Least Square Plane and Least Square Quadric Surface Approximation by Using Modified Lagrange’s Method

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Abstract: Now a days Surface fitting is applied all engineering and medical fields. Kamron Saniee, 2007 find a simple expression for multivariate Lagrange’s Interpolation. We derive a least square plane and least square quadric surface Approximation from a given N+1 tabular points when the function is unique. We used least square method technique. We can apply this method in surface fitting also.

Key words: Least square, quadric surface , Normal equations

1. Introduction

Least square principle method is one the best approximation method in numerical analysis for line and curve fitting, this method was invented by Lagrange’s. A function \( y = f(x) \) may be given in discrete data \( x_k, y_k \). The best approximation in the least square is defined as that for which the constants \( c_i, i = 0,1,2,3 \ldots n \) are determined so that the aggregate of \( w(x)E \) over given domain \( D \) is as small as possible, where \( w(x) > 0 \$ is the weight function for the function whose values are given at \( N + 1 \) points \( x_0, x_1 \ldots x_N \).

We have

\[
I(c_0, c_1 \ldots c_n) = \sum_{k=0}^{N} w(x_k)\left[f(x_k) - \sum_{i=0}^{n} \phi_i(x_k)\right]^2 = \text{minimum} \quad (1)
\]

Where \( \phi_i(x) = x^i, i = 0,1,2,3 \ldots n \) and \( w(x) = 1 \)

The necessary conditions for (1) to have a minimum value is that

\[
\frac{\partial I}{\partial c_i} = 0, \quad i = 0,1,2,3 \ldots n .
\]

This gives a system of \( n + 1 \)linear equations in \( n + 1 \)constants. These equations are called normal equations. Then we get approximated nth degree polynomial function of \( x \).

1. Least square plane: Given a discrete data \( (x_k, y_k), k = 0,1, \ldots N, \quad N \geq 2 \).

Consider \( \phi(x, y) = c_0 + c_1 x + c_2 y \)

Such that \( I(c_0, c_1, c_2) = \sum_{k=0}^{N} [z_k - (c_0 + c_1 x_k + c_2 y_k)]^2 = \text{minimum} \)

Then Normal equations \( \frac{\partial I}{\partial c_i} = 0, \quad i = 0,1,2 \).

ie.,

\[
c_0(N + 1) + c_1 \sum x_k + c_2 \sum y_k = \sum z_k
\]

\[
c_0 \sum x_k + c_1 \sum x_k^2 + c_2 \sum x_k y_k = \sum x_k z_k
\]

\[
c_0 \sum y_k + c_1 \sum x_k y_k + c_2 \sum y_k^2 = \sum y_k z_k
\]

Solving this system of equations we get \( c_0, c_1 \) and \( c_2 \).

Example 1: Suppose that given data points \((-1,1,-2),(1,2,3),(1,1,2)\)

that lie on \( z = f(x, y) \). These points define uniquely a linear function in two variables,

\[
s_{oz} = c_0 + c_1 x_k + c_2 y_k, \quad k = 0,1,2
\]
The coefficients satisfy the normal equations

\[3c_0 + 1c_1 + 4c_2 = 3\]
\[c_0 + 3c_1 + 2c_2 = 7\]
\[4c_0 + 2c_1 + 6c_2 = 6\]

Solving these equations we get \(c_0 = -1, c_1 = 2, c_2 = 1\)
Thus \(z = -1 + 2x + y\).

II. Least square quadric surface

Given a discrete data \((x_k, y_k), k = 0, 1, \ldots, N, N \geq 5\)
Consider \(\emptyset(x, y) = c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy\)
Such that
\[
I(c_0, c_1, c_2, c_3, c_4, c_5) = \sum_{k=0}^{N} [z_k - (c_0 + c_1 x_k + c_2 y_k + c_3 x_k^2 + c_4 y_k^2 + c_5 x_k y_k)]^2 = \text{minimum}
\]
The coefficients satisfy normal equations \(\frac{\partial I}{\partial c_i} = 0, i = 0, 1, 2, 3, 4, 5\).

\[c_0(N+1) + c_1 \sum x_k + c_2 \sum y_k + c_3 \sum x_k^2 + c_4 \sum y_k^2 + c_5 \sum x_k y_k = \sum z_k\]
\[c_0 \sum x_k + c_1 \sum x_k^2 + c_2 \sum x_k y_k + c_3 \sum x_k^3 + c_4 \sum x_k y_k^2 + c_5 \sum x_k^2 = \sum x_k z_k\]
\[c_0 \sum y_k + c_1 \sum x_k y_k + c_2 \sum y_k^2 + c_3 \sum x_k y_k^2 + c_4 \sum y_k^3 + c_5 \sum y_k y_k^2 = \sum y_k z_k\]
\[c_0 \sum x_k^2 + c_1 \sum x_k^3 + c_2 \sum x_k^2 y_k + c_3 \sum x_k^4 + c_4 \sum x_k^2 y_k^2 + c_5 \sum x_k^3 y_k = \sum x_k^2 z_k\]
\[c_0 \sum y_k^2 + c_1 \sum x_k y_k^2 + c_2 \sum y_k^3 + c_3 \sum x_k y_k^2 + c_4 \sum y_k^4 + c_5 \sum y_k y_k^3 = \sum y_k^2 z_k\]
\[c_0 \sum x_k y_k + c_1 \sum x_k y_k^2 + c_2 \sum x_k y_k^2 + c_3 \sum x_k^3 y_k + c_4 \sum x_k y_k^3 + c_5 \sum x_k^2 y_k^2 = \sum x_k y_k z_k\]
Solve these equations we get \(c_0, c_1, c_2, c_3, c_4, c_5\).

Example 2: Suppose that given data points \((0, 0, 0), (0, 1, -4), (1, -1, 1)\)
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(1, 2, −2), (2, 1, 4), (1, 3, −3) those lie on \( z = f(x, y) \). These data points satisfy uniquely a degree of two variable function.

\[ z_k = c_0 + c_1 x_k + c_2 x_k^2 + c_3 y_k + c_4 y_k^2 + c_5 x_k y_k, \quad k = 0, 1, 2, 3, 4, 5 \]

The coefficients \( c_0, c_1, c_2, c_3, c_4, c_5 \) satisfy the normal equations

\[
\begin{align*}
6c_0 + 5c_1 + 6c_2 + 7c_3 + 16c_4 + 6c_5 &= -4 \\
5c_0 + 7c_1 + 6c_2 + 11c_3 + 16c_4 + 8c_5 &= 4 \\
6c_0 + 6c_1 + 16c_2 + 8c_3 + 36c_4 + 16c_5 &= -14 \\
7c_0 + 11c_1 + 8c_2 + 19c_3 + 18c_4 + 12c_5 &= 12 \\
16c_0 + 16c_1 + 36c_2 + 18c_3 + 100c_4 + 36c_5 &= -34 \\
6c_0 + 8c_1 + 16c_2 + 12c_3 + 36c_4 + 18c_5 &= -6 \\
\end{align*}
\]

Solving this system of equations we get

\( c_0 = 0, c_1 = -1, c_2 = -4, c_3 = 1, c_4 = 0, c_5 = 3 \)

Thus \( z = -x - 4y + x^2 + 3xy \)

References


