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Finite Abelian Automata

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Abstract: *Finite Semigroup Automaton, Finite Monoid Automaton, Finite Group Automaton, Finite Abelian Automaton have been introduced. Cross Product of Finite Semigroup Automatons, Finite Monoid Automatons, Finite Group Automatons, Finite Abelian Automatons have been defined. If* $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ *and* $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ *are any two Finite Semigroup Automatons, then* $B_1 \times B_2$ *is also a finite Semigroup automaton. If* $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ *and* $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ *are any two Finite Moniod Automatons, then* $B_1 \times B_2$ *is also a finite Monoid automaton. If* $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ *and* $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ *are any two Finite Group Automatons, then* $B_1 \times B_2$ *is also a finite group automaton. If* $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ *and* $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ *are any two Finite Abelian Automatons, then* $B_1 \times B_2$ *is also a finite Abelian automaton. Some Propositions are found in a Finite Abelian Automaton.*

keywords: *Finite Semigroup Automaton, Finite Monoid Automaton, Finite Group Automaton, Finite Abelian Automaton*

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I. Introduction

As the theory of Automata plays an important role in many fields, the theory of AC Finite Binary Automata, Finite Semigroup Automata, Finite Monoid Automata, Finite Group Automata, Finite Abelian Automata will also play an important role in these fields. The theory of Automata has become a part of computer science. It is very useful in electrical engineering. It provides useful techniques in a wide variety of applications.

II. Finite Automata And Finite Binary Automata

2.0Finite Automaton: A Finite Automaton is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where Q is a finite set of states, Σ is a finite set of inputs, q_0 in Q is the initial state, $F \subseteq Q$ is the set of final states and δ is the transition function mapping $Q \times \Sigma$ to Q.

If Σ^* is the set of strings of inputs, then the transition function δ is extended as follows :

For $w \in \Sigma^*$ and $a \in \Sigma$, $\delta': Q \times \Sigma^* \to Q$ is defined by $\delta'(q,wa) = \delta(\delta'(q,w),a)$.

If no confusion arises δ' can be replaced by δ.

2.1Finite Binary Automaton: A Finite Binary Automaton B is a 6-tuple $(Q, *, \Sigma, \delta, q_0, F)$, where Q is a finite set of states, * is a mapping from $Q \times Q$ to Q, Σ is a finite set of integers, q_0 in Q is the initial state and $F \subseteq Q$ is the set of final states and δ is the transition function mapping from $Q \times \Sigma$ to Q defined by $\delta(q,n) = q^n$.

If Σ^* is the set of strings of inputs, then the transition function δ is extended as follows :

For $m \in \Sigma^*$ and $n \in \Sigma$, $\delta': Q \times \Sigma^* \to Q$ is defined by $\delta'(q,mn) = \delta(\delta'(q,m),n)$.

If no confusion arises δ' can be replaced by δ.

2.2Associative Finite Binary Automaton: A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be a associative finite binary automaton if $p * (q * r) = (p * q) * r$, for all p,q,r in Q.

2.3Commutative Finite Binary Automaton: A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be a commutative finite binary automaton if $p * q = q * p$, for all p,q in Q.

2.3.1AC Finite Binary Automaton: A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be a AC Finite Binary Automaton if it is both associative and commutative

2.4Cross Product of Finite Binary Automatons: Let $B_1 = (Q_1, \Delta_1 , \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two Finite Binary Automatons. Then we define $B_1 \times B_2 = (Q, *, \Sigma, \delta, r_0, F)$, where $Q = Q_1 \times Q_2$, * is a mapping from $Q \times Q$ to Q defined by for $p,q \in Q = Q_1 \times Q_2$, where $p=(p_1,p_2)$, $q=(q_1,q_2)$, $p * q = (p_1 \Delta_1 q_1 , p_2 \Delta_2 q_2)$ $\Sigma = \Sigma_1 \times \Sigma_2$, $r_0 = p_o \times q_o$ in Q is the initial state and $F = F_1 \times F_2 \subseteq Q$ is the set of final states and δ is the transition function mapping from $Q \times \Sigma$ to Q defined by $\delta((p,q,),n) = (p^n,q^n)$.

Proposition 2.4.1 : If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Binary Automatons, then $B_1 \times B_2$ is also a finite binary automaton.
Proposition 2.4.2 : Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two Associative Finite Binary Automatons. Then $B_1 \times B_2$ is also an associative finite binary automaton.
Proposition 2.4.3 : Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two commutative Finite Binary Automatons. Then $B_1 \times B_2$ is also a commutative finite binary automaton.
Proposition 2.4.4 : Let $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ be any two AC Finite Binary Automatons. Then $B_1 \times B_2$ is also an AC finite binary automaton.
Proposition 2.4.5 : Let $B = (Q, \Delta_1, \Sigma, \delta, p_0, F)$ be an AC Finite Binary Automaton. Then $\delta((a * b), n) = \delta(a, n) * \delta(b, n)$, for any $a, b \in Q$ and $n \in \Sigma$.

III. Finite Abelian Automata

3.1Finite Semi-group Automaton : A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be a Finite Semigroup Automaton if it is an Associative Finite Binary Automaton.
Proposition 3.1.1 : If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Semi-group Automatons, then $B_1 \times B_2$ is also a finite Semi-group automaton.
Proof : By Proposition 2.4.2 if $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Associative Finite Binary Automatons, then $B_1 \times B_2$ is also an associative finite binary automaton.
That is , $B_1 \times B_2$ is a finite Semi-group automaton.
3.2Finite Monoid Automaton : A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be a Finite Monoid Automaton if (i) $p * (q * r) = (p * q) * r$, for all p,q,r in Q. (ii) there exists a state denoted by 0 in Q such that $p * 0 = p = 0 * p$, for all p in Q.
If such a state exists in Q, then the state 0 is called the identity state of Q.
Proposition 3.2.1 : If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Monoid Automatons, then $B_1 \times B_2$ is also a finite monoid automaton.
Proof : By Proposition 3.1.1 if $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Semi-group Automatons, then $B_1 \times B_2$ is also a finite Semi-group automaton.
Let 0_1 and 0_2 be the identity states of Q_1 and Q_2 respectively.
Then $(0_1 , 0_2) \in Q_1 \times Q_2$
Let $0 = (0_1 , 0_2)$
Let $x = (p,q) \in Q_1 \times Q_2$
Then $x * 0 = (p,q) * (0_1 , 0_2)$
$= (p \Delta_1 0_1 , q \Delta_2 0_2)$
$= (p , q)$
$= x$
And $0 * x = (0_1 , 0_2) * (p,q)$
$= (0_1 \Delta_1 p , 0_2 \Delta_2 q)$
$= (p , q)$
$= x$
Therefore , $B_1 \times B_2$ is a finite monoid automaton.
3.3Finite Group Automaton : A Finite Binary Automaton $B = (Q, *, \Sigma, \delta, q_0, F)$ is said to be a Finite Group Automaton if (i) $p * (q * r) = (p * q) * r$, for all p,q,r in Q.
(ii) there exists a state denoted by 0 in Q such that $p * 0 = p = 0 * p$, for all p in Q
(iv) for each state p in Q there exists a state q in Q such that $p * q = 0 = q * p$
If for a state p in Q there exists a state q in Q such that $p * q = 0 = q * p$, then the state q is called the inverse state and the state p is called a invertible state in Q.
Proposition 3.3.1 : If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Group Automatons, then $B_1 \times B_2$ is also a finite group automaton.
Proof : By Proposition 3.2.1 if $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Monoid Automatons, then $B_1 \times B_2$ is also a Finite Monoid Automaton.
Let $x = (p,q) \in Q_1 \times Q_2$
Let 0_1 and 0_2 be the identity states of Q_1 and Q_2 respectively.
Since $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ is a Finite Group Automaton, for p in Q_1 there exists p' in Q such that $p \Delta_1 p' = 0_1 = p' \Delta_1 p$
Since $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Group Automaton, for q in Q_2 there exists q' in Q such that $q \Delta_2 q' = 0_2 = q' \Delta_2 q$
Therefore, $B_1 \times B_2$ is also a finite group automaton.

3.4Finite Abelian Automaton : An AC Finite Binary Automaton B = (Q, *, Σ, δ, q_0, F) is said to be a Finite Abelian Automaton if (ii) there exists a state denoted by 0 in Q such that p * 0 = p = 0 * p , for all p in Q
(iv) for each state p in Q there exists a state q in Q such that p * q = 0 = q * p

Proposition 3.4.1 : If $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Abelian Automatons, then $B_1 \times B_2$ is also a finite abelian automaton.

Proof : By Proposition 3.3.1 if $B_1 = (Q_1, \Delta_1, \Sigma_1, \delta_1, p_0, F_1)$ and $B_2 = (Q_2, \Delta_2, \Sigma_2, \delta_2, q_0, F_2)$ are any two Finite Group Automatons, then $B_1 \times B_2$ is also a Finite Group Automaton.
Let x, y ϵ $Q_1 \times Q_2$ where x = (p , q) and y = (r , s)
Then x * y = (p , q) * (r , s)
= ($p \Delta_1 r$, $q \Delta_2 s$)
= ($r \Delta_1 p$, $s \Delta_2 q$) (Since B_1 and B_2 are Finite Abelian Automatons)
= (r , s) * (p , q)
= y * x
Therefore, $B_1 \times B_2$ is also a Finite Abelian Automaton.

Proposition 3.4.2 : Let B = (Q, *, Σ, δ, q_0, F) be a Finite Group Automaton in which Σ contains 2. If δ(a , 2) = 0, for all a in Q, then B = (Q, *, Σ, δ, q_0, F) is a Finite Abelian Automaton.

Proof : Let B = (Q, *, Σ, δ, q_0, F) be a Finite Group Automaton in which Σ contains 2.
Suppose δ(a , 2) = 0, for all a in Q.
Let a , b ϵ Q
Then δ(a , 2) = 0 and δ(a , 2) = 0
By definition of Finite binary automata δ(a , 2) = a^2
Therefore, $a^2 = 0$
a * a = 0
Therefore, a is a invertible state and the inverse state of a is a itself.
That is, $a^{-1} = a$
Similarly $b^{-1} = b$
Therefore, $(a * b)^{-1} = a * b$
But in any Finite Group Automaton we have $(a * b)^{-1} = b^{-1} * a^{-1}$
Therefore, $a * b = (a * b)^{-1}$
$= b^{-1} * a^{-1}$
= b * a
Hence B = (Q, *, Σ, δ, q_0, F) is a Finite Abelian Automaton.

Proposition 3.4.3 : Let B = (Q, *, Σ, δ, q_0, F) be a Finite Group Automaton in which Σ contains three consecutive integers m, m+1, m+2 such that δ(a * b , m) = δ(a , m) * δ(b , m), δ(a * b , m+1) = δ(a , m+1) * δ(b , m+1) and δ(a * b , m+2) = δ(a , m+2) * δ(b , m+2), for all a, b ϵ Q. Then B = (Q, *, Σ, δ, q_0, F) is a Finite Abelian Automaton.

Proof : Let a , b ϵ Q
Now B = (Q, *, Σ, δ, q_0, F) is a Finite Group Automaton in which Σ contains three consecutive integers m, m+1, m+2 such that δ(a * b , m) = δ(a , m) * δ(b , m),
δ(a * b , m+1) = δ(a , m+1) * δ(b , m+1),
and δ(a * b , m+2) = δ(a , m+2) * δ(b , m+2)
Then δ(b , m) * δ(a , 1) = δ(a , 1) * δ(b , m)
and δ(b , m+1) * δ(a , 1) = δ(a , 1) * δ(b , m+1)
δ(b , m) * δ(b , 1) * δ(a , 1) = δ(a , 1) * δ(b , m) * δ(b , 1)
δ(b , m) * δ(b , 1) * δ(a , 1) = δ(b , m) * δ(a , 1) * δ(b , 1)
b^m * δ(b , 1) * δ(a , 1) = b^m * δ(a , 1) * δ(b , 1)
δ(b , 1) * δ(a , 1) = δ(a , 1) * δ(b , 1)
b * a = a * b
Hence B = (Q, *, Σ, δ, q_0, F) is a Finite Abelian Automaton.

IV. Conclusion

Even though Automata theory is mostly used in computer engineering, many Mathematicians are doing research in this field. They produce good results in this field. Automata Theory is also useful in electrical engineering and in many fields. The theory of Finite Abelian Automata will also be useful in these fields. The Researcher can develop these ideas and can find new results.

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