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Markov Decision Model For Maintenance Problem Of Deteriorating Equipment With Valueiteration

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Abstract: *This paper analyses a dynamic system which is reviewed at equidistant points of time and at each review, the system is classified into possible number of states and subsequently a decision has to be made. The economic consequences of the decisions taken at the review times are reflected in costs. These properties of Markov decision process are employed to study the maintenance condition of deteriorating equipment. The result could be used to study the status of equipment used in various organizations to determine their efficiency and productivity.*

Keywords: *Markov decision process, decision epochs, maintenance, deterioration, transition probability, probability, transition reward.*

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I. Introduction

In systems where the availability of the equipment is of concern, the efficient repair and/or replacement of this equipment are critical to the continued usefulness of the system. This repair and replacement of equipment is called maintenance. Maintenance has a definite influence on operating costs, either through its own (maintenance) labor or through its effect on system downtime and efficiency. Maintenance can also be used to increase the probability that a system will continue to operate efficiently, given that it is allowed a certain amount of downtime for repairs. The purpose of maintenance is to return a failed or deteriorating component to a satisfactory operating state. Deterioration is a process where the condition of a component gradually worsens. If left unattended, the process will lead to deterioration failure.

In general, there are two maintenance strategies that can be applied to return the component to a satisfactory operating state. The first strategy is to repair only when a component fails to operate or when its cost of operation becomes exorbitantly high. This is called corrective maintenance (CM), or emergency repair. The second strategy is to inspect periodically and then to repair and/or replace as is needed. This is called preventive maintenance (PM).

The purpose of PM is to eliminate the need for radical treatment sometime in the future (which is almost always much more expensive). PM, by its very nature, can be scheduled and controlled for a minimum cost. Clearly, too little maintenance may have very costly consequences but on the other hand, it may not be economical to perform it too frequently. The problem of replacement or overhaul of equipment, which deteriorates with usage, is one of the standard applications of Markov processes. Continuous operation, or daily start/stop operation, without failure requires a comprehensive plant preventive maintenance planning and diagnostic system for each equipment and component. In this paper, we analyze the optimal average cost and fraction of time that equipment is in bad condition in long-run, using Markov decision process with value iteration.

II. Markov Decision Models

A Markov model is a special type of dynamic model with which the probabilistic evolution of a system can be modeled in time. The main assumption underlying such a model is that all information about the future behavior is captured in the state description. In other words, the present state provides all relevant information about the future behavior and knowledge about previous states is not necessary. More formally, a Markov chain is a discrete time process governed by a discrete state space E (observed at discrete time points) and transition matrix P, for which the Markov property holds, i.e.

$$P_{ij} = P(X_{t+1} = j \mid X_0 = i_0, X_1 = i_1, \ldots\ldots\ldots, X_t = i) = P(X_{t+1} = j \mid X_t = i)$$

(1.0)

Development of Value Iteration Method

Let X_n denote the state of the process at time n and a_n the action chosen at time n, then the above is equivalent to stating that:

$$P(X_{n+1} = j \mid X_0, a_0, X_1, a_1, \ldots\ldots\ldots X_n = i, a_n = a) = P_{ij}(a) \tag{1.1}$$

We consider an aperiodic irreducible Markov chain with m states (m< ∞)and the transition probability matrix P with every transition, i to j associate a reward R_{ij} if we let $V_i^{(n)}$ be the expected total earnings (reward) in the next n transitions, given that the system is in state i at present.
A simple relation can be given

For $\left\{V_i^{(n)}\right\}_{n-1}^{\infty}$ as follows:

$$V_i^{(n)} = \sum_{j=1}^{m} P_{ij}\left[R_{ij} + V_j^{(n-1)}\right] \quad i = 1, 2, \ldots\ldots\ldots, m;\, n = 1, 2, 3, \ldots\ldots\ldots \tag{1.2}$$

Let $\sum_{j=1}^{m} P_{ij} R_{ij} = Q_i$ (1.3)

Equation (1.2) can now be written as:

$$V_i^{(n)} = Q_i + \sum_{j=1}^{m} P_{ij} V_j^{(n-1)} \tag{1.4}$$

Setting n = 1, 2 … we get

$$V_i^{(1)} = Q_i + \sum_{j-1}^{m} P_{ij} V^{(0)} \tag{1.5}$$

$$V_i^{(2)} = Q_i + \sum_{j=1}^{m} P_{ij}\left[Q_j + \sum_{k=1}^{m} P_{jk} V_k^{(0)}\right] = Q_i + \sum_{j=1}^{m} P_{ij} Q_j + \sum_{k=1}^{m}\sum_{j=1}^{m} P_{ij} P_{jk} V_k^{(0)}$$

$$= Q_i + \sum_{j=1}^{m} P_{ij} Q_j + \sum_{k=1}^{m}\sum_{j=1}^{m} P_{ij} P_{jk} V_k^{(0)} = Q_i + \sum_{j=1}^{m} P_{ij} Q_j + \sum_{k=1}^{m} P_{ik}^{(2)} V_k^{(0)} \tag{1.6}$$

Where $P_{ij}^{(n)}$ is the $(i, j)^{th}$ element of the matrix P^n

$$\text{Let } V^{(n)} = \begin{bmatrix} V_1^{(n)} \\ V_2^{(n)} \\ V_3^{(n)} \\ . \\ . \\ V_m^{(n)} \end{bmatrix} Q = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ . \\ . \\ Q_m \end{bmatrix} \tag{1.7}$$

Equation (1.4) can be put in matrix notation as

$$V^{(n)} = Q + PQ + P^2 V^{(0)} \tag{1.8}$$

Extending this to a nth term, we have

$$V^{(n)} = Q + PQ + P^2 Q + \ldots\ldots + P^{(n-1)} Q + P^n V^{(0)} = \left[1 + \sum_{k-1}^{n-1} PK\right] Q + P^n V^{(0)} \tag{1.9}$$

1.6 Model Formulation

We consider a company with several machines which must be regularly supervised and maintained for effective operation of the company. The company is faced with tough time in maintaining its equipment and would like to use analytical technique in making decisions between preventive maintenance, corrective maintenance and investing in the improved version of the machines. The equipment undergoes state changes between stable, unstable and bad condition based on the following transition probability and reward matrix.

$$p = \begin{bmatrix} p_{11} & p_{12} & \ldots\ldots p_{1m} \\ . & . & . \\ . & . & . \\ p_{m1} & p_{m2} & \ldots\ldots p_{mm} \end{bmatrix}$$

$$R = \begin{bmatrix} R_{11} & R_{12} & \ldots.R_{1m} \\ .. & .. & \ldots.. \\ R_{m1} & R_{m2} & \ldots.R_{mm} \end{bmatrix} \qquad (2.0)$$

Let the equipment status be described by a random variable (X). Suppose that equipment status is considered for several years (n). We obtain a stochastic process X_n; n=1, 2, 3……

We assume the condition of the equipment to be

i) Stable condition (state1)

ii) Unstable condition (state 2)

iii) Bad condition (state 3)

We consider the states to be mutually exclusive and exhaustive. It is further assume that the stochastic process is governed by a first order Markov chain mentioned in (1.0). The possible transition between the states are presented in fig (1) below

The transition between the states is described in by the following transition diagram.

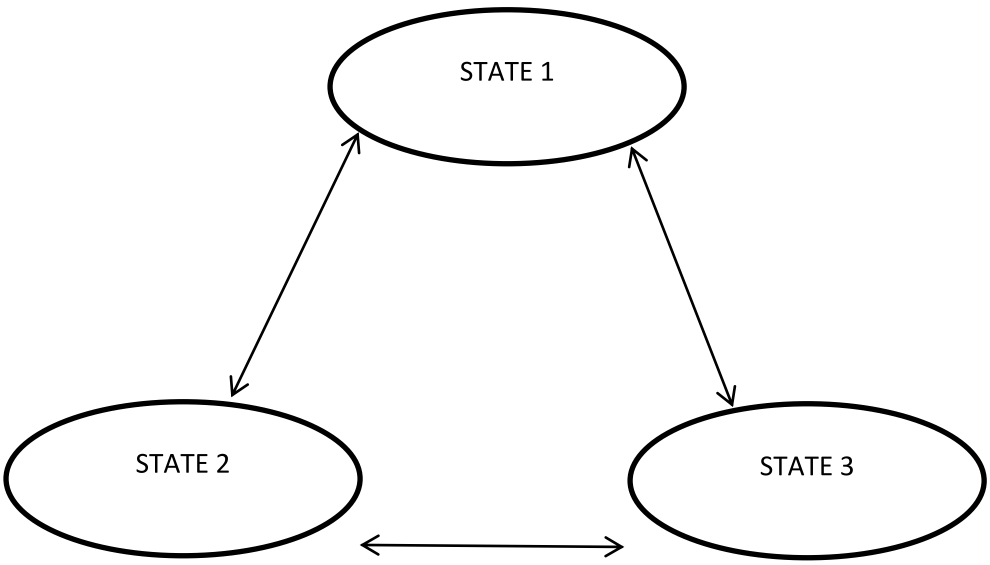

Figure 1.1 the transition diagram for the equipment.

From the transition diagram and equation (1.0) where m,n=1,2,3 we obtain a transition matrix p.

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} \qquad (2.1)$$

When the equipment condition is in state 1, let there be two alternatives open to the manufacturer:

Alternative 1: preventive maintenance

Alternative 2: continue without change

Let the corresponding transition probabilities and rewards be given as:

$$\begin{bmatrix} {}^{1}P_{11} & {}^{1}P_{12} & {}^{1}P_{13} \end{bmatrix} \qquad (2.2)$$

$$\begin{bmatrix} {}^1R_{11} & {}^1R_{12} & {}^1R_{13} \end{bmatrix} \tag{2.3}$$

and

$$\begin{bmatrix} {}^2P_{11} & {}^2P_{12} & {}^2P_{13} \end{bmatrix} \tag{2.4}$$

$$\begin{bmatrix} {}^2R_{11} & {}^2R_{12} & {}^2R_{13} \end{bmatrix} \tag{2.5}$$

When the equipment condition is in state 2, let the alternatives be given as:
Alternative 1: corrective maintenance
Alternative 2: investing in the improved version
Suppose that the corresponding transition probability and rewards be given as:

$$\begin{bmatrix} {}^1P_{21} & {}^1P_{22} & {}^1P_{23} \end{bmatrix} \tag{2.6}$$

$$\begin{bmatrix} {}^1R_{21} & {}^1R_{22} & {}^1R_{23} \end{bmatrix} \tag{2.7}$$

and

$$\begin{bmatrix} {}^2P_{21} & {}^2P_{22} & {}^2P_{23} \end{bmatrix} \tag{2.8}$$

$$\begin{bmatrix} {}^2R_{21} & {}^2R_{22} & {}^2R_{23} \end{bmatrix} \tag{2.9}$$

When theequipment conditionis in state 3, let the following alternatives be open to him.
Alternative 1: replacement of the bad parts
Alternative 2: change the equipment

$$\begin{bmatrix} {}^1P_{31} & {}^1P_{32} & {}^1P_{33} \end{bmatrix} \tag{3.0}$$

$$\begin{bmatrix} {}^1R_{31} & {}^1R_{32} & {}^1R_{33} \end{bmatrix} \tag{3.1}$$

and

$$\begin{bmatrix} {}^2P_{31} & {}^2P_{32} & {}^2P_{33} \end{bmatrix} \tag{3.2}$$

$$\begin{bmatrix} {}^2R_{31} & {}^2R_{32} & {}^2R_{33} \end{bmatrix}$$

Model application

The company is faced with tough time in maintaining its equipment and would like to use analytical technique in making decisions between preventive maintenance, corrective maintenance and investing in the improved version of the machines. The equipment undergoes state changes between stable, unstable and bad condition based on the following transition probability and reward matrix.

Let the transition probabilities (P_{ij}) and the corresponding reward (R_{ij}) be given as follows:

$$P = P_{ij} = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix}; i,\ j = 1, 2, 3 \tag{3.3}$$

$$R = R_{ij} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}; i,\ j = 1, 2, 3$$

Let D be the decision set and we have two alternative decisions available to the manufacturer. That is, Alternative 1; and Alternative 2; Thus in every state we have k = 1, 2 $\in$ D.
We shall determine when the equipment is best condition for every n using equation (1.2). Since our interest is to minimize cost and maximize profit, the alternative that yields more earnings constitutes the best option for the states and time.

The machine undergoes state changes base on the following transition probabilities and the corresponding reward matrices in (thousand naira) respectively.

Suppose we have the following transition probabilities corresponding reward matrices

$$P_{ij} = \begin{pmatrix} {}^1p_{11} & {}^1p_{12} & {}^1p_{13} \\ {}^1p_{21} & {}^1p_{22} & {}^1p_{23} \\ {}^1p_{31} & {}^1p_{32} & {}^1p_{33} \end{pmatrix} = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.3 & 0.5 & 0.2 \end{pmatrix} \qquad i, j = 1, 2, 3 \quad for \quad k = 1$$

$$R_{ij} = \begin{pmatrix} {}^1R_{11} & {}^1R_{12} & {}^1R_{13} \\ {}^1R_{21} & {}^1R_{22} & {}^1R_{23} \\ {}^1R_{31} & {}^1R_{32} & {}^1R_{33} \end{pmatrix} = \begin{pmatrix} 6 & 5 & 4 \\ 4 & 3 & -2 \\ 4 & -6 & 3 \end{pmatrix} \qquad i, j = 1, 2, 3 \quad for \quad k = 1 \qquad (3.4)$$

$$P_{ij} = \begin{pmatrix} {}^2p_{11} & {}^2p_{12} & {}^2p_{13} \\ {}^2p_{21} & {}^2p_{22} & {}^2p_{23} \\ {}^2p_{31} & {}^2p_{32} & {}^2p_{33} \end{pmatrix} = \begin{pmatrix} 0.6 & 0.1 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.1 & 0.3 & 0.6 \end{pmatrix} \qquad i, j = 1, 2, 3 \quad for \quad k = 2$$

$$R_{ij} = \begin{pmatrix} {}^2R_{11} & {}^2R_{12} & {}^2R_{13} \\ {}^2R_{21} & {}^2R_{22} & {}^2R_{23} \\ {}^2R_{31} & {}^2R_{32} & {}^2R_{33} \end{pmatrix} = \begin{pmatrix} 6 & 3 & 2 \\ -2 & -10 & 5 \\ -7 & -8 & -1 \end{pmatrix} \qquad i, j = 1, 2, 3 \quad for \quad k = 2$$

We substitute the above values into the optimality equation in (1.2) to obtain our iterations.

III. Discussion of results

Table 1: The summary result of the optimal policies and rewards

N	$d_1^{(n)}$	$d_2^{(n)}$	$d_3^{(n)}$	${}^oV_1^{(n)}$	${}^oV_2^{(n)}$	${}^oV_3^{(n)}$
1	1	1	1	540	160	-120
2	1	1	1	872	274	-8.2
3	1	2	1	1,433.6	675.8	300.2
4	1	2	1	2,488.96	1,655.38	1,128.22
5	1	2	1	4,539.06	3,622.12	2,928.24
6	1	2	1	8,572.57	7,563.93	6,686.67

The results indicate the best policies for each n. $d_i^{(n)} \; where \; n = 1, 2, 3, 4, 5, 6 \; and \; i = 1, 2, 3$. Thus, we have obtained the best options for the three states for six months. In addition to the best policies, the corresponding expected rewards are also provided.

For the first month, $d_1^{(1)} = 1 \; with \; {}^oV_1^{(1)} = 540$ means that the best option for state 1 is for the company owner to use preventive maintenance and the corresponding expected reward is five hundred and forty thousand naira.

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