Slip Hydromagnetic Radiating Flow In Wavy Isothermal Channel With Porous Medium Considering Joule And Frictional Heating

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Abstract: The problem of hydromagnetic mixed convection heat transfer across a porous medium due to buoyancy force, difference in temperature, the presence of pressure gradient and radiation is considered between two finitely long vertical isothermal walls, one of which is wavy. The presence of the heat source and combined effects of viscous dissipation and Joule heating in slip flow regime is taken into account. The resulting nonlinear dimensionless governing equations are linearized by perturbation technique and then solved numerically using the software MAPLE. As a result, the dimensionless velocity, temperature, skin friction coefficient and the Nusselt number are illustrated through the graphs and discussed for different physical parameters.

Keywords: Frictional heating, hydromagnetic slip flow, isothermal wavy channel, Joule heating, porous medium.

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I. Introduction

Investigation of flow of viscous fluid over or through wavy wall(s), in both horizontal and vertical orientations, has been of much interest to scholars because of its relevance to physical problems (see for example, Vajravelu and Sastri [1] and the literature cited therein), for different wall temperature conditions, where the flow and transfer processes are limited to buoyancy effect of thermal diffusion. Since then, many scholars have extended the work on the same geometry; Das and Ahmed [2] and Fasogbon [3] account for the presence of an external transverse magnetic field which is applied uniformly with constant heat source or sink, Taneja and Jain [4] examined the slip effects on hydromagnetic flow in presence of temperature-dependent heat source, Fasogbon and Omolehin [5] examined the radiation effect, Fasogbon [6] analysed the combined effects of thermal and mass diffusion in the presence of uniform internal heat source or sink, Devika et al. [7] discussed steady chemical reaction effects on MHD free convection flow through porous medium, Ahmed et al. [8] carried out the Soret and Dufour effects on an MHD free convective flow, Soret and Dufour effects on steady free convective MHD viscoelastic fluid flow was studied by Das [9], Amini et al. [10] considered effects of wall waviness and temperature variations on the fluid flow and natural convection, and most recently, Gbadeyan et al. [11] studied the problem of coupled heat and mass transfer by free convection of a chemically-reacting viscous incompressible and electrically conducting fluid in the presence of diffusion-thermo, thermal-diffusion and constant internal heat source or sink. This present work is primarily motivated by the flow of a viscous fluid over and through a porous medium. This is due to its natural occurrence in the movement of water and oil inside the earth and the flow of rivers through porous banks. Hydromagnetic laminar flow through a porous medium is also relevant for its wide range of application in many branches of science and technology, chemical engineering and petroleum technology (cf. Ravikumar et al. [12]).

Most of the previous studies of the flows of heat or heat and mass convection have focused mainly on no-slip boundary condition where the fluid velocity takes the velocity of the solid boundary. However, the non-adherence of the fluid to a solid boundary known as velocity slip, has been observed under certain practical problems, such as the walls of air-craft and rockets moving at very high altitude. When the particles adjacent to the wall no longer take the velocity of the wall but possess a finite tangential velocity which slips along the wall. The flow regime thus induced is referred to as slip flow regime, and its effect is relevant in this time of modern science, technology and fast growing industrialization. Other vital practical applications are where a thin film of light oils is attached to the moving walls or when the wall is coated with special coating such as thick monolayer of hydrophobic mechanical device where a thin of lubricant is attached to the wall slipping over one another or when the walls are coated with special coating to minimize the friction between them, and others abound. Another subject of interest is the combined action of viscous dissipation and Joule heating on the hydromagnetic flow and heat transfer for its application on heat exchanger designs, wire and glass fiber drawing and in nuclear engineering in connection with the cooling of reactors. Another vital application is in various devices which are subjected to
large variations of gravitational force. In view of these applications, Chauhan and Agrawal [13] presented hydrodynamic convection effects with viscous and Ohmic dissipation in a vertical channel partially filled by a porous medium. Jaber [14] investigated Joule heating and viscous dissipation effects on hydromagnetic flow over stretching porous sheet subjected to power law heat flux in presence of heat source. Ohmic heating and viscous dissipation effects over a vertical plate in the presence of porous medium was performed by Loganathan and Sivapoomanpria [15]. To the best of our knowledge, combined effects of viscous dissipation and Joule heating on free and force convective through a porous medium which is bounded by finitely long vertical isothermal walls (see [1-11]) and the present work demonstrate the issue.

II. Formulation Of The Problem

A steady, laminar, two-dimensional hydromagnetic mixed convection flow of a viscous incompressible electrically conducting dissipative fluid which is an optically thin non gray gas through a porous medium between two finitely long vertical walls of distance \( d \) apart is considered. A Cartesian co-ordinates system where X-axis is in the upward direction; direction of flow, and the axis of \( Y \) normal to it is chosen. The hot wavy wall at \( Y = 0 \) is sinusoidal \( Y = \epsilon \cos(k_x X) \) and has a uniform temperature \( T_1 \) while the wall at \( Y = d \) is cold smooth flat subjected to slip velocity \( u_s' \) and a uniform temperature \( T_0 \), where \( u_s' \) is proportional to the local wall shear stress and is given by \( u_s' = L', \frac{\partial u}{\partial y'} \) and \( L' = \left(\frac{2 - m_1}{m_1}\right) L \). Orthogonal to the channel walls is a uniform magnetic field of strength \( B_0 \). Properties of the fluid are assumed constant except that density change with temperature that brings about the buoyancy forces in a manner corresponding to the equation of state \( \rho_v = \rho \left(1 + \beta(T - T_v)\right) \). There is no applied magnetic field in the \( Y \)-direction and magnetic Reynold’s number is much less than unit, so that Hall, ion slip current and induced magnetic field are ignored. \( \nabla \cdot B = 0 \) (Gauss’s law of magnetism) gives \( B_y = \text{constant} = B_0 \) in the flow, thus \( B = (0, B_0, 0) \) and Lorentz force \( \vec{F} = \vec{J} \times \vec{B} = \sigma_e \left(\vec{u} \times \vec{B}\right) \times \vec{B} = -\sigma_e B_0^2 u_v, \nabla \times \vec{B} = \vec{J}, \nabla \cdot \vec{E} = 0 \) and \( \vec{J} = \sigma_e \vec{u} \times \vec{B} \) (Ohm’s law).

Omitted again are Maxwell currents displacement and free charges. Now, we assume the fluid is in the optically thin limit; the fluid does not absorb its own radiation but it only absorbs radiations emitted by the boundaries. Accordingly, Cogley et al. [16] showed that the relation \( \frac{\partial q_v}{\partial r} = 4I(T - T_v) \)

where \( I = \int_0^r \alpha_e \left(\frac{\partial B_0}{\partial T}\right) d\lambda \), evaluated at the temperature \( T_v \) holds.

The equations which govern the problem when the velocity and the temperature are functions of \( X \) and \( Y \) are in the dimensionless form as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial (P_0 - P)}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + G_\theta \theta - \frac{1}{A} u - Mu
\]

(2)

\[
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - \frac{1}{A}
\]

(3)

\[
p_v \left( u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} - R\theta + S\theta + je(\epsilon c) u^2 + p_v(\epsilon c) \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + 2 \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} - 4 \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \right)
\]

(4)

\[A = \frac{k d^2}{\rho v}, M = \frac{\sigma_e B_0^2 d^2}{\rho v}, G_v = \frac{d^3 g \beta (T_w - T_v)}{v^2}, je = \frac{e_j^2 B_0^2 d^2 c_p}{k}, e_c = \frac{v^2}{c_p (T_w - T_v) d^2}
\]

The above equations were non-dimensionalized upon using dimensionless variables.

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\[(x, y) = \frac{1}{d}(X, Y), (u, v) = \frac{d}{v}(u', v'), \theta = \frac{T - T_e}{T_w - T_e}, T_w > T_e, P = \frac{P'd^2}{\rho v^2}, P_e = \frac{P'e^2}{\rho v^2}, \lambda = k_w d, \]
\[
\varepsilon = \frac{\varepsilon^*}{d}, p_r = \frac{\mu c_p}{k}, S = \frac{Qd^2}{k}, n = \frac{T_1 - T_e}{T_w - T_e}, R = \frac{4Id^2}{k}
\]

The boundary conditions for the present problem are:
\[u = 0, v = 0, \theta = 1 \text{ at } y = \varepsilon \cos(\lambda x) \]

\[u = h \frac{\partial u}{\partial y}, v = 0, \theta = n \text{ at } y = 1 \]

III. Solution Of The Problem

To solve the equations (1) to (4), we assume
\[u(x, y) = u_0(y) + \varepsilon u_1(x, y), v = \varepsilon v_1(x, y) \]
\[p(x, y) = p_0(y) + \varepsilon p_1(x, y) \]
\[\theta(x, y) = \theta_0(y) + \varepsilon \theta_1(x, y) \]

where the zeroth- and perturbed quantities denoted by the subscripts 0 and 1 respectively.

Substituting the equation (6) in equations (1) to (4) and equating $\varepsilon^2$, we obtain
\[u_0''(y) - \left(M + \frac{1}{A}\right)u_0(y) = k_p - G_r \theta_0(y) \]
\[\theta_0''(y) - (R - S) \theta_0(y) + je(ec)\left(u_0(y)\right)^2 + p_r(ec)\left(u_0'(y)\right)^2 = k_p - G_r \theta_0(y) \]

Satisfying
\[u_0 = 0, \theta_0 = 1, \text{ on } y = 0 \quad u_0 = h u_0'(y), \theta_0 = n, \text{ on } y = 1 \]
\[u_{l,x} + v_{l,y} = 0 \]
\[u_0u_{l,x} + v_0u_{l,y} = -P_{l,x} + u_{l,xx} + u_{l,xy} \frac{1}{A}u_l + G_s \theta_l - Mu_l \]
\[u_0v_{l,x} = -P_{l,y} + v_{l,xx} + v_{l,xy} \frac{1}{A}v_l \]
\[p_r\left(u_0\theta_{l,x} + v_0\theta_{l,y}\right) = \theta_{l,xx} + \theta_{l,xy} - \frac{1}{A}u_l - R\theta_l - S\theta_l + 2je(ec)u_0u_l + 2p_r(ec)u_0'(u_l,v_l) \]

subject to the boundary conditions
\[u_0 = -u_0, v_1 = 0, \theta_l = -\theta_l'(y), \text{ on } y = 0 \]
\[u_l = h u_l'(y), v_1 = 0, \theta_l = 0, \text{ on } y = 1 \]

To solve (10) - (14), we introduce a stream function
\[\Psi_1 \]
\[u_l = -\Psi_{l,x}, v_1 = \Psi_{l,y} \]

so that (10) can be satisfied identically and $P_1$ eliminated. We then assume wave-like solutions of the form

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\[ \Psi'(x, y) = \epsilon e^{i\lambda \psi(x, y)} \] (16)

\[ \theta'(x, y) = \epsilon e^{i\lambda \phi(x, y)} \] (17)

So that (15) becomes

\[ u_i(x, y) = -\epsilon e^{i\lambda \psi'}(x, y) \quad v_i(x, y) = \epsilon e^{i\lambda \psi}(x, y) \]

and equations in terms of stream function reduce to

\[ \psi'' + \lambda^2 \left( \frac{1}{A} \psi - 2\psi'' \right) - \frac{1}{A} \psi'' - M \psi'' - G_i \psi' = i\lambda \left( u_0 \psi'' - u_0'' \psi \right) \]

\[ \phi'' - R\phi + S\phi + 2je(ec)u_0\psi' + 2p_r(ec)u_0' \left( -\psi'' - \lambda^2 \psi \right) - \lambda^2 \phi = i\lambda p_r \left( u_0\phi + \theta_0 \psi' \right) \]

where \( i \) is the complex unit. If we further assume that \( \lambda \) is much less than unity or \( k_w << 1 \), we write

\[ \psi(\lambda, y) = \sum_{j=0}^{\infty} \lambda^j \psi_j \]

\[ \psi(\lambda, y) = \sum_{j=0}^{\infty} \lambda^j \psi_j \quad j = (0, 1, 2, \ldots) \]

Putting (19) into (17) and (18) in terms of \( \lambda \) and comparing the like-power terms of \( \lambda \) to the order of \( \lambda^2 \), we have overall set of ordinary differential equations and the boundary conditions for this work:

\[ \frac{d^4}{dy^4} \psi_0(y) - \left( M + \frac{1}{A} \right) \left( \frac{d^2}{dy^2} \psi_0(y) \right) = G_r \left( \frac{d}{dy} \phi_0(y) \right) \] (20)

\[ \frac{d^2}{dy^2} \phi_0(y) - (R - S) \phi_0(y) + 2je ec u_0(y) \left( \frac{d}{dy} \psi_0(y) \right) - 2 p_r \left( \frac{d}{dy} u_0(y) \right) \left( \frac{d^2}{dy^2} \psi_0(y) \right) = 0 \] (21)

\[ \frac{d^4}{dy^4} \psi_1(y) - \left( M + \frac{1}{A} \right) \left( \frac{d^2}{dy^2} \psi_1(y) \right) = G_r \left( \frac{d}{dy} \phi_1(y) \right) + u_0(y) \left( \frac{d^2}{dy^2} \psi_0(y) \right) - \left( \frac{d^2}{dy^2} u_0(y) \right) \psi_0(y) \] (22)

\[ \frac{d^2}{dy^2} \phi_1(y) - (R - S) \phi_1(y) + 2je ec u_0(y) \left( \frac{d}{dy} \psi_1(y) \right) - 2 p_r \left( \frac{d}{dy} u_0(y) \right) \left( \frac{d^2}{dy^2} \psi_1(y) \right) = u_0(y) \phi_0(y) + \psi_0(y), \left( \frac{d}{dy} \theta_0(y) \right) \] (23)

\[ \frac{d^4}{dy^4} \psi_2(y) - \left( M + \frac{1}{A} \right) \left( \frac{d^2}{dy^2} \psi_2(y) \right) + \frac{\psi_0(y)}{A} = 2 \left( \frac{d^2}{dy^2} \psi_0(y) \right) + G_r \left( \frac{d}{dy} \phi_2(y) \right) - u_0(y) \left( \frac{d^2}{dy^2} \psi_1(y) \right) + \left( \frac{d^2}{dy^2} u_0(y) \right) \psi_1(y) \] (24)
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\[
\frac{d^2}{dy^2} \phi_2(y) - (R - S) \phi_2(y) + 2jej \psi_0(y) \left( \frac{d}{dy} \psi_2(y) \right) \\
- 2 P_r ec \left( \frac{d}{dy} u_0(y) \right) \left( \frac{d^2}{dy^2} \psi_2(y) \right) - 2 P_r ec \left( \frac{d}{dy} u_0(y) \right) \psi_0(y) = \phi_0(y) \\
- u_0(y) \phi_1(y) + \psi_1(y) \left( \frac{d}{dy} \theta_0(y) \right)
\]

The solutions for \( u \) and \( \theta \) have been obtained, and the physical quantities of interest at any point in the field; skin friction \( C_f \) and Nusselt number \( N_u \) emerged but are not presented here for the sake of brevity. From equations (6), (15), (16) and (19), after obvious simplification, on the assumption that 

\( u(y) = u_0(y) + \varepsilon \lambda_0 \psi_1(y) \)
\( \theta(y) = \theta_0(y) - \varepsilon \lambda \phi_1(y) \)
\( c_f(y) = u_0(y) + \varepsilon \lambda \psi_1''(y) \)
\( N_u(y) = \theta_0(y) - \varepsilon \lambda \phi_1'(y) \)

IV. Results And Discussion

To study the combined influence of friction and ohmic heating on the hydromagnetic free and force convection on heat transfer through a porous medium in finitely long vertical isothermal wavy channel, the dimensionless velocity \( u \), the temperature \( \theta \), skin friction coefficient \( C_f \) and Nusselt number \( N_u \) are illustrated graphically against \( y \) for selected values of physical parameters. Here we took the geometric parameters \( \lambda x = \frac{\pi}{2} \), \( \lambda = 0.01 \), and \( \varepsilon = 0.025 \), \( P_r = 0.71 \), \( h = 0 / 0.2 \), \( k_p = 0 / 2 \) for no slip/slip condition, means absence/presence of pressure gradient. The default parameters used throughout the numerical calculations, unless otherwise stated are: \( G_r = 5 \), \( A = 0.3 \), \( P_r = 0.71 \), \( h = 0.2 \), \( n = 0.4 \), \( ec = 0.5 \), \( je = 0.5 \), \( M = 3 \), \( k_p = 2 \), \( R = 3 \), \( S = 2 \)

Fig 1. Variation of \( S \) on \( u \) against \( y \) when \( k_p=0 \)  
Fig 2. Variation of \( S \) on \( u \) against \( y \) when \( k_p=2 \)
We set the values $\varepsilon$, $\gamma$, $A$, $R$, $M$, to zero and $\eta=0.4$, $A=1$ and $kp=0, 2$ introduced in the present study and obtained the results in Fig. 1-5. The results compared with the famous work of Vajravelu ans Sastri [1] and found to be in good agreement.
The study reveals that the rate of heat transfer is most effective in a sinusoidal wavy isothermal wall than in an isothermal smooth flat wall. In mixed convection, the influence of buoyancy forces on a forced flow or the influence of forced flow on a buoyant flow is significant, hence, immensely affected by increasing pressure gradient.

V. Conclusion

Fig. 6 and 7 showed that viscous and Joule heating has very slight influence on velocity profile both in no-slip and slip condition.

VI. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Permeability parameter</td>
</tr>
<tr>
<td>$B_0$</td>
<td>Intensity of the applied magnetic field</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>d</td>
<td>Distance between the walls</td>
</tr>
<tr>
<td>ec</td>
<td>Eckert number</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>$G_r$</td>
<td>Grashof number</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$q_r$</td>
<td>Radiation heat flux</td>
</tr>
<tr>
<td>R</td>
<td>Radiation parameter</td>
</tr>
<tr>
<td>S</td>
<td>Heat source/sink parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Thermal expansion coefficient</td>
</tr>
</tbody>
</table>

References


Fig 6. Variation of ec on u against y when kp=2

Fig 7. Variation of ec on theta against y when kp=2

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h  Velocity slip parameter \( h = \frac{L'}{d} \)  
\( \rho_e \)  density of the fluid in the equilibrium state  
\( \rho \)  Fluid density  
\( \mu \)  Dynamic viscosity  
\( \nu \)  Kinematic viscosity  
\( m_i \)  Maxwell’s reflection coefficient  
\( L \)  mean free path  
\( k_w \)  wave number  
\( \lambda \)  Wavelength  
\( \alpha_s \)  mean absorption coefficient  
je  Joule heating  
k  Thermal conductivity  
k'  Permeability of the porous medium  
k_p  Pressure gradient  
M  magnetic field parameter  
N  Temperature difference ratio  
P  pressure  
T  Fluid temperature  
\( T_w \)  Temperature of the wavy wall  
\( T_i \)  Temperature of the smooth wall  
\( T_{eq} \)  Temperature in the equilibrium state  
(\( u, v \))  Dimensionless velocity components  
(\( u', v' \))  velocity components in (\( x', y' \)) direction  
(X, Y)  Coordinate system  
(x, y)  dimensional coordinate  