Nano Regular Generalized Star Star B-Normal Space In Nano Topological Spaces

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Abstract: In this paper we introduce the concept of nano $rg^*=b$-regular and nano $rg^*=b$-normal spaces in nano topological spaces. Also some characterizations and several properties concerning of these spaces.

Key Words: nano $rg^*=b$-regular, nano $rg^*=b$-normal.

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I. Introduction

The concept of nano topology was introduced by Lellis Thivagar [5] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the weak forms of nano open sets namely nano $a$-open sets, nano-semi open sets and nano pre-open sets in a nano topological space.


This chapter contains Nano regular generalized star star $b$-regular spaces and Nano regular generalized star star $b$-normal space are introduced, and characterization of these spaces are studied.

II. Preliminaries

Definition 2.1 Let $U$ be a non-empty finite set of objects called the universe and $R$ be an equivalence relation on $U$ named as the indiscernibility relation, elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be the approximation space. Let $X \subseteq U$.

i) The lower approximation of $X$ with respect to $R$ is the set of all objects, which can be for certain classified as $X$ with respect to $R$ and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x \in U} R(x) : R(x) \subseteq X$, where $R(x)$ denotes the equivalence class determined by $x$.

ii) The upper approximation of $X$ with respect to $R$ is the set of all objects, which can be possibly classified as $X$ with respect to $R$ and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x \in U} R(x) : R(x) \cap X \neq \emptyset$ where $R(x)$ denotes the equivalence class determined by $x$.

iii) The boundary region of $X$ with respect to $R$ is set of all objects, which can be classified neither as $X$ nor as not $X$ with respect to $R$ and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 Let $U$ be the universe, $R$ be an equivalence relation on $U$ and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. $U$ and $\emptyset \in \tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on $U$ called as the nano topology on $U$ with respect to $X$. We call $(U, \tau_R(X))$ as the Nano topological space.

Definition 2.3 A map $f : (U, \tau_R(U)) \rightarrow (V, \tau_R(V))$ is said to be nano $rg^*=b$-continuous if $f^{-1}(V)$ is nano $rg^*=b$-closed in $(U, \tau_R(U))$ for each nano closed set $V$ in $(V, \tau_R(V))$.

Definition 2.4 A map $f : (U, \tau_R(U)) \rightarrow (V, \tau_R(V))$ is said to be nano $rg^*=b$-closed if the image of every nano closed set in $(U, \tau_R(U))$ is nano $rg^*=b$-closed in $(V, \tau_R(V))$.

Definition 2.5 A map $f : (U, \tau_R(U)) \rightarrow (V, \tau_R(V))$ is said to be nano $rg^*=b$-open if $f(A)$ is nano $rg^*=b$-open for each nano open set $A$ in $(U, \tau_R(U))$. 

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**Definition 2.6** A map \( f : (U, \tau_g(X)) \to (V, \tau_g(Y)) \) is said to be strongly nano \( rg^{**}b \)-closed if the image \( f(A) \) is nano \( rg^{**}b \)-closed in \((V, \tau_g(Y))\) for each nano \( rg^{**}b \)-closed in \((U, \tau_g(X))\).

**Definition 2.7** A map \( f : (U, \tau_g(X)) \to (V, \tau_g(Y)) \) is said to be strongly nano \( rg^{**}b \)-open if the image \( f(A) \) is nano \( rg^{**}b \)-open in \((V, \tau_g(Y))\) for each nano \( rg^{**}b \)-open in \((U, \tau_g(X))\).

**Definition 2.8** A map \( f : (U, \tau_g(X)) \to (V, \tau_g(Y)) \) is said to be Contra nano \( rg^{**}b \)-closed map (resp. Contra nano \( rg^{**}b \)-open) if the image of every nano closed (resp. nano open) set in \((U, \tau_g(X))\) is nano \( rg^{**}b \)-closed (resp. nano \( rg^{**}b \)-closed) set in \((V, \tau_g(Y))\).

**Definition 2.9** A map \( f : (U, \tau_g(X)) \to (V, \tau_g(Y)) \) is said to be \( Nrg^{**}b \)-Irresolute if the inverse image of every \( Nrg^{**}b \)-closed set in \((V, \tau_g(Y))\) is \( Nrg^{**}b \)-closed set in \((U, \tau_g(X))\).

### III. Nano Regular Generalized Star Star b-Regular Space

**Definition 3.1** A nano topological space \((U, \tau_g(X))\) is said to be nano regular generalized star b-regular space (briefly \( Nrg^{**}b \)-regular), if for each nano-closed subset \( \rho \) of \( U \) and each point \( x \in U - \rho \), there exist two disjoint \( Nrg^{**}b \)-open sets one containing \( \rho \) and other containing \( x \).

**Example 3.2** Let \( U = V = \{a, b, c\} \) with \( U/R = \{\{a\}, \{b, c\}\} \) and \( X = \{a, b\} \subseteq U \). Then the nano topology is defined as \( \tau_g(X) = \{U, \emptyset, \{a\}, U, \{b, c\}\} \).

- \( i \) Let \( \rho = \{b, c\} \subseteq U \) and the point \( a \in U - \rho \) there exist \( Nrg^{**}b \)-open sets are \( \{b, c\} \subseteq \{b, c\} \).
- \( a \in \{a\} \) and \( \{b, c\} \cap \{a\} = \emptyset \).

- \( ii \) Let \( \rho = \{a\} \subseteq U \), and the point \( b \in U - \rho \), there exist \( Nrg^{**}b \)-open sets are \( \{a\} \subseteq \{a\}, b \in \{b, c\} \) and \( \{b, c\} \cap \{a\} = \emptyset \) and the point \( c \in U - \rho \) there exist \( Nrg^{**}b \)-open sets are \( \{a\} \subseteq \{a\} \), \( c \in \{b, c\} \) and \( \{b, c\} \cap \{a\} = \emptyset \). Therefore \((U, \tau_g(X)) \) \( Nrg^{**}b \)-regular.

**Theorem 3.3** Every nano regular space is \( Nrg^{**}b \)-regular.

The proof is obvious.

**Example 3.4** Let \( U = \{a, b, c, d\} \) with \( U/R = \{\{a\}, \{b\}, \{c, d\}\} \) and \( X = \{a, b\} \subseteq U \). Then the nano topology is defined as \( \tau_g(X) = \{U, \emptyset, \{a\}, \{b\}, \emptyset\} \) here only one nano-closed proper subsets of \( U \) consists of \( \rho = \{c, d\} \) and corresponding to \( \{c, d\} \subseteq \{b, c, d\} \) and the point \( a \in \{a\} \) such that \( \{b, c, d\} \cap \{a\} = \emptyset \). Again \( \{c, d\} \subseteq \{a, c, d\} \) and the point \( b \in \{b\} \) such that \( \{a, c, d\} \cap \{b\} = \emptyset \). Therefore \((U, \tau_g(X)) \) \( Nrg^{**}b \)-regular but not nano-regular.

**Theorem 3.5** If \( f : (U, \tau_g(X)) \to (V, \tau_g(Y)) \) is \( Nrg^{**}b \)-irresolute, nano-closed injection and \( V \) is \( Nrg^{**}b \)-regular space then \( U \) is \( Nrg^{**}b \)-regular space.

**Proof:** Let \( \rho \) be nano-closed set in \( U \) and \( u \in \rho \). since \( f \) is nano-closed injection, \( f(\rho) \) is nano-closed set in \( V \) such that \( f(u) \in f(\rho) \). Now \( V \) is \( Nrg^{**}b \)-regular, there exist disjoint \( Nrg^{**}b \)-open sets \( E \) and \( F \) such that \( f(u) \in E \) and \( f(\rho) \in F \), which implies \( u \in f^{-1}(E) \) and \( \rho \in f^{-1}(F) \). Since \( f \) is \( Nrg^{**}b \)-irresolute, where \( f^{-1}(E) \) and \( f^{-1}(F) \) are disjoint \( Nrg^{**}b \)-open sets in \( U \). Therefore \( U \) is \( Nrg^{**}b \)-regular space.

**Theorem 3.6** If \( f : (U, \tau_g(X)) \to (V, \tau_g(Y)) \) is nano-continuous surjective, strongly \( Nrg^{**}b \)-open function and \( U \) is \( Nrg^{**}b \)-regular space then \( V \) is \( Nrg^{**}b \)-regular space.

**Proof:** Let \( \rho \) be nano-closed set in \( V \) and \( v \in \rho \). Take \( v = f(u) \) for some \( u \in U \). since \( f \) is nano-continuous surjective, \( f^{-1}(\rho) \) is nano-closed set in \( U \) such that \( u \notin f^{-1}(\rho) \). Now \( U \) is \( Nrg^{**}b \)-regular, there exist disjoint \( Nrg^{**}b \)-open sets \( G \) and \( H \) such that \( u \in G \) and \( f^{-1}(\rho) \subseteq H \). That is \( v = f(u) \in f(G) \) and \( \rho \in f(H) \). Since \( f \) is strongly \( Nrg^{**}b \)-open function, \( f(G) \) and \( f(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( U \). Therefore \( V \) is \( Nrg^{**}b \)-regular space.

**Theorem 3.7** If \( f : (U, \tau_g(X)) \to (V, \tau_g(Y)) \) is \( Nrg^{**}b \)-continuous, nano-closed injection and \( V \) is ultra regular space then \( U \) is \( Nrg^{**}b \)-regular space.

**Proof:** Let \( \rho \) be nano-closed set in \( U \) and \( u \notin \rho \). since \( f \) is nano-closed injection, \( f(\rho) \) is nano-closed set in \( V \) such that \( f(u) \notin f(\rho) \). Now \( V \) is ultra regular, there exist disjoint nano clopen open sets \( E \) and \( F \) such that \( f(u) \in E \) and \( f(\rho) \subseteq H \), which implies \( u \in f^{-1}(G) \) and \( \rho \in f^{-1}(H) \). Since \( f \) is \( Nrg^{**}b \)-continuous, where \( f^{-1}(G) \) and \( f^{-1}(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( U \). Therefore \( U \) is \( Nrg^{**}b \)-regular space.

**Theorem 3.8** If \( f : (U, \tau_g(X)) \to (V, \tau_g(Y)) \) is nano-continuous bijective, \( Nrg^{**}b \)-open function and \( U \) is nano regular space then \( V \) is \( Nrg^{**}b \)-regular space.
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**Proof:** Let $\rho$ be nano-closed set in $V$ and $v \notin \rho$. Take $v = f(u)$ for some $u \in U$. Since $f$ is nano-continuous bijective, $f^{-1}(\rho)$ is nano-closed set in $U$ such that $u \notin f^{-1}(\rho)$. Now $U$ is nano regular space, there exist disjoint open sets $G$ and $H$ such that $u \in G$ and $f^{-1}(\rho) \subset H$. That is $v = f(u) \in f(G)$ and $\rho \subset f(H)$. Since $f$ is $Nrg**b-open function, $f(G)$ and $f(H)$ are disjoint $Nrg**b-open sets in $V$. Therefore $V$ is $Nrg**b-normal space.

**Theorem 3.9** If $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nano-continuous bijective, contra $Nrg**b-function and $U$ is ultra regular space then $V$ is $Nrg**b-normal space.

**Proof:** Let $\rho$ be nano-closed set in $V$ and $v \notin \rho$. Take $v = f(u)$ for some $u \in U$. Since $f$ is nano-continuous bijective, $f^{-1}(\rho)$ is nano-closed set in $U$ such that $u \notin f^{-1}(\rho)$. Now $U$ is ultra regular, there exist disjoint clopen sets $G$ and $H$ such that $u \in G$ and $f^{-1}(\rho) \subset H$. That is $v = f(u) \in f(G)$ and $\rho \subset f(H)$. Since $f$ is contra $Nrg**b-function, $f(G)$ and $f(H)$ are disjoint $Nrg**b-open sets in $V$. Therefore $V$ is $Nrg**b-normal space.

**Theorem 3.10** If $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is $Nrg**b-continuous$, nano-closed injection and $V$ is nano regular space then $U$ is $Nrg**b-normal space.

**Proof:** Let $\rho$ be nano-closed set in $U$ and $u \notin \rho$. Since $f$ is nano-continuous injection, $f(\rho)$ is nano-closed set in $V$ such that $f(u) \notin f(\rho)$. Now $V$ is nano regular, there exist disjoint nano open-open sets $E$ and $F$ such that $f(u) \in E$ and $f(\rho) \subset H$. Therefore $U$ is $Nrg**b-normal space.

**Theorem 3.11** If $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nano-continuous bijective, $Nrg**b-open function and $U$ is ultra regular space then $V$ is $Nrg**b-normal space.

**Proof:** Let $\rho$ be nano-closed set in $U$ and $u \notin \rho$. Take $v = f(u)$ for some $u \in U$. Since $f$ is nano-continuous bijective, $f^{-1}(\rho)$ is nano-closed set in $U$ such that $u \notin f^{-1}(\rho)$. Now $U$ is ultra regular, there exist disjoint clopen sets $G$ and $H$ such that $u \in G$ and $f^{-1}(\rho) \subset H$. That is $v = f(u) \in f(G)$ and $\rho \subset f(H)$. Since $f$ is $Nrg**b-open function, $f(G)$ and $f(H)$ are disjoint $Nrg**b-open sets in $V$. Therefore $V$ is $Nrg**b-normal space.

**Theorem 3.12** If $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nano-continuous surjective, strongly $Nrg**b-open function and $U$ is ultra regular space then $V$ is $Nrg**b-normal space.

**IV. Nano Regular Generalized Star Star B-Normal Space**

**Definition 4.1** A nano topological space $(U, \tau_R(X))$ is said to be nano regular generalized star star $b$-normal space(briefly $Nrg**b-normal$), if for every pair of disjoint nano-closed subsets $\rho_1$ and $\rho_2$ of $U$ there exist $Nrg**b-open sets $G$ and $H$ such that $\rho_1 \subseteq G$, and $\rho_2 \subseteq H$ and $G \cap H = \emptyset$.

**Example 4.2** Let $U = V = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, b\} \subseteq U$. Then the nano topology is defined as $\tau_R(X) = \{U, \emptyset, \{a\}, \{b,c\}\}$. Here the only one pair of non-empty disjoint nano-closed proper subset of $U$ consists of $\rho_1 = \{b, c\}$ and $\rho_2 = \{a\}$ and corresponding to $Nrg**b-open sets $\{b,c\}$ and $\{a\}$ such that $\{b,c\} \subseteq \{b, c\}$, $\{a\} \subseteq \{a\}$ and $\{b,c\} \cap \{a\} = \emptyset$.

**Theorem 4.3** If $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is $Nrg**b-irresolute$, nano-closed injection and $V$ is $Nrg**b-normal space then $U$ is $Nrg**b-normal space.

**Proof:** Let $\rho_1$ and $\rho_2$ be disjoint nano-closed sets in $U$ then $f(\rho_1)$ and $f(\rho_2)$ are disjoint nano-closed sets in $V$. Since $f$ is nano-closed injection. Now $V$ is $Nrg**b-normal, there exist disjoint $Nrg**b-open sets $G$ and $H$ such that $f(\rho_1) \subset G$ and $f(\rho_2) \subset H$. Therefore $U$ is $Nrg**b-normal space.

**Theorem 4.4** If $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nano-continuous surjective, strongly $Nrg**b-open function and $U$ is $Nrg**b-normal space then $V$ is $Nrg**b-normal space.

**Proof:** Let $\rho_1$ and $\rho_2$ be disjoint nano-closed sets in $U$ then $f^{-1}(\rho_1)$ and $f^{-1}(\rho_2)$ are disjoint nano-closed set in $U$. Since $f$ is nano-continuous. Now $U$ is $Nrg**b-normal, there exist disjoint $Nrg**b-open sets $G$ and $H$ such

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that \( f^{-1}(\rho_1) \subseteq G \) and \( f^{-1}(\rho_2) \subseteq H \). That is \( \rho_1 \subseteq f(G) \) and \( \rho_2 \subseteq f(H) \), \( f(G) \) and \( f(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( V \). Since \( f \) is strongly \( Nrg^{**}b \)-open function. Therefore \( V \) is \( Nrg^{**}b \)-regular space.

**Theorem 4.5** If \( f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y)) \) is \( Nrg^{**}b \)-continuous, nano-closed injection and \( V \) is ultra normal space then \( U \) is \( Nrg^{**}b \)-normal space.

**Proof:** Let \( \rho_1 \) and \( \rho_2 \) be disjoint nano-closed sets in \( U \) then \( f(\rho_1) \) and \( f(\rho_2) \) are disjoint nano-closed sets in \( V \). Since \( f \) is nano-closed injection. Now \( V \) is ultra normal, there exist disjoint nano clopen sets \( G \) and \( H \) such that \( f(\rho_1) \subseteq G \) and \( f(\rho_2) \subseteq H \). Which implies \( \rho_1 \subseteq f^{-1}(G) \) and \( \rho_2 \subseteq f^{-1}(H) \). Since \( f \) is \( Nrg^{**}b \)-continuous, where \( f^{-1}(G) \) and \( f^{-1}(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( U \). Therefore \( U \) is \( Nrg^{**}b \)-normal space.

**Theorem 4.6** If \( f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y)) \) is nano-continuous bijective, \( Nrg^{**}b \)-open function and \( U \) is nano normal space then \( V \) is \( Nrg^{**}b \)-normal space.

**Proof:** Let \( \rho_1 \) and \( \rho_2 \) be disjoint nano-closed sets in \( V \) then \( f^{-1}(\rho_1) \) and \( f^{-1}(\rho_2) \) are disjoint nano-closed set in \( U \). Since \( f \) is nano-continuous bijective. Now \( U \) is ultra normal, there exist disjoint nano clopen sets \( G \) and \( H \) such that \( f^{-1}(\rho_1) \subseteq G \) and \( f^{-1}(\rho_2) \subseteq H \). That is \( \rho_1 \subseteq f(G) \) and \( \rho_2 \subseteq f(H) \), \( f(G) \) and \( f(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( V \). Since \( f \) is \( Nrg^{**}b \)-open function. Therefore \( V \) is \( Nrg^{**}b \)-normal space.

**Theorem 4.7** If \( f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y)) \) is nano-continuous bijective, contra \( Nrg^{**}b \)-closed function and \( U \) is ultra normal space then \( V \) is \( Nrg^{**}b \)-normal space.

**Proof:** Let \( \rho_1 \) and \( \rho_2 \) be disjoint nano-closed sets in \( V \) then \( f^{-1}(\rho_1) \) and \( f^{-1}(\rho_2) \) are disjoint nano-closed set in \( U \). Since \( f \) is nano-continuous bijective. Now \( U \) is ultra normal, there exist disjoint nano clopen sets \( G \) and \( H \) such that \( f^{-1}(\rho_1) \subseteq G \) and \( f^{-1}(\rho_2) \subseteq H \). That is \( \rho_1 \subseteq f(G) \) and \( \rho_2 \subseteq f(H) \), \( f(G) \) and \( f(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( V \). Since \( f \) is contra \( Nrg^{**}b \)-closed function. Therefore \( V \) is \( Nrg^{**}b \)-normal space.

**Theorem 4.8** If \( f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y)) \) is \( Nrg^{**}b \)-continuous, nano-closed injection and \( V \) is nano normal space then \( U \) is \( Nrg^{**}b \)-normal space.

**Proof:** Let \( \rho_1 \) and \( \rho_2 \) be disjoint nano-closed sets in \( U \) then \( f(\rho_1) \) and \( f(\rho_2) \) are disjoint nano-closed sets in \( V \). Since \( f \) is nano-closed injection. Now \( V \) is normal, there exist disjoint nano open sets \( G \) and \( H \) such that \( f(\rho_1) \subseteq G \) and \( f(\rho_2) \subseteq H \). Which implies \( \rho_1 \subseteq f^{-1}(G) \) and \( \rho_2 \subseteq f^{-1}(H) \). Since \( f \) is \( Nrg^{**}b \)-continuous, where \( f^{-1}(G) \) and \( f^{-1}(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( U \). Therefore \( U \) is \( Nrg^{**}b \)-normal space.

**Theorem 4.9** If \( f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y)) \) is nano-continuous bijective, \( Nrg^{**}b \)-open function and \( U \) is normal space then \( V \) is \( Nrg^{**}b \)-normal space.
**Proof:** Let \( \rho_1 \) and \( \rho_2 \) be disjoint nano-closed sets in \( V \) then \( f^{-1}(\rho_1) \) and \( f^{-1}(\rho_2) \) are disjoint nano-closed set in \( U \). Since \( f \) is nano-continuous bijective. Now \( U \) is normal, there exist disjoint nano-open sets \( G \) and \( H \) such that \( f^{-1}(\rho_1) \subseteq G \) and \( f^{-1}(\rho_2) \subseteq H \). That is \( \rho_1 \subseteq f(G) \) and \( \rho_2 \subseteq f(H) \), \( f(G) \) and \( f(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( V \). Since \( f \) is \( Nrg^{**}b \)-open function. Therefore \( V \) is \( Nrg^{**}b \)-normal space.

**Theorem 4.10** If \( f: (U, \tau_B(X)) \rightarrow (V, \tau_B(Y)) \) is nano-continuous surjective, strongly \( Nrg^{**}b \)-open function and \( U \) is ultra regular space then \( V \) is \( Nrg^{**}b \)-regular space.

**Proof:** Let \( \rho_1 \) and \( \rho_2 \) be disjoint nano-closed sets in \( V \) then \( f^{-1}(\rho_1) \) and \( f^{-1}(\rho_2) \) are disjoint nano-closed set in \( U \). Since \( f \) is nano-continuous bijective. Now \( U \) is normal, there exist disjoint nano-clopen sets \( G \) and \( H \) such that \( f^{-1}(\rho_1) \subseteq G \) and \( f^{-1}(\rho_2) \subseteq H \). We know that every nano-open set is \( Nrg^{**}b \)-open. That is \( \rho_1 \subseteq f(G) \) and \( \rho_2 \subseteq f(H) \), \( f(G) \) and \( f(H) \) are disjoint \( Nrg^{**}b \)-open sets in \( V \). Since \( f \) is strongly \( Nrg^{**}b \)-open function. Therefore \( V \) is \( Nrg^{**}b \)-normal space.

**References**


