IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 2 Ver. IV (Mar. - Apr. 2017), PP 38-41
www.iosrjournals.org

Bölcsföldi-Birkás Prime Numbers

József Bölcsföldi[1], György Birkás[2]
[1](*Eötvös Loránd University Budapest and Perczel Mór Secondary Grammar School Siófok, Hungary)*
[2](*Baross Gábor Secondary Technical School Siófok, Hungary)*

Abstract: *After defining, Bölcsföldi-Birkás prime numbers will be presented from 23 to 2327753. How many Bölcsföldi-Birkás prime numbers are there in the interval (10^{p-1}, 10^p) (where p is a prime number)? On the one hand, it has been counted by computer among the prime numbers with up to 13-digits. On the other hand, the function (1) gives the approximate number of Bölcsföldi-Birkás prime numbers in the interval (10^{p-1},10^p). The function (2) gives the approximate number of Bölcsföldi-Birkás prime numbers where all digits are 3 or 7 in the interval (10^{p-1},10^p). Near-proof reasonig has emerged from the conformity of Mills' prime numbers with* ***Bölcsföldi:*** *Birkás prime numbers. The set of Bölcsföldi-Birkás prime numbers is probably infinite.*

I. Introduction

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ($F_0=0$, $F_1=1$, $F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where $1\leq x\leq y$), Pell-primes ($P_0=0$, $P_1=1$, $P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found a further set of special prime numbers within the set of prime numbers. It is the set of Bölcsföldi-Birkás prime numbers.

II. Bölcsföldi-Birkás Prime Numbers [3], [9], [10].

Definition:a positive integer number is a Bölcsföldi-Birkás prime number, if
a/ the positive integer number is prime, b/ all digits are prime, c/ the number of digits is prime,
d/ the sum of digits is prime.

Positive integer numbers that meet the conditions a/ and b/ have been known for a long time: these are prime numbers containing prime digits [1]. The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all three conditions (a/, b/, c/) at the same time are known: they are full prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Bölcsföldi-Birkás prime numbers (Fig.1, Fig.2).

Bölcsföldi-Birkás prime number p has the following sum form:

2.1 $$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) \in \{2, 3, 5, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3, 7\} \text{ and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The Bölcsföldi-Birkás prime numbers are as follows (the last digit can only be 3 or 7):
23,
223, 227, 337, 353, 373, 557, 577, 733, 757, 773,
22573, 23327, 25237, 25253, 25523, 27253, 27527, 32233,32237, 32257, 32323, 32327 ,33223, 33353, 33377,
33533, 33773, 35227, 35353, 35533, 35537, 35573, 35753, 37223, 37337, 52237, 52253, 52727, 53353, 53777,
55333, 55337, 55373, 55733, 57223, 57557, 57737, 57773, 72253, 73553, 73757, 75223, 75227, 75353, 75377,
75533, 75557, 75577, 75773,77377, 77557, 77573, 77773,
2222333, 2222533, 2222537, 2222573, 2223233, 2223253, 2225233, 2225323, 2227727, 2232323, 2232523,
2233223, 2233337, 2233373, 2233757, 2235227, 2235353, 2235377, 2235557, 2235773, 2237537, 2237773,
2252233, 2252273, 2253257, 2253323, 2253353, 2253557, 2253773, 2255333, 2255575, 2257373, 2257553,
2257733, 2272253, 2272727, 2273357, 2275733, 2277553, 2277733, 2322253, 2323337, 2323733, 2323777,
2325377,2 325773, 2327737, 2327753, etc.

2
T(p) is the factual frequency of Bölcsföldi-Birkás prime numbers in the interval (10^{p-1}, 10^p). T(2)=1, T(3)=10, T(5)=53, T(7)=496, T(11)=75119, T(13)=934734. etc.
S(p) function gives the number of Bölcsföldi-Birkás prime numbers in the interval (10^{p-1},10^p). We think that

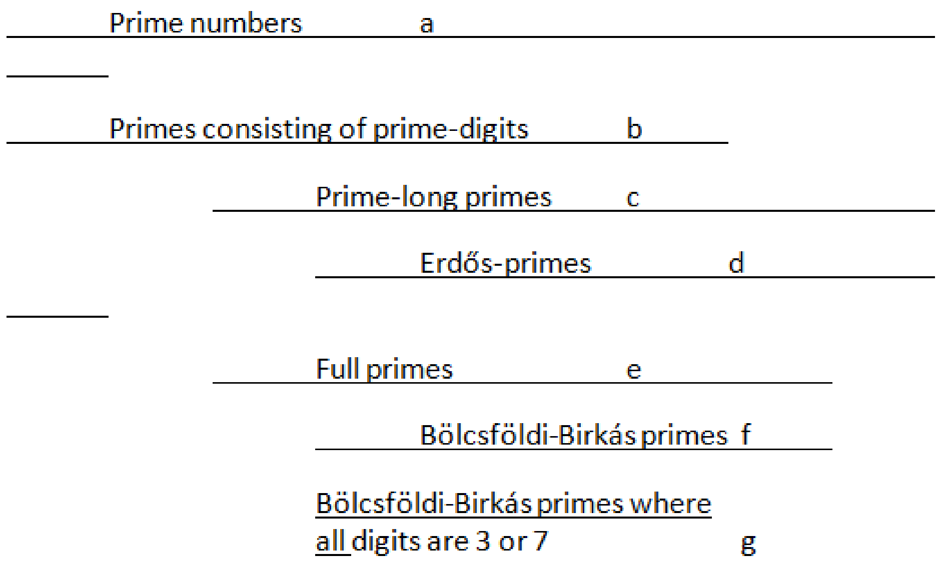
$\mathbf{S(p)=0.21x4^{p-2}}$, where p is prime. (1)

The factual number of Bölcsföldi-Birkás primes and the number of Bölcsföldi-Birkás primes calculated according to function (1) are as follows:

Number of digits p	The factual number of Bölcsföldi_Birkás primes in the interval $(10^{p-1},10^p)$ T(p)	The number of Bölcsföldi_Birkás primes according to function $S(p)=0.21x4^{p-2}$	T(p)/S(p)
2	1	0.21	4,76
3	10	0,84	11,90
5	53	13	4,08
7	496	215	2,31
11	75119	55050	1,36
13	934734	880804	1,06

Fig.1

Fig.2



where g C ? C ? C ?,? C ? and f C d.

3. The Bölcsföldi-Birkás prime numbers, where all digits are 3 or 7, are as follows:
337, 373, 733, 773, 33377, 33773, 37337, 77377, 77773, etc.

These numbers have the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \text{ where } e_j(p) \in \{3,7\} \text{ and } k(p)+1 \text{ is prime and } \sum_{j=0}^{k(p)} e_j(p) \text{ is prime.}$$

We have found a Bölcsföldi-Birkás number with 109-digits where the digits are only 3 or 7: ………………………………………………..77337373337733737. The dots mean digit 3. The sum of digits is the prime number 359.

V(p) is the factual frequency of Bölcsföldi-Birkás prime numbers in the interval $(10^{p-1},10^p)$, where all digits are 3 or 7: V(2)=0, V(3)=4, V(5)=5, V(7)=13, V(11)=129, V(13)=501, V(17)=4748, V(19)=21248.

W(p) function gives the number of Bölcsföldi-Birkás prime numbers in the interval $(10^{p-1},10^p)$, where all digits

are 3 or 7. We think that
$W(p)=1/7x2^{p-2}$ where p is prime. (2)
The V(p) factual number of Bölcsföldi-Birkás primes and the number of Bölcsföldi-Birkás primes calculated according to function (2) (where all digits are 3 or 7) are as follows:

p	The V(p) factual number in the interval $(10^{p-1},10^p)$	The number according to function $W(p)=1/7x2^{p-2}$	V(p)/W(p)
2	0	0,14	-
3	4	0,29	13,79
5	5	1	5
7	13	5	2,6
11	129	73	1,77
13	501	293	1,71
17	4748	4681	1,01
19	21248	18725	1,13

The set of Bölcsföldi-Birkás primes where all digits are 3 or 7 is a real subset of the set of Bölcsföldi-Birkás primes: $g \subset f$.

In the twin prime-pair (p-2, p) the value of V (p) will considerably increase: V(13)~4xV(11), V(19)~4xV(17), etc.

$\lim_{p\to\infty} V(p)=\infty$ is probably, consequently $\lim_{p\to\infty} T(p)=\infty$ is probably, where p is prime.

4. Number of the elements of the set of Bölcsföldi-Birkás prime numbers [3], [9],[10].

4.1 In the twin prime-pair (p-2,p) the number of Bölcsföldi-Birkás primes increases approximately ten times: T(5)~10xT(3), T(7)~10xT(5), T(13)~10xT(11),…T(p)~10xT(p-2). The number of the twin prime-pairs is probably infinite (Jitang Chang, 2013). Thus, it can be stated that the number of the set of Bölcsföldi-Birkás prime numbers is probably infinite: $\lim_{p\to\infty} T(p)=\infty$ is probably, where p is prime.

4.2Let's take the set of Mills' prime numbers!

Definition: The number $m=[M \text{ ad } 3^n]$ is a prime number, where M=1,306377883863080690468614492602 is the Mills' constant, and n=1,2,3,… is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…

The connection n→m is the following:1→2, 2→11, 3→1361, 4→2521008887,… The Mills' prime number $m=[M \text{ ad } 3^n]$ corresponds with the interval $(10^{m-1},10^m)$ and vice versa. For instance:2→$(10, 10^2)$, 11→$(10^{10},10^{11})$, 1361→$(10^{1360},10^{1361})$, etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals $(10^{m-1}, 10^m)$ that contain at least one Mills' prime number is infinite. The number of Bölcsföldi-Birkás primes in the interval $(10^{m-1},10^m)$ is $S(m)=0.21x4^{m-2}$.

The number of Bölcsföldi-Birkás prime numbers is probably infinite:$\lim_{p\to\infty} T(p)=\infty$ is probably where p is prime

III. Conclusion

65 different sets of special prime numbers have been known. We have found the 66th set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

Acknowledgement

The authors would like to thank you for publishing this article.

References

[1]. http://oeis.org/A019546
[2]. Freud, Robert – Gyarmati, Edit: *Number theory* (in Hungarian), Budapest, 2000
[3]. http://ac.inf.elte.hu → VOLUMES → VOLUME 44 (2015)→ VOLLPRIMZAHLENMENGE→FULL TEXT
[4]. http://primes.utm.edu/largest.html
[5]. http://mathworld.wolfram.com/SmarandacheSequences.html
[6]. Dubner, H.:"Fw:(Prime Numbers) Rekord Primes All Prime digits" Februar 17. 2002
[7]. http://listserv.nodak.edu/scripts/wa.exe?A2=ind0202&L=nmbrthry&P=1697
[8]. Harman, Glyn: Counting Primes whose Sum of Digits is Prime.Journal of Integer Sequences (2012. , Vol. 15, 12.2.2.)
[9]. ANNALES Universitatis Scientiarum Budapestiensis de Rolando Eötvös Nominate Sectio Computatorica,2015, pp 221-22
[10]. International Journal of Mathematics and Statistics Invention, February 2017: http://www. ijmsi.org/Papers/ Volume.5.Issue.2/B05020407.pdf

József Bölcsföldi
Perczel Mór Secondary Grammar School
H-8600 Siófok
Március 15 park 1
Hungary
bolcsteri@gmail.com

György Birkás
Baross Gábor Secondary Technical School
H-8600 Siófok
Kardvirágköz 7/a
Hungary
birkasgy@enternet.hu