On Decomposition of \( \beta g^* \) Closed Sets in Topological Spaces

C.Dhanapakyam, K.Indirani
Department of Mathematics Rathnavel subramanian College of Arts & Science Coimbatore-, India
Nirmala College for women Red fields,Coimbatore-,India
Corresponding Author: C.Dhanapakyam

Abstract: The aim of this paper is to introduce and study the classes of \( \beta g^* \)-locally closed set and different notions of generalization of continuous functions namely \( \beta g^*lc \)-continuity, \( \beta g^*lc^* \)-continuity and \( \beta g^*lc^{**} \)-continuity and their corresponding irresoluteness were studied.

Keywords: \( \beta g^* \)-separated, \( \beta g^*-dense \), \( \beta g^*-submaximal \), \( \beta g^*lc \)-continuity, \( \beta g^*lc^* \)-continuity \( \beta g^*lc^{**} \)-continuity.

Date of Submission: 02-07-2018               Date of acceptance: 18-07-2018

I. Introduction:
The first step of locally closedness was done by Bourbaki [2]. He defined a set \( A \) to be locally closed if it is the intersection of an open and a closed set. In literature many general topologists introduced the studies of locally closed sets. Extensive research on locally closedness and generalizing locally closedness were done in recent years. Stone [7] used the term LC for a locally closed set. Ganster and Reilly used locally closed sets in [4] to define LC-continuity and LC-irresoluteness. Balachandran et al [1] introduced the concept of generalized locally closed sets. The aim of this paper is to introduce and study the classes of \( \beta g^* \) locally closed set and different notions of generalization of continuous functions namely \( \beta g^*lc \)-continuity, \( \beta g^*lc^* \)-continuity and \( \beta g^*lc^{**} \)-continuity and their corresponding irresoluteness were studied.

II. Preliminary Notes
Throughout this paper \((X,\tau),(Y,\sigma)\) are topological spaces with no separation axioms assumed unless otherwise stated. Let \( A \subseteq X \). The closure of \( A \) and the interior of \( A \) will be denoted by \( Cl(A) \) and \( Int(A) \) respectively.

Definition 2.1. A Subset \( S \) of a space \((X,\tau)\) is called
(i) locally closed (briefly lc) \([6]\) if \( S=U\cap F \), where \( U \) is open and \( F \) is closed in \((X,\tau)\).
(ii) \( r \)-locally closed (briefly rlc) \( [6] \) if \( S=U\cap F \), where \( U \) is \( r \)-open and \( F \) is \( r \)-closed in \((X,\tau)\).
(iii) generalized locally closed (briefly glc) \( [1] \) if \( S=U\cap F \), where \( U \) is \( g \)-open and \( F \) is \( g \)-closed in \((X,\tau)\).

Definition 2.2. \([4]\) A subset \( A \) of a topological space \((X,\tau)\) is called \( \beta g \)-closed if \( gcl(A)\subseteq U \) whenever \( A\subseteq U \) and \( U \) is \( \beta \)-open subset of \( X \).

Definition 2.3. For a subset \( A \) of a space \( X \), \( \beta g^*-cl(A)=\cap \{F: A\subseteq F, F \text{ is } \beta g^* \text{ closed in } X \} \) is called the \( \beta g^* \)-closure of \( A \).

Remark 2.4. For a topological space \((X,\tau),(Y,\sigma)\), the following statements hold:
(1) Every closed set is \( \beta g^* \)-closed but not conversely \([4]\).
(2) Every \( g \)-closed set is \( \beta g^* \)-closed but not conversely \([4]\).
(3) Every \( g^* \)-closed set is \( \beta g^* \)-closed but not conversely \([4]\).
(4) A subset \( A \) of \( X \) is \( \beta g^* \)-closed if and only if \( \beta g^*-cl(A)=A \).
(5) A subset \( A \) of \( X \) is \( \beta g^* \)-open if and only if \( \beta g^*-int(A)=A \).

Corollary 2.5. If \( A \) is a \( \beta g^* \)-closed set and \( F \) is a closed set, then \( A\cap F \) is a \( \beta g^* \)-closed set.

Definition 2.6. \([5]\): A function \( f:(X,\tau)\rightarrow(Y,\sigma) \) is called \( \beta g^* \) continuous if \( f^\tau(V) \) is \( \beta g^* \)-closed subset of \( (X,\tau) \) for every closed subset \( V \) of \( (Y,\sigma) \).

Definition 2.7. A function \( f:(X,\tau)\rightarrow(Y,\sigma) \) is called
i) LC-continuous \([6]\) if \( f^\tau(V)\subseteq LC(X,\tau) \) for every \( V\in\sigma \).
ii) GLC-continuous \([1]\) if \( f^\tau(V)\subseteq GLC(X,\tau) \) for every \( V\in\sigma \).

Definition 2.8. A subset \( S \) of a space \((X,\tau)\) is called
(i) submaximal \([3]\) if every dense subset is open.
(ii) \( g \)-submaximal \([1]\) if every dense subset is \( g \)-open.

DOI: 10.9790/5728-1404014751 www.iosrjournals.org 47 | Page
On Decomposition of $\beta g^*$ Closed Sets in Topological Spaces

III. $\beta g^*$ Locally Closed Set

**Definition 3.1:** A subset $A$ of $(X, \tau)$ is said to be $\beta g^*$ locally closed set (briefly $\beta g^*lc$) if $A=L\cap M$ where $L$ is $\beta g^*$-open and $M$ is $\beta g^*$-closed in $(X, \tau)$.

**Definition 3.2:** A subset $A$ of $(X, \tau)$ is said to be $\beta g^*lc$ set if there exists a $\beta g^*$-open set $L$ and a closed set $M$ of $(X, \tau)$ such that $A=L\cap M$.

**Definition 3.3:** A subset $B$ of $(X, \tau)$ is said to be $\beta g^*lc^*$ set if there exists an open set $L$ and a $\beta g^*$-closed set $M$ such that $A=L\cap M$.

The class of all $\beta g^*lc$ (resp. $\beta g^*lc^*$ & $\beta g^*lc^{**}$) sets in $X$ is denoted by $\beta g^*LC(X)$ (resp. $\beta g^*LC^*(X)$ & $\beta g^*LC^{**}(X)$).

From the above definitions we have the following results.

**Proposition 3.4**

i) Every locally closed set is $\beta g^*$-lc.
ii) Every glc-set is $\beta g^*$-lc.
iii) Every $g^*lc$-set is $\beta g^*$-lc.
iv) Every $\beta g^*$-lc set is $\beta g^*$-lc.
v) Every glc-set is $\beta g^*$-lc.
vii) Every $\beta g^*lc^*$-set is $\beta g^*lc^{**}$.
viii) Every $\beta g^*lc^{**}$-set is $\beta g^*lc^{***}$.

However the converses of the above are not true as seen by the following examples

**Example 3.5.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},X\}$. Then $A=\{a\}$ is $\beta g^*$-lc-set but not locally closed.

**Example 3.6.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},X\}$. Then $A=\{a\}$ is $\beta g^*$-lc-set but not glc-set.

**Example 3.7.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},X\}$. Then $A=\{a\}$ is $\beta g^*$-lc-set but not $g^*lc$-set.

**Example 3.8.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},X\}$. Then $A=\{a\}$ is $\beta g^*$-lc-set but not $\beta g^*lc^*$-set.

**Example 3.9.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},X\}$. Then $A=\{a\}$ is $\beta g^*lc$ set but not glc-set.

**Example 3.10.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},X\}$. Then $A=\{a\}$ is $\beta g^*lc^*$-set but not rlc-set.

**Example 3.11.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},X\}$. Then $A=\{a\}$ is $\beta g^*lc^*$-set but not $\beta g^*lc^{**}$-set.

**Remark 3.12.** In example 3.8, Let $A=\{b\}$ is $\beta g^*lc^*$-set but not $\beta g^*lc^{**}$-set.

**Remark 3.13.** The concepts of $\beta g^*lc$ set and $\beta g^*lc^{**}$ sets are independent of each other as seen from the following example.

**Example 3.14.** In example 3.6, Let $A=\{b,c\}$ is $\beta g^*lc^*$-set but not $\beta g^*lc^{**}$-set and Let $A=\{a\}$ is $\beta g^*lc^{**}$-set but not $\beta g^*lc^*$-set.

**Remark 3.15.** Union of two $\beta g^*lc$-sets are $\beta g^*lc$-sets.

IV. $\beta g^*$-DENSE SETS AND $\beta g^*$-SUBMAXIMAL SPACES

**Definition 4.1.** A subset $A$ of $(X, \tau)$ is called $\beta g^*$-dense if $\beta g^-cl(A)=X$.

**Example 4.2.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},\{b,c\},\{a,b,c\},X\}$. Then the set $A=\{a,b,c\}$ is $\beta g^*$-dense in $(X, \tau)$.

Recall that a subset $A$ of a space $(X, \tau)$ is called dense if $cl(A)=X$.

**Proposition 4.3.** Every $\beta g^*$-dense set is dense.

Let $A$ be a $\beta g^*$-dense set in $(X, \tau)$. Then $\beta g^-cl(A)=X$. Since $\beta g^-cl(A)\subseteq cl(A)\subseteq cl(A)$. We have $cl(A)=X$ and so $A$ is dense.

The converse of the above proposition need not be true as seen from the following example.

**Example 4.4.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{b\},\{c\},\{b,c\},X\}$. Then the set $A=\{b,c\}$ is a dense in $(X, \tau)$ but it is not $\beta g^*$-dense in $(X, \tau)$.

**Definition 4.5.** A topological space $(X, \tau)$ is called $\beta g^*$-submaximal if every dense subset in it is $\beta g^*$-open in $(X, \tau)$.

**Proposition 4.6.** Every $\beta g^*$-submaximal space is $\beta g^*$-submaximal.

Let $(X, \tau)$ be a $\beta g^*$-submaximal space and $A$ be a dense subset of $(X, \tau)$. Then $A$ is open. But every open set is $\beta g^*$-open and so $A$ is $\beta g^*$-open. Therefore $(X, \tau)$ is $\beta g^*$-submaximal.

However, the converse of the above proposition need not be true as seen from the following example.

**Example 4.7.** Let $X=\{a,b,c\}$ with $\tau=\{\phi,\{a\},\{b,c\}\}$. Then $\beta g^*O(X)=P(X)$. We have every dense subset is $\beta g^*$-open and hence $(X, \tau)$ is $\beta g^*$-submaximal. However, the set $A=\{c\}$ is dense in $(X, \tau)$, but it is not open in $(X, \tau)$. Therefore $(X, \tau)$ is not submaximal.

**Proposition 4.8.** Every $g$-submaximal space is $\beta g^*$-submaximal.

Let $(X, \tau)$ be a $g$-submaximal space and $A$ be a dense subset of $(X, \tau)$. Then $A$ is $g$-open. But every $g$-open set is $\beta g^*$-open and $A$ is $\beta g^*$-open. Therefore $(X, \tau)$ is $\beta g^*$-submaximal.

The converse of the above proposition need not be true as seen from the following example.
Example 4.9. Let $X = \{a,b,c,d\}$ with $\tau = \{\emptyset, \{d\}, \{a,b,c\}, X\}$. Then $G(O(X)) = P(X)$ and $G(O(X)) = \{\emptyset, \{d\}, \{a,b,c\}, X\}$, we have every dense subset is $\beta g$-open and hence $(X, \tau)$ is $\beta g$-submaximal. However, the set $A = \{a\}$ is dense in $(X, \tau)$, but it is not $g$-open in $(X, \tau)$. Therefore $(X, \tau)$ is not $g$-submaximal.

Proposition 4.10. Every r-submaximal space is $\beta g$-submaximal.

The converse of the above proposition need not be true as seen from the following example.

Example 4.11. Let $X = \{a,b,c,d\}$ with $\tau = \{\emptyset, \{a\}, \{b,c\}, X\}$. Then $RO(X) = P(X)$ and $\beta g(O(X)) = \{\emptyset, \{b,c\}, \{a\}, X\}$. Every dense subset is r-open and hence $(X, \tau)$ is r-submaximal. However, the set $A = \{a\}$ is dense in $(X, \tau)$, but it is not $\beta g$-open in $(X, \tau)$. Therefore $(X, \tau)$ is not $\beta g$-submaximal.

Theorem 4.12. Assume that $\beta g C(X)$ is closed under finite intersections. For a subset $A$ of $(X, \tau)$ the following statements are equivalent:

1. $A \subseteq \beta g L C(X)$,
2. $A = S \cap \beta g - cl(A)$ for some $\beta g$-open set $S$,
3. $\beta g - cl(A) - A$ is $\beta g$-closed,
4. $A \subseteq \beta g^* - int(AU(\beta g - cl(A)^*))$.

Proof. (1) $\Rightarrow$ (2). Let $A \subseteq \beta g L C(X)$. Then $A = S \cap \beta g - cl(A)$ is $\beta g^*$-open and $G = \beta g - cl(A)$ is $\beta g^*$-closed. Since $\beta g^* \subseteq G \subseteq \beta g - cl(A)$ and $S \cap \beta g - cl(A) \subseteq A$, we get $A = S \cap \beta g - cl(A)$. Also $A \subseteq S$ and $A \subseteq \beta g - cl(A)$ implies $A \cap S \cap \beta g - cl(A) \subseteq A$ and therefore $A = S \cap \beta g - cl(A)$.

Theorem 4.13. For a subset $A$ of $(X, \tau)$, the following statements are equivalent:

1. $A \subseteq \beta g L C(X)$,
2. $A = S \cap \beta g - cl(A)$ for some $\beta g^*$-open set $S$,
3. $\beta g - cl(A)$ is $\beta g$-closed,
4. $A \subseteq A \subseteq \beta g^* - int(AU(\beta g - cl(A)^*))$.

Proof. (1) $\Rightarrow$ (2). Let $A \subseteq \beta g L C(X)$. Then there exist an $\beta g^*$-open set $S$ and a closed set $G$ such that $A = S \cap G$. Since $\beta g^* \subseteq G \subseteq \beta g - cl(A)$ and $S \cap \beta g - cl(A) \subseteq A$. Also $S \subseteq S$ and $\beta g - cl(A)$ implies $S \cap \beta g - cl(A) \subseteq A$ and therefore $A = S \cap \beta g - cl(A)$.

Theorem 4.14. A space $(X, \tau)$ is $\beta g^*$-submaximal if and only if for any $P(X) = \beta g^* L C(X)$.

Proof. Necessity. Let $A \subseteq \beta g L C(X)$. Then there exist an $\beta g^*$-open set $S$ and a closed set $G$ such that $A = S \cap G$. Since $S \cap \beta g - cl(A) \subseteq A$ and $A \subseteq \beta g - cl(A)$ implies $S \cap \beta g - cl(A) \subseteq A$ and therefore $A = S \cap \beta g - cl(A)$.

Sufficiency. Let $A$ be a dense subset of $(X, \tau)$. Then this implies $A \cap \beta g L C(X)$ implies $A = A \cap \beta g L C(X)$ implies $A = A \cap \beta g L C(X)$ implies $A = \beta g L C(X)$.

Theorem 4.15. Let $A$ be a subset of $(X, \tau)$. Then $A \subseteq \beta g L C^*(X)$ if and only if for any $P(X) = \beta g L C^*(X)$.

Proof. Necessity. Let $A \subseteq \beta g L C^*(X)$. Then there exist an $\beta g^*$-open set $S$ and a closed set $G$ such that $A = S \cap G$. Since $S \cap \beta g - cl(A) \subseteq A$ and $A \subseteq \beta g - cl(A)$ implies $S \cap \beta g - cl(A) \subseteq A$ and therefore $A = S \cap \beta g - cl(A)$.

Sufficiency. Let $A$ be a dense subset of $(X, \tau)$. Then this implies $A \cap \beta g L C^*(X)$ implies $A = A \cap \beta g L C^*(X)$ implies $A = \beta g L C^*(X)$.

Theorem 4.16. Let $A$ be a subset of $(X, \tau)$. If $A \subseteq \beta g L C^*(X)$, then $A \subseteq A \subseteq \beta g - cl(A)$ implies $A \subseteq \beta g - cl(A)$ implies $A \subseteq \beta g - cl(A)$ implies $A \subseteq \beta g - cl(A)$.

Proof. Let $A \subseteq \beta g L C^*(X)$, then by Theorem 4.15, $A = S \cap \beta g - cl(A)$ for some open set $S$ and $\beta g - cl(A) \subseteq S$. Also $A \subseteq S \cap \beta g - cl(A) \subseteq A$ and $A \subseteq \beta g - cl(A)$.

Converse part is trivial.

Theorem 4.17. Assume that $\beta g O(X)$ forms a topology. For subsets $A$ and $B$ in $(X, \tau)$, the following are true:

1. If $A \subseteq B \subseteq \beta g L C(X)$, then $A \subseteq \beta g L C(X)$.
2. If $A \subseteq B \subseteq \beta g L C(X)$, then $A \subseteq \beta g L C(X)$.
3. If $A \subseteq B \subseteq \beta g L C(X)$, then $A \subseteq \beta g L C(X)$.
4. If $A \subseteq \beta g L C(X)$ and $B$ is $\beta g^*$-open (resp. $\beta g$-closed), then $A \subseteq \beta g L C(X)$.

DOI: 10.9790/5728-1404014751 www.iosrjournals.org 49 | Page
On Decomposition of $\beta g^*$ Closed Sets in Topological Spaces

(5) If $A \in \beta g^* LC(X)$ and $B$ is $\beta g^*$-open (resp. closed), then $A \cap B \in \beta g^* LC(X)$. 
(6) If $A \in \beta g^* LC^+(X)$ and $B$ is $\beta g^*$-closed (resp. open), then $A \cap B \in \beta g^* LC^+(X)$.
(7) If $A \in \beta g^* LC(X)$ and $B$ is $\beta g^*$-closed, then $A \cap B \in \beta g^* LC(X)$.
(8) If $A \in \beta g^* LC^+(X)$ and $B$ is $\beta g^*$-open, then $A \cap B \in \beta g^* LC(X)$.
(9) If $A \in \beta g^* LC(X)$ and $B \in \beta g^* LC(X)$, then $A \cap B \in \beta g^* LC(X)$.

Proof. By Remark 2.4, (1) to (8) hold.

(9) Let $A=\cap S \cap G$ where $S$ is open and $G$ is $\beta g^*$-closed and $B=P \cap Q$ where $P$ is $\beta g^*$-open and $Q$ is closed. Then $A \cap B=\cap (S \cap G) \cap (P \cap Q)$ where $S \cap G$ is $\beta g^*$-open and $P \cap Q$ is $\beta g^*$-closed, therefore $A \cap B \in \beta g^* LC(X)$.

Definition 4.18. Let $A$ and $B$ be subsets of $(X, \tau)$. Then $A$ and $B$ are said to be $\beta g^*$-separated if $A \cap \beta g^*-cl(B)=\phi$ and $\beta g^*-cl(A) \cap B=\phi$.

Example 4.19. Let $X=\{a, b, c\}$ with $\tau=\{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{X\}\}$. Let $A=\{a\}$ and $B=\{b\}$. Then $\beta g^*-cl(A)=\{a\}$ and $\beta g^*-cl(B)=\{b\}$ and so the sets $A$ and $B$ are $\beta g^*$-separated.

Proposition 4.20. Assume that $\beta g^* O(X)$ forms a topology. For a topological space $(X, \tau)$, the following are true:

1. Let $A, B \in \beta g^* LC(X)$. If $A$ and $B$ are $\beta g^*$-separated then $A \cup B \in \beta g^* LC(X)$.
2. Let $A, B \in \beta g^* LC(X)$. If $A$ and $B$ are $\beta g^*$-separated (i.e., $A \cap \beta g^*-cl(B)=\phi$ and $\beta g^*-cl(A) \cap B=\phi$), then $A \cup B \in \beta g^* LC^+(X)$.
3. Let $A, B \in \beta g^* LC^+(X)$. If $A$ and $B$ are $\beta g^*$-separated then $A \cup B \in \beta g^* LC^+(X)$.

Proof. (1) Since $A, B \in \beta g^* LC(X)$, by theorem 4.13, there exists $\beta g^*$-open sets $U$ and $V$ of $(X, \tau)$ such that $A=V \cap \beta g^*-cl(B)$ and $B=V \cap \beta g^*-cl(A)$. Now $G=V \cap (X-\beta g^*-cl(B))$ and $H=V \cap (X-\beta g^*-cl(A))$ are $\beta g^*$-open subsets of $(X, \tau)$. Since $\beta g^*-cl(B)=\beta g^* cl(A)$, $A=\beta g^*-cl(A)$ becomes $A=\beta g^*-cl(B) \neq \beta g^*-cl(B) \neq cl(A)$. Similarly $B=\beta g^*-cl(A)$. Moreover $G \cap \beta g^*-cl(B)=\phi$ and $H \cap \beta g^*-cl(A)=\phi$. Since $G$ and $H$ are $\beta g^*$-open sets of $(X, \tau)$, $GUH$ is $\beta g^*$-open. Therefore $A \cup B=(GUH) \cap \beta g^*-cl(AUB)$ and hence $A \cup B \in \beta g^* LC(X)$.

(2) and (3) are similar to (1), using Theorems 4.13 and 4.14.

Lemma 4.21. If $A$ is $\beta g^*$-closed in $(X, \tau)$ and $B$ is $\beta g^*$-closed in $(Y, \sigma)$, then $A \cup B$ is $\beta g^*$-closed in $(X \times Y, \tau \times \sigma)$.

Theorem 4.22. Let $(X, \tau)$ and $(Y, \sigma)$ be any two topological spaces. Then

i) If $A \in \beta g^* LC(X, \tau)$ and $B \in \beta g^* LC(Y, \sigma)$, then $A \times B \in \beta g^* LC(X \times Y, \tau \times \sigma)$.
ii) If $A \in \beta g^* LC^+(X, \tau)$ and $B \in \beta g^* LC^+(Y, \sigma)$, then $A \times B \in \beta g^* LC^+(X \times Y, \tau \times \sigma)$.
iii) If $A \in \beta g^* LC^+(X, \tau)$ and $B \in \beta g^* LC^*(Y, \sigma)$, then $A \times B \in \beta g^* LC+(X \times Y, \tau \times \sigma)$.

Proof. Let $A \in \beta g^* LC(X, \tau)$ and $B \in \beta g^* LC(Y, \sigma)$. Then there exists $\beta g^*$-open sets $V$ and $V'$ of $(X, \tau)$ and $(Y, \sigma)$ respectively and $\beta g^*$-closed sets $W$ and $W'$ of $(X, \tau)$ and $(Y, \sigma)$ respectively such that $V=A \times W$ and $V'=A \times W'$. Then $A \times B=(V \cap W) \cap (V' \cap W') \cap (W \cap W')$ holds and hence $A \times B \in \beta g^* LC(X \times Y, \tau \times \sigma)$.

The proofs of (ii) and (iii) are similar to (i).

V. $\beta g^* LC$-CONTINUOUS AND $\beta g^* LC$-IRRESOLUTE FUNCTIONS

In this section, we define $\beta g^*$-continuous and $\beta g^*$-LC-irresolute functions and obtain a pasting lemma for $\beta g^*$-LC*-continuous functions and irresolute functions.

Definition 5.1. A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called

i) $\beta g^*$-LC-continuous if $f^{-1}(V) \in \beta g^* LC(X, \tau)$ for every $V \in \sigma$.
ii) $\beta g^*$-LC*-continuous if $f^{-1}(V) \in \beta g^* LC^+(X, \tau)$ for every $V \in \sigma$.
iii) $\beta g^*$-LC*-continuous if $f^{-1}(V) \in \beta g^* LC^*(X, \tau)$ for every $V \in \sigma$.
iv) $\beta g^*$-LC-irresolute if $f^{-1}(V) \in \beta g^* LC(Y, \sigma)$ for every $V \in \beta g^* LC(X, \tau)$.
v) $\beta g^*$-LC*-irresolute if $f^{-1}(V) \in \beta g^* LC^+(Y, \sigma)$ for every $V \in \beta g^* LC^+(X, \tau)$.
vi) $\beta g^*$-LC*-irresolute if $f^{-1}(V) \in \beta g^* LC^+(Y, \sigma)$ for every $V \in \beta g^* LC^*(X, \tau)$.

Proposition 5.2. If $f:(X, \tau) \rightarrow (Y, \sigma)$ is $\beta g^*$-LC-irresolute, then it is $\beta g^*$-LC-continuous.

Proof. Let $V$ be open in $Y$. Then $f^{-1}(V) \in \beta g^* LC(X, \tau)$. Hence $f$ is $\beta g^*$-LC-continuous.

Proposition 5.3. Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a function, then

1. If $f$ is $\beta g^*$-LC-continuous, then $f$ is $\beta g^*$-LC-continuous.
2. If $f$ is $\beta g^*$-LC*-continuous, then $f$ is $\beta g^*$-LC-continuous.
3. If $f$ is $\beta g^*$-LC*-continuous, then $f$ is $\beta g^*$-LC-continuous.
4. If $f$ is $\beta g^*$-CL-continuous, then $f$ is $\beta g^*$-LC-continuous.

Remark 5.4. The above set of conclusions are true only when $X$ is locally closed in $Y$.

Example 5.5. 1. Let $X=Y=\{a, b, c, d\}$, $\tau=\{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{X\}\}$ and $\sigma=\{\phi, \{a\}, \{a, d\}, \{X\}\}$. Let $f$: $X \rightarrow Y$ be the identity map. Then $f$ is $\beta g^*$-LC-continuous but not $\beta g^*$-LC*-continuous. Since for the open set $\{a, d\}$, $f^{-1}(\{a, d\})=\{a, d\}$ is not locally closed in $X$.

2. Let $X=Y=\{a, b, c, d\}$, $\tau=\{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{X\}\}$ and $\sigma=\{\phi, \{d\}, \{a, d\}, \{a, c, d\}, \{X\}\}$. Let $f$: $X \rightarrow Y$ be the identity map. Then $f$ is $\beta g^*$-LC-continuous but not $\beta g^*$-LC*-continuous. Since for the open set $\{a, c, d\}$, $f^{-1}(\{a, c, d\})=\{a, c, d\}$ is not $\beta g^*$-LC-closed in $X$.
3. Let X=Y=\{a,b,c,d\}, \tau=\{\phi,\{c\},\{a,b\},\{a,b,c\}\},X and \sigma=\{\phi,\{a\},\{a,c\}\},X. Let f: X→Y be the identity map. Then f is \beta^g^* LC-continuous but not \beta^g^* LC^+-continuous. Since for the open set \{a,c\}, f^{-1}\{a,c\} = \{a,c\} is not \beta^g^* LC^-closed in X.

4. Let X=Y=\{a,b,c,d\}, \tau=\{\phi,\{a\},\{b,c\},\{a,b,c\}\},X and \sigma=\{\phi,\{a\},\{a,b\},\{a,b,d\}\},X. Let f: X→Y be the identity map. Then f is \beta^g^* LC-continuous but not glc-continuous. Since for the open set \{a,b,d\}, f^{-1}\{a,b,d\} = \{a,b,d\} is not \beta^g^* LC^-set in X.

We recall the definition of the combination of two functions: Let X=AUB and f:A→Y and h:B→Y be two functions. We say that f and h are compatible if f/A∩B=h/A∩B. If f:A→Y and h:B→Y are compatible, then the functions (f\Delta h):X→Y for every x∈B is called the combination of f and h.

Next we have the theorem concerning the composition of functions.

**Theorem 5.6.** Let X=A∪B, where A and B are \beta^g^* -closed and regular open subsets of (X,τ) and f:(A, τ\upharpoonright A)→(Y,σ) and h:(B,τ\upharpoonright B)→(Y,σ) be compatible functions.

a) If f and h are \beta^g^* LC^-continuous, then (f\Delta h):X→Y is \beta^g^* LC^-continuous.

b) If f and h are \beta^g^* LC^- irresolute, then (f\Delta h):X→Y is \beta^g^* LC^- irresolute.

Next we have the theorem concerning the combination of functions.

**Theorem 5.7.** Let f:(X,τ)→(Y,σ) and g:(Y,σ)→(Z,η) be two functions, then

a) g\circ f is \beta^g^* LC- irresolute if f and g are \beta^g^* LC- irresolute.

b) g\circ f is \beta^g^* LC^- irresolute if f and g are \beta^g^* LC^- irresolute.

c) g\circ f is \beta^g^* LC^- irresolute if f and g are \beta^g^* LC^- irresolute.

d) g\circ f is \beta^g^* LC-continuous if f is \beta^g^* LC-continuous and g is \beta^g^* LC-continuous.

e) g\circ f is \beta^g^* LC^-continuous if f is \beta^g^* LC^-continuous and g is \beta^g^* LC^- irresolute.

References


[5]. C.Dhanapakyam .K.Indirani,On \beta^g^* continuity in topological spaces (communicated).
