Bipolar Intuitionistic M Fuzzy Group and Anti M Fuzzy Group.

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Abstract: The concept of a Bipolar intuitionistic M fuzzy group is a new algebraic structure of a bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group are defined and some related properties are investigated. The purpose of the study is to implement the fuzzy set theory and group theory of bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group. The relation between of a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group are established.

Keywords: M fuzzy group, anti M fuzzy group, bipolar intuitionistic fuzzy set, bipolar intuitionistic M fuzzy group, bipolar intuitionistic anti M fuzzy group.

I. Introduction

The concept of fuzzy sets was initiated by L.A. Zadeh [13] then it has become a vigorous area of research in engineering, medical science, graph theory. Rosenfeld [12] gave the idea of fuzzy subgroups. Bipolar valued fuzzy sets was introduced by K.M. Lee [5] are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1,1]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1) indicates that elements somewhat satisfy the property and the membership degree [-1,0) indicates that elements somewhat satisfy the implicit counter property. The author W. R. Zhang [15] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. The author Mourad Oqla [6] commenced the concept of an intuitionistic anti M fuzzy group. Chakrabartey and R.Nandra [1] investigated note on union and intersection of intuitionistic fuzzy sets. P.S. Das, A. Rajeshkumar [2,3] were analyzed fuzzy groups and level subgroups. R. Muthuraj [8,9] introduced the concept of bipolar fuzzy subgroup of a M fuzzy group and bipolar anti M fuzzy group. He was introduced the notion of an image and pre-image of a bipolar fuzzy subset of a bipolar fuzzy subgroup of a group and also discuss some of its properties of bipolar M fuzzy subgroup under M homomorphism and M anti homomorphism. We discuss some of its properties with bipolar intuitionistic M fuzzy subgroup of M fuzzy group and anti M fuzzy group are established under M homomorphism and M anti homomorphism.

II. Preliminaries

In this paper $G = (G,*)$ is a finite groups, e is the identity element of G, and $xy$ mean $x*y$ the fundamental definitions that will be used in the sequel.

Definition 2.1 Let G be a non empty group, A bipolar intuitionistic fuzzy set (IFS) A in G is an object of the form $A = \{x, \mu_A^+(x), \mu_A^-(x), \nu_A^+(x), \nu_A^-(x) / x \in G\}$ where $\mu_A^+: G \to [0,1]$ and $\nu_A^+: G \to [0,1]$, $\mu_A^- : G \to [-1,0]$ and $\nu_A^- : G \to [-1,0]$ is called degree of positive membership, degree of negative membership and the degree of positive non membership, degree of negative non membership respectively.

Definition 2.2 [8] Let G be a group. A bipolar valued intuitionistic fuzzy set (IFS) A of G is called a bipolar intuitionistic fuzzy subgroup of G, if for all $x, y \in G$

i) $\mu_A^+(xy) \geq \min(\mu_A^+(x), \mu_A^+(y))$ and $\nu_A^+(xy) \leq \max(\nu_A^+(x), \nu_A^+(y))$

ii) $\mu_A^-(xy) \leq \max(\mu_A^-(x), \mu_A^-(y))$ and $\nu_A^-(xy) \geq \min(\nu_A^-(x), \nu_A^-(y))$

iii) $\mu_A^+(x^{-1}) = \mu_A^-(x), \mu_A^-(x^{-1}) = \mu_A^+(x)$ and $\nu_A^+(x^{-1}) = \nu_A^-(x), \nu_A^-(x^{-1}) = \nu_A^+(x)$.
Example 2.3

\[ \mu^+(x) = \begin{cases} 
0.7 & \text{if } x = 1 \\
0.6 & \text{if } x = -1 \\
0.4 & \text{if } x = i, -i 
\end{cases} \]

\[ \nu^+(x) = \begin{cases} 
0.2 & \text{if } x = 1 \\
0.3 & \text{if } x = -1 \\
0.5 & \text{if } x = i, -i 
\end{cases} \]

Definition 2.4 [7] Let \( G \) be a group. A bipolar valued IFS (or) bipolar IFS \( A \) of \( G \) is called a bipolar intuitionistic anti fuzzy subgroup of \( G \), if for all \( x, y \in G \)

i) \( \mu^+_A(xy) \leq \max(\mu^+_A(x), \mu^+_A(y)) \) and \( \nu^+_A(xy) \geq \min(\nu^+_A(x), \nu^+_A(y)) \)

ii) \( \mu^-_A(xy) \geq \min(\mu^-_A(x), \mu^-_A(y)) \) and \( \nu^-_A(xy) \leq \max(\nu^-_A(x), \nu^-_A(y)) \)

iii) \( \mu^+_A(x^{-1}) = \mu^-_A(x), \mu^-_A(x^{-1}) = \mu^+_A(x) \) and \( \nu^+_A(x^{-1}) = \nu^-_A(x), \nu^-_A(x^{-1}) = \nu^+_A(x) \).

Example 2.5

\[ \mu^+(x) = \begin{cases} 
0.4 & \text{if } x = 1 \\
0.6 & \text{if } x = -1 \\
0.7 & \text{if } x = i, -i 
\end{cases} \]

\[ \nu^+(x) = \begin{cases} 
0.5 & \text{if } x = 1 \\
0.3 & \text{if } x = -1 \\
0.2 & \text{if } x = i, -i 
\end{cases} \]

Definition 2.6 Let \( G \) be an \( M \) group and \( A \) be a bipolar intuitionistic fuzzy subgroup of \( G \), then \( A \) is called a bipolar intuitionistic \( M \) fuzzy group of \( G \), if for all \( x \in G \) and \( m \in M \) then,

i) \( \mu^+_A(mx) \geq \mu^+_A(x) \) and \( \nu^+_A(mx) \leq \nu^+_A(x) \). ii) \( \mu^-_A(mx) \leq \mu^-_A(x) \) and \( \nu^-_A(mx) \geq \nu^-_A(x) \).

Example 2.7

Consider \( 1 \in M \)

\[ \mu^+(x) = \begin{cases} 
0.7 & \text{if } x = 1 \\
0.6 & \text{if } x = -1 \\
0.4 & \text{if } x = i, -i 
\end{cases} \]

\[ \nu^+(x) = \begin{cases} 
0.2 & \text{if } x = 1 \\
0.3 & \text{if } x = -1 \\
0.5 & \text{if } x = i, -i 
\end{cases} \]

Definition 2.8 Let \( G \) be an \( M \) group and \( A \) be a bipolar intuitionistic anti fuzzy subgroup of \( G \), then \( A \) is called a bipolar intuitionistic anti \( M \) fuzzy group of \( G \), if for all \( x \in G \) and \( m \in M \) then,

i) \( \mu^+_A(mx) \leq \mu^+_A(x) \) and \( \nu^+_A(mx) \geq \nu^+_A(x) \). ii) \( \mu^-_A(mx) \geq \mu^-_A(x) \) and \( \nu^-_A(mx) \leq \nu^-_A(x) \).

Example 2.9

Consider \( 1 \in M \)

\[ \mu^+(x) = \begin{cases} 
0.4 & \text{if } x = 1 \\
0.6 & \text{if } x = -1 \\
0.7 & \text{if } x = i, -i 
\end{cases} \]

\[ \nu^+(x) = \begin{cases} 
0.5 & \text{if } x = 1 \\
0.3 & \text{if } x = -1 \\
0.2 & \text{if } x = i, -i 
\end{cases} \]

Theorem 2.10 If \( A \) and \( B \) are bipolar intuitionistic \( M \) fuzzy group of \( G \), then \( A \cap B \) is a bipolar intuitionistic \( M \) fuzzy group of \( G \). Proof Consider \( m \in M \) and \( x \in A \cap B \) implies \( x \in A, x \in B \)

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Consider \( \mu_{A \cap B}^+ (mx) = \min (\mu_A^+ (mx), \mu_B^+ (mx)) \geq \min (\mu_A^+ (x), \mu_B^+ (x)) = \mu_{A \cap B}^+ (x) \).
Therefore \( \mu_{A \cap B}^+ (mx) \geq \mu_{A \cap B}^+ (x) \).

Consider \( \nu_{A \cap B}^+ (mx) = \max (\nu_A^+ (mx), \nu_B^+ (mx)) \leq \max (\nu_A^+ (x), \nu_B^+ (x)) = \nu_{A \cap B}^+ (x) \).
Therefore \( \nu_{A \cap B}^+ (mx) \leq \nu_{A \cap B}^+ (x) \).

Consider \( \mu_{A \cup B}^- (mx) = \max (\mu_A^- (mx), \mu_B^- (mx)) \leq \max (\mu_A^- (x), \mu_B^- (x)) = \mu_{A \cup B}^- (x) \).
Therefore \( \mu_{A \cup B}^- (mx) \leq \mu_{A \cup B}^- (x) \).

Consider \( \nu_{A \cup B}^- (mx) = \min (\nu_A^- (mx), \nu_B^- (mx)) \geq \min (\nu_A^- (x), \nu_B^- (x)) = \nu_{A \cup B}^- (x) \).
Therefore \( \nu_{A \cup B}^- (mx) \geq \nu_{A \cup B}^- (x) \).

Therefore \( A \cap B \) is a bipolar intuitionistic M fuzzy group of \( G \).

**Theorem 2.11** If \( A \) is a bipolar intuitionistic M fuzzy group of \( G \), then \( \overline{A} = A \) is also a bipolar intuitionistic M fuzzy group of \( G \).

**Proof** Let \( m \in M \) and \( x \in A \)

Consider \( \mu_A^+ (mx) = \nu_A^+ (mx) = \mu_A^+ (mx) \geq \mu_A^+ (x) \). Therefore \( \mu_A^+ (mx) \geq \mu_A^+ (x) \).

Consider \( \nu_A^+ (mx) = \mu_A^+ (mx) = \nu_A^+ (mx) \leq \nu_A^+ (x) \). Therefore \( \nu_A^+ (mx) \leq \nu_A^+ (x) \).

Consider \( \mu_A^- (mx) = \nu_A^- (mx) = \mu_A^- (mx) \leq \mu_A^- (x) \). Therefore \( \mu_A^- (mx) \leq \mu_A^- (x) \).

Consider \( \nu_A^- (mx) = \mu_A^- (mx) = \nu_A^- (mx) \geq \nu_A^- (x) \). Therefore \( \nu_A^- (mx) \geq \nu_A^- (x) \).

Therefore \( \overline{A} = A \) is a bipolar intuitionistic M fuzzy group of \( G \).

**Theorem 2.12** Union of any two bipolar intuitionistic M fuzzy group is also a bipolar intuitionistic M fuzzy group if either is contained in the other.

**Proof** Let \( A \) and \( B \) be a bipolar intuitionistic M fuzzy group of \( G \).

To prove that \( A \cup B \) is a bipolar intuitionistic M fuzzy group of \( G \) if \( A \subseteq B \) (or) \( B \subseteq A \) \( \Rightarrow A \cup B = A \)

If \( A \subseteq B \Rightarrow A \cup B = B \) (or) \( B \subseteq A \Rightarrow A \cup B = A \)

Let \( m \in M \) & \( x \in A \cup B \)

Consider \( \mu_{A \cup B}^+ (mx) = \max (\mu_A^+ (mx), \mu_B^+ (mx)) \geq \max (\mu_A^+ (x), \mu_B^+ (x)) = \mu_{A \cup B}^+ (x) \).
Therefore \( \mu_{A \cup B}^+ (mx) \geq \mu_{A \cup B}^+ (x) \).

Consider \( \nu_{A \cup B}^+ (mx) = \min (\nu_A^+ (mx), \nu_B^+ (mx)) \leq \min (\nu_A^+ (x), \nu_B^+ (x)) = \nu_{A \cup B}^+ (x) \).
Therefore \( \nu_{A \cup B}^+ (mx) \leq \nu_{A \cup B}^+ (x) \).
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Consider \( \mu_{\text{A}\cup\text{B}}(mx) = \min(\mu_{\text{A}}(mx), \mu_{\text{B}}(mx)) \leq \min(\mu_{\text{A}}(x), \mu_{\text{B}}(x)) = \mu_{\text{A}\cup\text{B}}(x) \).

Therefore \( \mu_{\text{A}\cup\text{B}}(mx) \leq \mu_{\text{A}\cup\text{B}}(x) \).

Consider \( v_{\text{A}\cup\text{B}}(mx) = \max(v_{\text{A}}(mx), v_{\text{B}}(mx)) \geq \max(v_{\text{A}}(x), v_{\text{B}}(x)) = v_{\text{A}\cup\text{B}}(x) \).

Therefore \( v_{\text{A}\cup\text{B}}(mx) \geq v_{\text{A}\cup\text{B}}(x) \).

Hence union of any two bipolar intuitionistic M fuzzy group is also a bipolar intuitionistic M fuzzy group if either is contained in the other.

**Theorem 2.13** If \( A \) is a bipolar intuitionistic anti M fuzzy group of \( G \), then \( A = A \) is also a bipolar intuitionistic anti M fuzzy group of \( G \).

**Proof** Consider \( m \in M \) and \( x \in A \)

Consider \( \mu_{\text{A}}^+(mx) = v_{\text{A}}^+(mx) = \mu_{\text{A}}^+(mx) \leq \mu_{\text{A}}^+(x) \). Therefore \( \mu_{\text{A}}^+(mx) \leq \mu_{\text{A}}^+(x) \).

Consider \( \mu_{\text{A}}^-(mx) = v_{\text{A}}^-(mx) = v_{\text{A}}^+(mx) \geq v_{\text{A}}^+(x) \). Therefore \( \mu_{\text{A}}^-(mx) \geq v_{\text{A}}^+(x) \).

Therefore \( A = A \) is a bipolar intuitionistic anti M fuzzy group of \( G \).

**Theorem 2.14** Union of any two bipolar intuitionistic anti M fuzzy group is also a bipolar intuitionistic anti M fuzzy group if either is contained in the other.

**Proof** Let \( A \) and \( B \) be a bipolar intuitionistic anti M fuzzy group of \( G \). To prove that \( A \cup B \) is also a bipolar intuitionistic anti M fuzzy group of \( G \) if \( A \subseteq B \) (or) \( B \subseteq A \Rightarrow A \cup B = A \).

Consider \( m \in M \) and \( x \in A \cup B \).

Consider \( \mu_{\text{A}\cup\text{B}}^+(mx) = \max(\mu_{\text{A}}^+(mx), \mu_{\text{B}}^+(mx)) \leq \max(\mu_{\text{A}}^+(x), \mu_{\text{B}}^+(x)) = \mu_{\text{A}\cup\text{B}}^+(x) \).

Therefore \( \mu_{\text{A}\cup\text{B}}^+(mx) \leq \mu_{\text{A}\cup\text{B}}^+(x) \).

Consider \( v_{\text{A}\cup\text{B}}^+(mx) = \min(v_{\text{A}}^+(mx), v_{\text{B}}^+(mx)) \leq \min(v_{\text{A}}^+(x), v_{\text{B}}^+(x)) = v_{\text{A}\cup\text{B}}^+(x) \).

Therefore \( v_{\text{A}\cup\text{B}}^+(mx) \leq v_{\text{A}\cup\text{B}}^+(x) \).

Therefore union of any two bipolar intuitionistic anti M fuzzy group is also a bipolar intuitionistic anti M fuzzy group if either is contained in the other.
III. Some Result Based On Bipolar Intuitionistic M Fuzzy Group And Anti M Fuzzy Group Of G.

Theorem 3.1 Let $\mu$ and $\nu$ be a bipolar intuitionistic fuzzy subset of an M fuzzy group then $\mu = (\mu^+, \mu^-)$ is a bipolar intuitionistic M fuzzy group of G if and only if $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti M fuzzy group of G.

Proof Let $\mu = (\mu^+, \mu^-)$ be a bipolar intuitionistic M fuzzy group of G. To prove $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti M fuzzy group of G.

i) $\mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\} \Rightarrow 1 - \nu^+(xy) \geq 1 - \min\{1 - \nu^+(x), 1 - \nu^+(y)\}$

$\Rightarrow \nu^+(xy) \leq 1 - \min\{1 - \nu^+(x), 1 - \nu^+(y)\}$

$\Rightarrow \nu^+(xy) \leq \max\{\nu^+(x), \nu^+(y)\}$

Therefore $\mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\} \Rightarrow \nu^+(xy) \leq \max\{\nu^+(x), \nu^+(y)\}$.

ii) $\mu^-(xy) \leq \max\{\mu^-(x), \mu^-(y)\} \Rightarrow -1 - \nu^-(xy) \leq -1 - \max\{-1 - \nu^-(x), -1 - \nu^-(y)\}$

$\Rightarrow \nu^-(xy) \geq -1 - \max\{-1 - \nu^-(x), -1 - \nu^-(y)\}$

$\Rightarrow \nu^-(xy) \geq \min\{-\nu^-(x), -\nu^-(y)\}$

Therefore $\mu^-(xy) \leq \max\{\mu^-(x), \mu^-(y)\} \Rightarrow \nu^-(xy) \geq \min\{-\nu^-(x), -\nu^-(y)\}$.

iii) $\mu^+(x^{-1}) = \mu^+(x) \Rightarrow 1 - \nu^+(x^{-1}) = 1 - \nu^+(x) \Rightarrow \nu^+(x^{-1}) = \nu^+(x)$ and $\mu^-(x^{-1}) = \mu^-(x) \Rightarrow -1 - \nu^-(x^{-1}) = -1 - \nu^-(x) \Rightarrow \nu^-(x^{-1}) = \nu^-(x)$.

iv) $\mu^+(mx) \geq \mu^+(x) \Rightarrow 1 - \mu^+(mx) \leq 1 - \mu^+(x) \Rightarrow \nu^+(mx) \leq \nu^+(x)$.

Therefore $\mu^+(mx) \geq \mu^+(x)$ and $\nu^+(mx) \leq \nu^+(x)$.

v) $\mu^-(mx) \leq \mu^-(x) \Rightarrow -1 - \mu^-(mx) \geq -1 - \mu^-(x) \Rightarrow \nu^-(mx) \geq \nu^-(x)$.

Therefore $\mu^-(mx) \leq \mu^-(x)$ and $\nu^-(mx) \geq \nu^-(x)$.

Therefore $\mu = (\mu^+, \mu^-)$ is a bipolar intuitionistic M fuzzy group of G if and only if $\nu = (\nu^+, \nu^-)$ is a bipolar intuitionistic anti M fuzzy group of G.

Definition 3.2 [9] Let f and g be a mapping from a group $G_1$ to a group $G_2$. Let $\mu = (\mu^+, \mu^-)$ and $\phi = (\phi^+, \phi^-)$ and $\nu = (\nu^+, \nu^-)$, $\psi = (\psi^+, \psi^-)$ are bipolar intuitionistic fuzzy subset in $G_1$ and $G_2$ respectively, then the image $f(\mu)$ and $g(\nu)$ is a bipolar intuitionistic fuzzy subset is defined by $f(\mu) = (f(\mu^+), f(\mu^-))$ and $g(\nu) = (g(\nu^+), g(\nu^-))$ of $G_2$ for all $u, v \in G_2$.
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\[
f(\mu^+)(u) = \max\{\mu^+(x); x \in f^{-1}(u)\} \text{ if } f^{-1}(u) \neq \emptyset, 0 \quad \text{and} \quad g(\nu^+)(v) = \min\{\nu^+(x); x \in g^{-1}(v)\} \text{ if } g^{-1}(v) \neq \emptyset, 0 \quad \text{and} \quad f(\mu^-)(u) = \min\{\mu^-(x); x \in f^{-1}(u)\}, \text{ if } f^{-1}(u) \neq \emptyset, 0 \quad g(\nu^-)(v) = \max\{\nu^-(x); x \in g^{-1}(v)\} \text{ if } g^{-1}(v) \neq \emptyset, 0.
\]

The preimage \(f^{-1}(\phi)\) is under \(f\) and \(g^{-1}(\psi)\) is under \(g\) is defined by the bipolar intuitionistic fuzzy subset of \(G_1\) for all \(x \in G_1, (f^{-1}(\phi^+))(x) = \phi^+(f(x)); (f^{-1}(\phi^-))(x) = \phi^-(f(x))\) and \((g^{-1}(\psi^+))(x) = \psi^+(g(x)); (g^{-1}(\psi^-))(x) = \psi^-(g(x)).\)

**Definition 3.3** [8] Let \(G_1\) and \(G_2\) be any two bipolar intuitionistic M groups then the function \(f: G_1 \rightarrow G_2\) and \(g: G_1 \rightarrow G_2\) is said to be an intuitionistic M homomorphism if,

i) \( f(xy) = f(x)f(y) \) for all \( x, y \in G_1 \)

ii) \( f(mx) = m f(x) \) for all \( m \in M \) and \( x \in G_1 \)

iii) \( g(xy) = g(x)g(y) \) for all \( x, y \in G_1 \)

iv) \( g(mx) = m g(x) \) for all \( m \in M \) and \( x \in G_1 \).

**Definition 3.4** [8] Let \(G_1\) and \(G_2\) be any two bipolar intuitionistic M groups (not necessarily commutative) then the function \(f: G_1 \rightarrow G_2\) and \(g: G_1 \rightarrow G_2\) is said to be an intuitionistic M anti homomorphism if,

i) \( f(xy) = f(x)f(y) \) for all \( x, y \in G_1 \)

ii) \( f(mx) = m f(x) \) for all \( m \in M \) and \( x \in G_1 \)

iii) \( g(xy) = g(x)g(y) \) for all \( x, y \in G_1 \)

iv) \( g(mx) = m g(x) \) for all \( m \in M \) and \( x \in G_1 \).

**Theorem 3.5** Let \(f\) and \(g\) be an intuitionistic M homomorphism from an M fuzzy group of \(G_1\) onto an M fuzzy group of \(G_2\). If \(\mu = (\mu^+, \mu^-)\) is a bipolar intuitionistic M fuzzy group of \(G_1\) then \(f(\mu)\) the image of \(\mu\) under \(f\) is a bipolar intuitionistic M fuzzy group of \(G_2\) if and only if \(\nu = (\nu^+, \nu^-)\) is a bipolar intuitionistic anti M fuzzy group of \(G_1\) then \(g(\nu)\) is the image of \(\nu\) under \(g\) is a bipolar intuitionistic anti M fuzzy group of \(G_2\).

**Proof** Let \(f: G_1 \rightarrow G_2\) and \(g: G_1 \rightarrow G_2\) be an intuitionistic M homomorphism.

Let \(\mu = (\mu^+, \mu^-)\) and \(\nu = (\nu^+, \nu^-)\) is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group of \(G_1\). To prove a bipolar intuitionistic fuzzy subset \(f(\mu) = (f(\mu^+), f(\mu^-))\) and \(g(\nu) = (g(\nu^+), g(\nu^-))\) on \(G_2\) is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group.

Let \(u, v \in G_2\) since \(f\) is a intuitionistic M homomorphism and so there exist \(x, y \in G_1\) such that \(f(x) = u \& f(y) = v\) it follows that \(xy \in f^{-1}(uv)\). We have to prove that \(g\) is an intuitionistic M homomorphism so there exist \(x, y \in G_1\) such that \(g(x) = u \& g(y) = v\) it follows that \(xy \in g^{-1}(uv)\).
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\[ i) \ (\mu^+)(uv) = \max\{\mu(x): x \in f^{-1}(uv)\} \]
\[ \geq \max\{\min\{\mu(x), \mu^+(y)\}: x \in f^{-1}(u), y \in f^{-1}(v)\} \]
\[ = \min\{f(\mu^+)(u), f(\mu^+)(v)\}. \]

Therefore \( f(\mu^+)(uv) \geq \min\{f(\mu^+)(u), f(\mu^+)(v)\} \)
\[ \Leftrightarrow \min\{1 - g(\mu^+)(uv)\} \geq \min\{1 - g(\mu^+)(u), 1 - g(\mu^+)(v)\} \]
\[ \Leftrightarrow g(\mu^+)(uv) \leq 1 - \min\{1 - g(\mu^+)(u), 1 - g(\mu^+)(v)\} \]
\[ \Leftrightarrow g(\mu^+)(uv) \leq \max\{g(\mu^+)(u), g(\mu^+)(v)\}. \]

Hence \( f(\mu^+)(uv) \geq \min\{f(\mu^+)(u), f(\mu^+)(v)\} \)
\[ \Leftrightarrow g(\mu^+)(uv) \leq \max\{g(\mu^+)(u), g(\mu^+)(v)\}. \]

\[ ii) \ (\mu^-)(uv) = \max\{\mu^+(z): z = xy \in f^{-1}(uv)\} \]
\[ \leq \max\{\max\{\mu^-(x), \mu^-(y)\}: x \in f^{-1}(u), y \in f^{-1}(v)\} \]
\[ = \max\{f(\mu^-)(u), f(\mu^-)(v)\}. \]

Therefore \( f(\mu^-)(uv) \leq \max\{f(\mu^-)(u), f(\mu^-)(v)\} \)
\[ \Leftrightarrow (-1 - g(\mu^-)(uv)) \leq \max\{-1 - g(\mu^-)(u), -1 - g(\mu^-)(v)\} \]
\[ \Leftrightarrow g(\mu^-)(uv) \geq -1 - \max\{-1 - g(\mu^-)(u), -1 - g(\mu^-)(v)\} \]
\[ \Leftrightarrow g(\mu^-)(uv) \geq \min\{g(\mu^-)(u), g(\mu^-)(v)\}. \]

Hence \( f(\mu^-)(uv) \leq \max\{f(\mu^-)(u), f(\mu^-)(v)\} \)
\[ \Leftrightarrow g(\mu^-)(uv) \geq \min\{g(\mu^-)(u), g(\mu^-)(v)\}. \]

\[ iii) \ \text{Now } f(\mu^+)(u^{-1}) = \max\{\mu^+(x): x \in f^{-1}(u^{-1})\} = \max\{\mu^+(x^{-1}): x^{-1} \in f^{-1}(u)\} = f(\mu^+)(u) \]

Therefore \( f(\mu^+)(u^{-1}) = f(\mu^+)(u) \Leftrightarrow \min\{1 - g(\mu^+)(u^{-1})\} = (1 - g(\mu^+)(u)) \)
\[ \Leftrightarrow g(\mu^+)(u^{-1}) = g(\mu^+)(u). \]

Hence \( f(\mu^+)(u^{-1}) = f(\mu^+)(u) \Leftrightarrow g(\mu^+)(u^{-1}) = g(\mu^+)(u). \)

\[ iv) \ f(\mu^-)(u^{-1}) = \min\{\mu^-(x): x \in f^{-1}(u^{-1})\} = \min\{\mu^-(x^{-1}): x^{-1} \in f^{-1}(u)\} = f(\mu^-)(u). \]

Therefore \( f(\mu^-)(u^{-1}) = f(\mu^-)(u) \Leftrightarrow (-1 - g(\mu^-)(u^{-1})) = (-1 - g(\mu^-)(u)) \)
\[ \Leftrightarrow g(\mu^-)(u^{-1}) = g(\mu^-)(u). \]
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Hence \( f(\mu^- (u^{-1}) = f(\mu^- (u) \Leftrightarrow g(\nu^- (u^{-1}) = g(\nu^- (u) \)

Therefore \( f(\mu) \) and \( g(\nu) \) is a bipolar fuzzy subgroup of \( G_2 \).

v) Let \( m \in M \) and \( u \in G_2 \),

\[
f(\mu^+(mu)) \geq \max\{\mu^+(x) : x \in f^{-1}(u)\} = f(\mu^+(u)).
\]

Therefore \( f(\mu^+(mu)) \geq f(\mu^+(u)) \Leftrightarrow (1 - g(\nu^+)(mu)) \geq (1 - g(\nu^+)(u))
\]

\[
\Rightarrow g(\nu^+(mu)) \leq g(\nu^+(u)).
\]

Hence \( f(\mu^+(mu)) \geq f(\mu^+(u)) \Leftrightarrow g(\nu^+(mu)) \leq g(\nu^+(u)).
\]

vi) \( f(\mu^- (mu)) \leq \min\{\mu^-(x) : x \in f^{-1}(u)\} = f(\mu^- (u)).
\]

Therefore \( f(\mu^- (mu)) \leq f(\mu^- (u)) \Leftrightarrow (1 - g(\nu^-)(mu)) \leq (1 - g(\nu^-)(u))
\]

\[
\Rightarrow g(\nu^-(mu)) \geq g(\nu^-(u)).
\]

Hence \( f(\mu^- (mu)) \leq f(\mu^- (u)) \Leftrightarrow g(\nu^- (mu)) \geq g(\nu^- (u)).
\]

Therefore if \( \mu \) be a bipolar intuitionistic M fuzzy group of \( G_1 \) then \( f(\mu) \) is a bipolar intuitionistic M fuzzy group of \( G_2 \) if and only if \( \nu \) be a bipolar intuitionistic anti M fuzzy group of \( G_1 \) then \( g(\nu) \) be a bipolar intuitionistic anti M fuzzy group of \( G_2 \).

**Theorem 3.6** The M homomorphic preimage of a bipolar intuitionistic M fuzzy group of \( G_2 \) is a bipolar intuitionistic M fuzzy group of \( G_1 \) if and only if M homomorphic preimage of a bipolar intuitionistic anti M fuzzy group of \( G_2 \) is a bipolar intuitionistic anti M fuzzy group of \( G_1 \).

**Proof** Let \( f : G_1 \to G_2 \) and \( g : G_1 \to G_2 \) be an intuitionistic M homomorphism. let \( \phi = (\phi^+, \phi^-) \) is a bipolar intuitionistic M fuzzy group of \( G_2 \) and \( \psi = (\psi^+, \psi^-) \) is a bipolar intuitionistic anti M fuzzy group of \( G_2 \),to prove a bipolar fuzzy subset \( \mu = (\mu^+, \mu^-) \) and \( \nu = (\nu^+, \nu^-) \) on \( G_1 \) is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group where \( \mu = f^{-1}(\phi) & \nu = g^{-1}(\psi) \)

i) Consider \( x, y \in G_1 \)

\[
(f^{-1}(\phi))^+(xy) = \phi^+(f(xy))
\]

\[
\geq \min\{\phi^+(f(x)), \phi^+(f(y))\}
\]

\[
= \min\{(f^{-1}(\phi))^+(x), (f^{-1}(\phi))^+(y)\}.
\]

Therefore \( (f^{-1}(\phi))^+(xy) \geq \min\{(f^{-1}(\phi))^+(x), (f^{-1}(\phi))^+(y)\} \).
\[ (g^{-1}(\psi))^{+}(xy) = \psi^{+}(g(xy)) \leq \max\{\psi^{+}(g(x)), \psi^{+}(g(y))\} \]
\[ = \max\{(g^{-1}(\psi))^{+}(x), (g^{-1}(\psi))^{+}(y)\}. \]

Hence \((f^{-1}(\phi))^{+}(xy) \geq \min\{(f^{-1}(\phi))^{+}(x), (f^{-1}(\phi))^{+}(y)\} \]
\[ \Leftrightarrow (g^{-1}(\psi))^{+}(xy) \leq \max\{(g^{-1}(\psi))^{+}(x), (g^{-1}(\psi))^{+}(y)\}. \]

ii) Let \( x, y \in G_1 \)
\[ (f^{-1}(\phi))^{-}(xy) = \phi^{-}(f(xy)) \leq \max\{\phi^{-}(f(x)), \phi^{-}(f(y))\} \]
\[ = \max\{(f^{-1}(\phi))^{-}(x), (f^{-1}(\phi))^{-}(y)\}. \]

Therefore \((f^{-1}(\phi))^{-}(xy) \leq \max\{(f^{-1}(\phi))^{-}(x), (f^{-1}(\phi))^{-}(y)\} \]
\[ \Leftrightarrow (g^{-1}(\psi))^{-}(xy) = \psi^{-}(g(xy)) \]
\[ \geq \min\{\psi^{-}(g(x)), \psi^{-}(g(y))\} \]
\[ = \min\{(g^{-1}(\psi))^{+}(x), (g^{-1}(\psi))^{+}(y)\}. \]

Hence \((f^{-1}(\phi))^{-}(xy) \leq \max\{(f^{-1}(\phi))^{-}(x), (f^{-1}(\phi))^{-}(y)\} \]
\[ \Leftrightarrow (g^{-1}(\psi))^{-}(xy) \geq \min\{(g^{-1}(\psi))^{-}(x), (g^{-1}(\psi))^{-}(y)\}. \]

iii) Consider \( x \in G_1 \)
\[ (f^{-1}(\phi))^{+}(x^{-1}) = \phi^{+}(f(x^{-1})) \]
\[ = \phi^{+}(f(x)) \text{ as } \phi \text{ is a bipolar M fuzzy group} \]
\[ = (f^{-1}(\phi))^{+}(x). \]

Therefore \((f^{-1}(\phi))^{+}(x^{-1}) = (f^{-1}(\phi))^{+}(x) \)
\[ \Leftrightarrow (g^{-1}(\psi))^{+}(x^{-1}) = \psi^{+}(g(x^{-1})) \]
\[ = \psi^{+}(g(x)) \text{ as } g \text{ is an M homomorphism} \]
\[ = (g^{-1}(\psi))^{+}(x). \]

Hence \((f^{-1}(\phi))^{+}(x^{-1}) = (f^{-1}(\phi))^{+}(x) \Leftrightarrow (g^{-1}(\psi))^{+}(x^{-1}) = (g^{-1}(\psi))^{+}(x). \]

iv) \((f^{-1}(\phi))^{-}(x^{-1}) = \phi^{-}(f(x^{-1})) \)
\[ = \phi^{-}(f(x)) \text{ as } \phi \text{ is a bipolar M fuzzy group} \]
\[ = (f^{-1}(\phi))^{-}(x). \]
Therefore \((f^{-1}(\phi))^-(x^{-1}) = (f^{-1}(\phi))^-(x)\)
\[
\Leftrightarrow (g^{-1}(\psi))^-(x^{-1}) = \psi^-(g(x^{-1}))
\]
\[= \psi^-(g(x)) \text{ as } g \text{ is an M homomorphism}
\]
\[= \psi^-(g(x)) \text{ as } \psi \text{ is a bipolar anti M fuzzy group}
\]
\[= (g^{-1}(\psi))^-(x).
\]
Hence \((f^{-1}(\phi))^-(x^{-1}) = (f^{-1}(\phi))^-(x) \Leftrightarrow (g^{-1}(\psi))^-(x^{-1}) = (g^{-1}(\psi))^-(x).
\]

v) \((f^{-1}(\phi))^+(mx) = \phi^+(f(mx))\)
\[\geq \phi^+(f(x)) \text{ as } \phi \text{ is bipolar M fuzzy group}
\]
\[= (f^{-1}(\phi))^+(x).
\]
Therefore \((f^{-1}(\phi))^+(mx) \geq (f^{-1}(\phi))^+(x)\)
\[
\Leftrightarrow (g^{-1}(\psi))^+(mx) = \psi^+(g(mx))
\]
\[= \psi^+(mg(x)) \text{ as } g \text{ is an M homomorphism}
\]
\[\leq \psi^+(g(x)) \text{ as } \psi \text{ is a bipolar anti M fuzzy group}
\]
\[= (g^{-1}(\psi))^+(x).
\]
Hence \((f^{-1}(\phi))^+(mx) \geq (f^{-1}(\phi))^+(x) \Leftrightarrow (g^{-1}(\psi))^+(mx) \leq (g^{-1}(\psi))^+(x).
\]

vi) \((f^{-1}(\phi))^-(mx) = \phi^-(f(mx))\)
\[\leq \phi^-(f(x)) \text{ as } \phi \text{ is bipolar M fuzzy group}
\]
\[= (f^{-1}(\phi))^-(x).
\]
Therefore \((f^{-1}(\phi))^-(mx) \leq (f^{-1}(\phi))^-(x) \Leftrightarrow (g^{-1}(\psi))^-(mx) = \psi^-(g(mx))
\]
\[= \psi^-(mg(x)) \text{ as } g \text{ is an M homomorphism}
\]
\[\geq \psi^-(g(x)) \text{ as } \psi \text{ is a bipolar anti M fuzzy group}
\]
\[= (g^{-1}(\psi))^-(x).
\]
Hence \((f^{-1}(\phi))^-(mx) \leq (f^{-1}(\phi))^-(x) \Leftrightarrow (g^{-1}(\psi))^-(mx) \geq (g^{-1}(\psi))^-(x).
\]

Hence \(f^{-1}(\phi) = \mu\) is a bipolar intuitionistic M fuzzy group of \(G_1\) and \(g^{-1}(\psi) = \nu\) is a bipolar intuitionistic anti M fuzzy group of \(G_1\).

**Theorem 3.7** Let \(f\) and \(g\) be an intuitionistic M anti homomorphism from an M fuzzy group of \(G_1\) onto an M fuzzy group of \(G_2\). If \(\mu = (\mu^+, \mu^-)\) is a bipolar intuitionistic M fuzzy group of \(G_1\) then \(f(\mu)\) the image of \(\mu\) under \(f\) is a bipolar intuitionistic M fuzzy group of \(G_2\) if and only if

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\( \nu = (\nu^+, \nu^-) \) is a bipolar intuitionistic anti M fuzzy group of \( G_1 \) then \( g(\nu) \) the image of \( \nu \) under \( g \) is a bipolar intuitionistic anti M fuzzy group of \( G_2 \).

**Proof** Let \( f: G_1 \to G_2 \) and \( g: G_1 \to G_2 \) be an intuitionistic M anti homomorphism and let \( \mu = (\mu^+, \mu^-) \) and \( \nu = (\nu^+, \nu^-) \) is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group of \( G_1 \). \( \mu^+: G_1 \to [0,1] \& \nu^- : G_1 \to [-1,0] \& G \to [0,1] \& : G \to [-1,0] \) are mappings, to prove a bipolar intuitionistic fuzzy subset \( f(\mu) = (f(\mu)^+, f(\mu)^-) \) and \( g(\nu) = (g(\nu)^+, g(\nu)^-) \) on \( G_2 \) is a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group.

Let \( u, v \in G_2 \) since \( f \) is an intuitionistic M anti homomorphism so there exist \( x, y \in G_1 \) such that \( f(x) = u \) and \( f(y) = v \), it follows that \( xy \in f^{-1}(uv) \) that \( g \) is intuitionistic M anti homomorphism so there exist \( x, y \in G_1 \) such that \( g(x) = u, g(y) = v \) which implies \( xy \in g^{-1}(uv) \)

i) Let \( f(\mu)^+(uv) \geq \max\{\mu^+(xy): x \in f^{-1}(u), y \in f^{-1}(v)\} \)

\[ \geq \max\{\min\{\mu^+(x), \mu^+(y)\}: x \in f^{-1}(u), y \in f^{-1}(v)\} \]

\[ = \min\{f(\mu)^+(u), f(\mu)^+(v)\}. \]

Therefore \( f(\mu)^+(uv) \geq \min\{f(\mu)^+(u), f(\mu)^+(v)\} \)

\[ \Leftrightarrow (1 - g(\nu)^+)(uv) \geq \min\{(1 - g(\nu)^+)(u), (1 - g(\nu)^+)(v)\} \]

\[ \Leftrightarrow g(\nu)^+(uv) \leq 1 - \min\{(1 - g(\nu)^+)(u), (1 - g(\nu)^+)(v)\} \]

\[ \Leftrightarrow g(\nu)^+(uv) \leq \max\{g(\nu)^+(u), g(\nu)^+(v)\}. \]

Hence \( (f(\mu)^+(uv) \geq \min\{f(\mu)^+(u), f(\mu)^+(v)\} \)

\[ \Leftrightarrow g(\nu)^+(uv) \leq \max\{g(\nu)^+(u), g(\nu)^+(v)\}. \]

ii) Let \( f(\mu)^-(uv) \leq \max\{\mu^-(xy): x \in f^{-1}(u), y \in f^{-1}(v)\} \)

\[ \leq \max\{\min\{\mu^-(x), \mu^-(y)\}: x \in f^{-1}(u), y \in f^{-1}(v)\} \]

\[ = \max\{f(\mu)^-(u), f(\mu)^-(v)\}. \]

Therefore \( f(\mu)^-(uv) \leq \max\{f(\mu)^-(u), f(\mu)^-(v)\} \)

\[ \Leftrightarrow (-1 - g(\nu)^-)(uv) \leq \max\{(-1 - g(\nu)^-)(u), (-1 - g(\nu)^-)(v)\} \]

\[ \Leftrightarrow g(\nu)^-(uv) \geq -1 - \max\{(-1 - g(\nu)^-)(u), (-1 - g(\nu)^-)(v)\} \]

\[ \Leftrightarrow g(\nu)^-(uv) \geq \min\{g(\nu)^-(u), g(\nu)^-(v)\}. \]

Hence \( f(\mu)^-(uv) \leq \max\{f(\mu)^-(u), f(\mu)^-(v)\} \)

\[ \Leftrightarrow g(\nu)^-(uv) \geq \min\{g(\nu)^-(u), g(\nu)^-(v)\}. \]
iii) Consider $f(\mu^+(u^{-1})) = \max\{\mu^+(x); x \in f^{-1}(u^{-1})\}$

$$= \max\{\mu^+(x^{-1}); x^{-1} \in f^{-1}(u)\}$$

$$= f(\mu^+(u)).$$

Therefore $f(\mu^+(u^{-1})) = f(\mu^+(u)) \iff (1 - g(\nu^+))(u^{-1}) = (1 - g(\nu^+))(u)$

$$\iff g(\nu^+)(u^{-1}) = g(\nu^+)(u).$$

Hence $f(\mu^+(u^{-1})) = f(\mu^+(u)) \iff g(\nu^+)(u^{-1}) = g(\nu^+)(u)$.

iv) Consider $f(\mu^- (u^{-1})) = \min\{\mu^-(x); x \in f^{-1}(u^{-1})\} = \min\{\mu^-(x^{-1}); x^{-1} \in f^{-1}(u)\}$

$$= (f(\mu^-))(u).$$

Therefore $f(\mu^- (u^{-1})) = f(\mu^- (u)) \iff (-1 - g(\nu^-))(u^{-1}) = (-1 - g(\nu^-))(u)$

$$\iff g(\nu^-)(u^{-1}) = g(\nu^-)(u).$$

Hence $f(\mu^- (u^{-1})) = f(\mu^- (u)) \iff g(\nu^-)(u^{-1}) = g(\nu^-)(u)$.

Therefore $f(\mu)$ and $g(\nu)$ is a bipolar fuzzy subgroup of $G_2$.

v) Consider $m \in M$ and $u \in G_2$

$$f(\mu)(mu) = \max\{\mu^+(mu); x \in f^{-1}(u)\} \geq \max\{\mu^+(x); x \in f^{-1}(u)\}$$

$$= f(\mu^+)(u).$$

Therefore $f(\mu)(mu) \geq f(\mu^+)(u) \iff (1 - g(\nu^+))(mu) \geq (1 - g(\nu^+))(u)$

$$\iff g(\nu^+)(mu) \leq g(\nu^+)(u).$$

Hence $f(\mu)(mu) \geq f(\mu^+)(u) \iff g(\nu^+)(mu) \leq g(\nu^+)(u)$.

vi) Consider $m \in M$ and $u \in G_2$

$$f(\mu^-)(mu) = \min\{\mu^-(mu); x \in f^{-1}(u)\} \leq \min\{\mu^-(x); x \in f^{-1}(u)\}$$

$$= f(\mu^-)(u).$$

Therefore $f(\mu^-)(mu) \leq f(\mu^-)(u) \iff (-1 - g(\nu^-))(mu) \leq (-1 - g(\nu^-))(u)$

$$\iff g(\nu^-)(mu) \geq g(\nu^-)(u).$$

Hence $f(\mu^-)(mu) \leq f(\mu^-)(u) \iff g(\nu^-)(mu) \geq g(\nu^-)(u)$.

Hence if $\mu$ be a bipolar intuitionistic M fuzzy group of $G_1$ then $f(\mu)$ is a bipolar M fuzzy group of $G_2$ if and only if $\nu$ be a bipolar anti M fuzzy group of $G_1$ then $g(\nu)$ be a bipolar intuitionistic anti M fuzzy group of $G_2$.
IV. Conclusion

The concept of a bipolar intuitionistic M fuzzy group is a new algebraic structure of a bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group are defined and some related properties are investigated. The purpose of the study is to implement the fuzzy set theory and group theory of bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group. The relation between of a bipolar intuitionistic M fuzzy group and bipolar intuitionistic anti M fuzzy group are established. We hope that our results can also be extended to other algebraic system.

References

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