Electronic Spin Orbit Coupling

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Abstract: In this note we will see how representations of the two-dimensional Unitary Group \( U(2) \) used long ago by the Author for the Many Electron Problem, gives rise to an equilateral triangle in a torus that also defines a Theta Function with Equiharmonic frequencies and whose vertices are up and down spins leading to spin-orbit coupling.

Keywords: Equiharmonics, Jacobi Theta Function, Coupling Constant, Lattice.

I. Introduction

In this note we see how representations of the 2 dimensional Unitary Group \( U(2) \), used long ago by [3] de Wet for the ‘Many Electron Problem’, leads to a lattice that is an equilateral triangle in a torus Fig. 1 that has been shown [4] to define a Theta Function nome \( q \) with equiharmonic frequencies \( \tau = \sin 120^\circ = \sin \omega \) or \( \sin 60^\circ \) Furthermore \( \omega \) is also the angle of the tritangent to a cubic surface defining the Exceptional Lie algebra \( E_6 \) [5] that has been used by de Wet [4] to map the Standard Model. Specifically there are equilateral triangles that rotate the quarks \( uud \), \( ddu \) into one another as shown by Fig. 1 of [4]. In this way the nome \( q \) is a quark coupling constant. It has the value \( q = 0.06583 \) that is close to the constant 0.118 found by Davies et. al. [2]. If we now concentrate on the complimentary angle \( \tan 60^\circ \) where \( \tau = \sqrt{3} \) then we find a possible nuclear coupling constant of 0.0043 that is the same order of magnitude as suggested by Rees [8], and 4.

We can now study electronic spin-orbit coupling in the same light.

II. Spin-Orbit Coupling

In this section we will rely heavily on the excellent book of Mumford [7] who considered the 2d-Complex Group in some detail in Ch. 1. Specifically he considered representations of \( U(2) \) and showed on pg. 42 that if \( \gamma = U(2) \) then \( i \tau \geq \frac{\sqrt{3}}{2} = \sin \omega \) which is the equilateral torus of Fig. 1 with boundaries \( AB \), \( BC \) defined by \( \cos 60^\circ = \frac{1}{2} \) and \( \sin \omega = 120^\circ \) of a fundamental domain. Therefore if we label the 3 apices by \( A=\text{up} \), \( B=\text{down} \) and \( C=\text{up} \) cyclic, then a rotation of \( 60^\circ \) will lead to \( udu \) then \( dud \) that imply spin-orbit coupling, because one spin must be in a different orbit by the Exclusion Principle. Then on pg.74 Mumford finds the Theta Function

\[
\Theta(0, \tau) = q + q^4 + q^9 + \cdots \tag{1}
\]

Where the nome \( q = \exp (\pi \tau) = \exp \left( \frac{\pi i}{\sqrt{3}} \right) \), and the dependence on \( z \) is carried by the lattice of Fig. 1[6]. The series converges very rapidly if \( q = 0.06583 \). Equation (1) is also the representation of an integer as the sum of squares ([1] p. 42) and on pages 62 and 63 Mumford shows that Fig.1 is a complex torus.

![Fig 1](Spin Coupling Lattice)
References

[6]. Lukas Lewark, Theta Functions, Seminar on Modular Forms (Jan 2007) online

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