \in \mathbb{N}^*$.Given the initial and possible value for $n = 1$, we have Fig. 1 that represents it.

Figure 1: square with side n = 1



In this case, we observe that in the figure we observe only a square bounded by the edges $\overline{ABCD}$. Next we go to the second option of measuring sides for a square, which will be $n = 2$. Fig. 2 shows this construction.

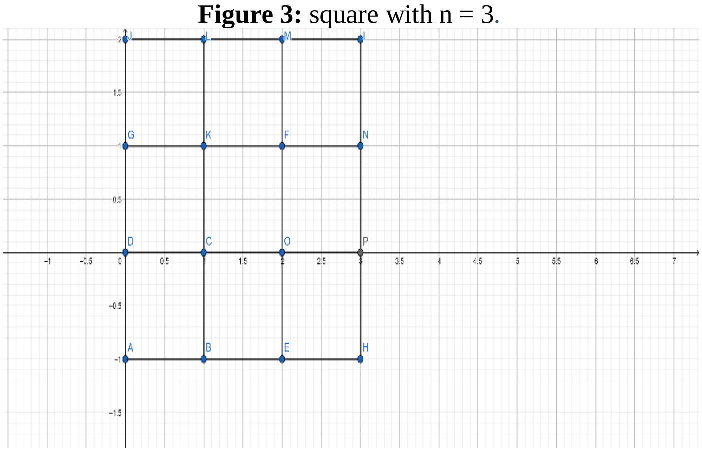
Figure 2: square with side n = 2



In this case, there are some different situations to be analyzed. Note that in addition to the larger square $\overline{AEGI}$, there are 4 smaller ones that are: $\overline{ABCD}, \overline{BCEF}, \overline{FGHC} and \overline{CDHI}$. Therefore, with side measuring 2 units, one can construct 5 squares within the original figure.

Now, Figure 3 will be introduced, which will present a square with side of 3 measurement units.

Figure 3: square with n = 3.



This is the third figure, it presents several configurations of squares that it presents being; the large square of side with 3 units of measure, $\overline{AHIJ}$, we have 4 squares of side n = 2 which are: $\overline{AEFG}, \overline{BHNK}, \overline{CPIL} and \overline{DOMJ}$; we have also the 9 squares of sides n = 1, being $\overline{ABCD}, \overline{BEOC}, \overline{EHPO}, \overline{OPNF}, \overline{COFK}, \overline{DCKG}, \overline{IJGK}, \overline{KFML}, and \overline{IJGK}$. Thus, for n = 3 one can observe 14 possible squares within its structure.

Fig. 4 shows a construction with **n = 4**, where all transformations are possible:

Figure 4: square with n = 4

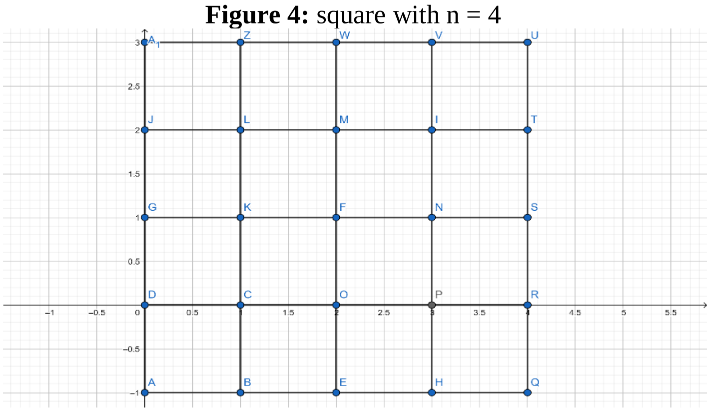


Fig. 4 presents a large square with n = 4, 4 squares with n = 3, 9 squares with n = 2 and 16 squares with n = 1. Thus, in this structure we have 30 possible squares with the permutation of the sides.

In the next item, it will present define theorem that generalizes the concept developed here.

III. General Equation for Perforating Figures

Theorem: $\boldsymbol{Q_n^R = \sum_{i=0}^{n}(n-i)^R}$
Where:
Q → represents the number of squares;
n →is the measure of the side;
R →is the dimension (plane) of the figure.

Returning to the concept presented in Fig. 1 through Fig. 4, which exemplify geometric forms in two dimensions, that is, R^2, when describing each of the parts of these figures, there is an interesting regularity, which will allow generalization is presented as Theorem:

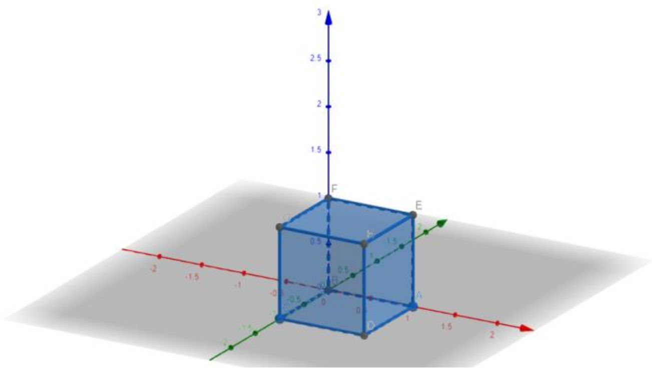
$$n = 1 \rightarrow 0^2 + 1^2 = 1$$
$$n = 2 \rightarrow 0^2 + 1^2 + 2^2 = 5$$
$$n = 3 \rightarrow 0^2 + 1^2 + 2^2 + 3^2 = 14$$
$$n = 4 \rightarrow 0^2 + 1^2 + 2^2 + 3^2 + 4^2 = 30$$
$$\vdots$$
$$n = n \rightarrow (n-n)^2 + (n-(n-1))^2 + (n-(n-2))^2(+\cdots+(n-0))^2$$
$$\vdots$$
$$n = n \rightarrow 0^2 + 1^2 + 2^2 + \cdots + n^2$$
$$\vdots$$
$$n = n \rightarrow \mathrm{Q_n^R} = \sum_{\mathrm{i=0}}^{\mathrm{n}}(\mathrm{n-i})^{\mathrm{R}} \qquad (1)$$

Where **Q** represents the number of regular figures, it can be obtained with the larger figure, **n** is the value referring to the side of the figure to be investigated, **R** refers to the dimension in which the figure is.

IV. Generalizations for 3 Dimensions

It can now be seen that the **theorem (1)** presented in the previous section extends its application to other dimensions, so its application for the third dimension will be presented here. The approach will be similar to the principle presented earlier. Fig. 5 shows this representation:

Figure 5: Side cube n = 1



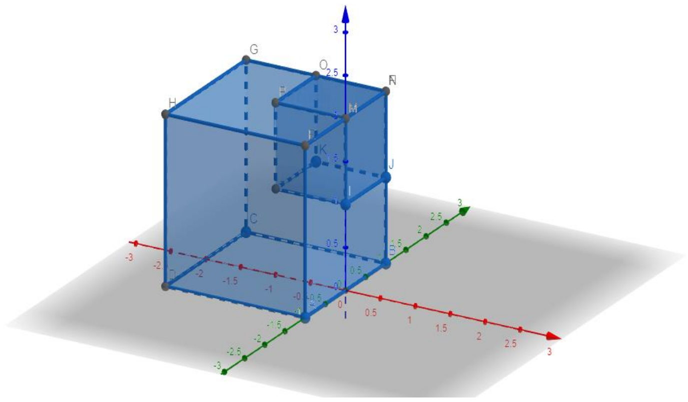
Now, using **Theorem (1)** will calculate the number of cubes that can be generated with this figure. Note theo perations:

$n = 1, \;\; R = 3, \;\; soon:$

$$\mathrm{Q_n^R} = \sum_{\mathrm{i=0}}^{\mathrm{n}}(\mathrm{n-i})^{\mathrm{R}} \rightarrow \mathrm{Q_1^3} = \sum_{\mathrm{i=0}}^{1}(1-\mathrm{i})^3$$
$$\mathrm{Q_1^3} = (1-0)^3 + (1-1)^3 \;\rightarrow\; \mathrm{Q_1^3} = (1)^3 + (0)^3 \;\rightarrow\; \mathrm{Q_1^3} = 1$$

See that **Theorem (1)** easily solves this condition. Fig. (6) will present a cube with a measurement side larger than the previous one. Watch:

Figure 6: Cube with side n = 2



This figure shows two possible types of cubes, when one has a larger one with side **n = 2** and another inside this figure other smaller cubes, with side **n = 1**. Thus with **Theorem (1)** it is shown how many cubes exist besides the larger cube. Note the development:

$n = 2, \;\; R = 3, \;\; soon$:

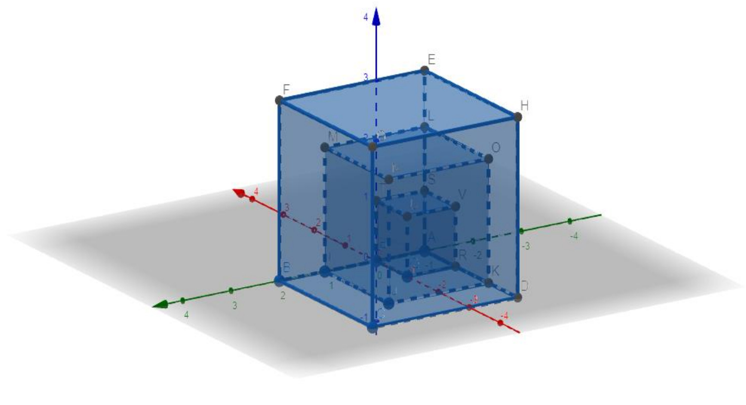
$$\mathrm{Q}_{\mathrm{n}}^{\mathrm{R}} = \sum_{\mathrm{i}=0}^{\mathrm{n}} (\mathrm{n} - \mathrm{i})^{\mathrm{R}} \rightarrow \mathrm{Q}_2^3 = \sum_{\mathrm{i}=0}^{2} (2 - \mathrm{i})^3$$

$$\mathrm{Q}_2^3 = (2-0)^3 + (2-1)^3 + (2-2)^3 \;\rightarrow\; \mathrm{Q}_1^3 = (2)^3 + (1)^3 + (0)^3 \;\rightarrow\; \mathrm{Q}_1^3 = 9$$

It is possible to see that if there are 9 cubes formed with this figure, there will be 8 stacked cubes of side n = 1 that form a larger cube of side n = 2. Thus the Theorem (1) presents this value easily.

Now another example shown by Fig. 7, which deals with a cube of side 3 units of measurements. See the figure:

Figure 7: side cube n = 3.



Using Theorem (1), see:

$n = 3, \;\; R = 3, \;\; soon$:

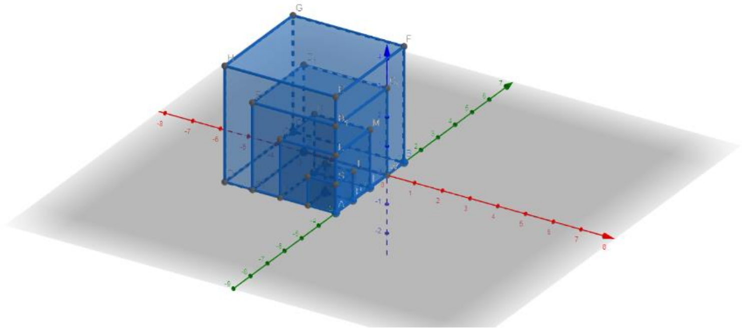
$$\mathrm{Q}_{\mathrm{n}}^{\mathrm{R}} = \sum_{\mathrm{i}=0}^{\mathrm{n}} (\mathrm{n} - \mathrm{i})^{\mathrm{R}} \rightarrow \mathrm{Q}_3^3 = \sum_{\mathrm{i}=0}^{3} (3 - \mathrm{i})^3$$

$$\mathrm{Q}_3^3 = (3-0)^3 + (3-1)^3 + (3-2)^3 + (3-3)^3 \;\rightarrow\; \mathrm{Q}_3^3 = (3)^3 + (2)^3 + (1)^3 + (0)^3 \;\rightarrow\; \mathrm{Q}_1^3 = 36$$

Theorem (1) presents 27 cubes of sides n = 1, 8 cubes of side n = 2 and 1 cube of side n = 3. Thus, the sum of all possible cubes with a three-dimensional shape with n = 3 side is 36.

We present a last visual example that allows us to understand a figure with n = 4. Fig. 8 shows this resolution:

Figure 08: Side cube n = 4



With Theorem (1) we have:

$n = 4, \ \ R = 3, \ \ soon$:

$$\mathrm{Q}_{\mathrm{n}}^{\mathrm{R}} = \sum_{\mathrm{i}=0}^{\mathrm{n}} (\mathrm{n}-\mathrm{i})^{\mathrm{R}} \rightarrow \mathrm{Q}_3^3 = \sum_{\mathrm{i}=0}^{4} (4-\mathrm{i})^3$$
$$\mathrm{Q}_4^3 = (4-0)^3 + (4-1)^3 + (4-2)^3 + (4-3)^3 + (4-4)^3$$
$$\mathrm{Q}_4^3 = (4)^3 + (3)^3 + (2)^3 + (1)^3 + (0)^3 = 100$$

Thus, there are 64 cubes of n = 1, 27 cubes of n = 2, 8 cubes of n = 3 and 1 cube of n = 4, making a total of 100 possible cubes within the figure.

The generalizations can expand to other dimensions, allowing to advance concepts of the Dictionary of Descartes, described by Santoy [2]. It is possible to find the coordinates of a figure in another dimension above the three-dimensional. In this case, it is observed a way to expand these concepts with Theorem (1), making it possible to understand the quantity of other figures, inserted in the larger figure, in any dimension. We will see two more cases that illustrate these possibilities.

How many figures can be constructed with a figure of 3 units of measure aside in the space of 4 dimensions?

$n = 3, \ \ R = 4, \ \ soon$:

$$\mathrm{Q}_{\mathrm{n}}^{\mathrm{R}} = \sum_{\mathrm{i}=0}^{\mathrm{n}} (\mathrm{n}-\mathrm{i})^{\mathrm{R}} \rightarrow \mathrm{Q}_3^4 = \sum_{\mathrm{i}=0}^{3} (3-\mathrm{i})^4$$
$$\mathrm{Q}_3^4 = (3-0)^4 + (3-1)^4 + (3-2)^4 + (3-3)^4$$
$$\mathrm{Q}_3^4 = (3)^4 + (2)^4 + (1)^4 + (0)^4 = 98$$

Thus there are 98 4-dimensional figures within a figure with n = 3. In the next condition is to explore the concepts and check the following question: in a figure of side n = 5, being in a space of 6 dimensions, how many other figures can be occupied by their space, in the same dimension?

$n = 5, \ \ R = 6, \ \ soon$:

$$\mathrm{Q}_{\mathrm{n}}^{\mathrm{R}} = \sum_{\mathrm{i}=0}^{\mathrm{n}} (\mathrm{n}-\mathrm{i})^{\mathrm{R}} \rightarrow \mathrm{Q}_5^6 = \sum_{\mathrm{i}=0}^{5} (5-\mathrm{i})^6$$
$$\mathrm{Q}_5^6 = (5-0)^6 + (5-1)^6 + (5-2)^6 + (5-3)^6 + (5-4)^6 + (5-5)^6$$
$$\mathrm{Q}_5^6 = (5)^6 + (4)^6 + (3)^6 + (2)^6 + (1)^6 + (0)^6 = 20515$$

This result presents the force of action of Theorem (1), which together with the Descartes Dictionary, can be used by people to advance their studies within the topology and Cohomology of geometric forms..

V. Conclusion

The structure presented in Theorem (1) allows us to observe how the geometric shapes behave within the construction of a basic unit. We could consider the structures described here as being a geometric DNA, very similar to the decomposition in prime factors used for natural numbers.

Nevertheless, in finding all the building blocks of a geometric figure, we are making possible an analysis of all the factors that can construct it, not being limited to the specific dimension.

Future authors will be able to advance the works described here, as well as to use them as tools in researches they are developing. Academic and academic applications will enable us to gain a greater understanding of how the universe presents itself to its observers.

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