On Weak m-power Commutative Near – rings and weak (m,n) power Commutative Near – rings

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Abstract: A right near – ring N is called weak commutative if $xyz = xyz$ for every $x,y,z \in N$ (Definition 9.4 [10]). A right near – ring N is called pseudo commutative (Definition 2.1 [11]) if $xyz = yzx$ for all $x,y,z \in N$. A right near – ring N is called quasi – weak commutative (Definition 2.1 [7]) if $xyz = yxz$ for all $x,y,z \in N$. We call a right near – ring N to be weak m – power commutative if $x^m y = y^m x$ for all $x,y,z \in N$. N is said to be weak (m,n) power commutative near – ring if $x^m y^n = x^n y^m$ for all $x,y,z \in N$. In this paper we study and establish various results of weak m – power commutative near – ring and weak (m,n) power commutative near – ring.

I. Introduction

S.Uma, R.Balakrishnan and T.Tamizhchelvam [11] called a near – ring N to be pseudo commutative if $xyz = zyx$ for every $x,y,z \in N$. G.Gopalakrishnamoorthy and S.Geetha [4] called a ring R to be m power commutative if $x^m y = y^m x$ for all $x,y \in R$ where $m \geq 1$ is a fixed integer. They also called a ring R to be (m,n) power commutative if $x^m y^n = y^n x^m$ for all $x,y \in R$ where $m \geq 1$ and $n \geq 1$ are fixed integers. G.Gopalakrishnamoorthy and R.Veega [6] called a near – ring N to be pseudo m- power commutative if $x^m y z = z y^m x$ for all $x,y,z \in N$ where $m \geq 1$ is a fixed integer. G.Gopalakrishnamoorthy, N.Kamaraj and S.Geetha [7] defined a near – ring N to be quasi – weak commutative if $xyz = yxz$ for all $x,y,z \in N$. In this paper we define weak m – power commutative near – ring and weak (m,n) power commutative near – ring and establish some results.

II. Preliminaries

Throughout this paper N denotes a near – ring with atleast two elements. For any non-empty set A $\subseteq N$, we denote $A - \{0\}$ by $A^*$. In this section we present some known definitions and results which are useful in the development of this paper.

2.1 Definition [10]:
A near – ring N is called weak-commutative if $xyz = zyx$ for every $x,y,z \in N$.

2.2 Definition:
A right near- ring N is called weak anti-commutative if $xyz = -zyx$ for every $x,y,z \in N$.

III. Weak m- power commutative near - rings

3.1 Definition:
Let N be a near – ring. N is said to be weak m- power commutative if $x^m y z = xz^m y$ for all $x,y,z \in N$, where $m \geq 1$ is a fixed integer.

3.2 Definition:
Let N be a near – ring. N is said to be weak m- power anti-commutative if $x^m y z = -xz^m y$ for all $x,y,z \in N$, where $m \geq 1$ is a fixed integer.

3.3 Lemma:
Let N be a distributive near – ring. If $xyz = \pm xyz$ for all $x,y,z \in N$ then N is either Weak Commutative or Weak anti – Commutative.

Proof:
For each $a \in N$, let
$C_a = \{ x \in N / xaz = xza \ \forall \ \ z \in N \}$
$A_a = \{ x \in N / xaz = -xza \ \forall \ \ z \in N \}$

By the hypothesis of the lemma,
$N = C_a \cup A_a$

We note that if $x,y \in C_a$, then $x - y \in C_a$

For $x,y \in C_a$, implies $xaz = +xza \ \forall \ \ z \in N$ and $yaz = +yza \ \forall \ \ z \in N$ → (1)

which further gives $→ (2)$
On Weak \( m \)-power Commutative Near rings and weak \((m,n)\) power Commutative Near rings

\[(x - y)az = (x - y)za \quad \forall z \in N\]

which implies \((x - y) \in C_a\).

Similarly, if \(x, y \in A_b\), then \(x - y \in A_b\).

We claim that either \(N = C_a\) or \(N = A_b\).

Suppose \(N \neq C_a\) and \(N \neq A_b\), then there are elements \(b \in C_a - A_b\) and \(d \in A_b - C_a\).

Now \(b + d \in N = C_a \cup A_b\).

If \(b + d \in C_a\) then \(d = (b + d) - b \in C_a\), a contradiction.

If \(b + d \in A_b\) then \(b = (b + d) - d \in A_b\), again a contradiction.

Hence either \(N = C_a\) or \(N = A_b\).

Let \(A = \{a \in N / C_a = N\}\) and \(B = \{a \in N / A_b = N\}\)

Clearly \(N = A \cup B\).

We note that that if \(x, y \in A\), then \(x - y \in A\).

For if \(x, y \in A\) \(\Rightarrow\) \(C_a = N\) and \(C_y = N\),

This implies \(xza = xaz\) and \(yza = yaz\) for all \(a, z \in N\),

So \((x - y)za = (x - y)az\) for all \(a, z \in N\), which proves that \(x - y \in A\).

Similarly, if \(x, y \in B\), then \(x - y \in B\).

We claim that either \(N = A\) or \(N = B\).

Suppose \(N \neq A\) and \(N \neq B\), there are elements \(u \in A - B\) and \(v \in B - A\).

Now, \(u + v \in N = A \cup B\).

If \(u + v \in A\), then \(v = (u + v) - u \in A\), a contradiction.

If \(u + v \in B\), then \(u = (u + v) - v \in B\), again a contradiction.

Hence either \(N = A\) or \(N = B\).

This proves that \(N\) is either weak commutative or weak anti–commutative.

**3.4 Lemma:**

Let \(N\) be a near ring (not necessarily associative). If \(x \ y^mz = \pm x \ z^n y\) for all \(x, y, z \in N\), then \(N\) is either weak \(m\)–power commutative or weak \(m\)–power anti–commutative.

**Proof:**

For each \(a \in N\), let \(C_a = \{x \in N / xa^m = xz^m a \quad \forall z \in N\}\)

\(A_a = \{x \in N / xa^m = xz^m a \quad \forall z \in N\}\)

By the hypothesis of the lemma, \(N = C_a \cup A_a\).

We note that, if \(x, y \in C_a\) then \(x - y \in C_a\).

For \(x, y \in C_a\) implies \(xa^m = xz^m a \quad \forall z \in N\) \(\Rightarrow \) \(1\)

and \(ya^m = yz^m a \quad \forall z \in N\) \(\Rightarrow \) \(2\)

Equation (1) \(-\) (2) gives,

\[(x - y)a^m = (x - y)z^m a \quad \forall z \in N.
\]

\(\Rightarrow (x - y) \in C_a\).

Similarly \(x, y \in A_a\) implies \(x - y \in A_a\).

We claim that either \(N = C_a\) or \(N = A_a\).

Suppose \(N \neq C_a\) and \(N \neq A_a\), there are elements \(b \in C_a - A_a\) and \(d \in A_a - C_a\).

Now, \(b + d \in N = C_a \cup A_a\).

If \(b + d \in C_a\) then \(d = (b + d) - b \in C_a\), a contradiction.

Similarly, if \(b + d \in A_a\), then \(b = (b + d) - d \in A_a\), again a contradiction.

Hence either \(N = C_a\) or \(N = A_a\).

Let \(A = \{a \in N / C_a = N\}\)

and \(B = \{a \in N / A_a = N\}\)

Clearly \(N = A \cup B\).

We note that if \(x, y \in A\) implies \(x - y \in A\).

For if \(x, y \in A\) implies \(C_a = N\) and \(C_y = N\).

This implies \(xa^m = xa^m z\) and \(yz^m a = ya^m z\) for all \(a, z \in N\).

So, \((x - y) z^m a = (x - y) a^m z\) for all \(a, z \in N\), which proves that \(x - y \in A\).

Similarly \(x, y \in B\) implies \(x - y \in B\).

We claim that either \(N = A\) or \(N = B\).

Suppose \(N \neq A\) and \(N \neq B\), there are elements \(u \in A - B\) and \(v \in B - A\).

Now, \(u + v \in N = A \cup B\).

If \(u + v \in A\), then \(v = (u + v) - u \in A\), a contradiction.
If $u + v \in B$, then $u = (u + v) - v \in B$, again a contradiction. Hence either $N = A$ or $N = B$. This proves that $N$ is either weak $m$– power commutative or weak $m$– power anti–commutative.

3.5 Note: When $m = 1$, we get Lemma 3.3.

3.6 Definition: Let $N$ be a near-ring and $m \geq 1$ and $n \geq 1$ be fixed integers. $N$ is said to be weak $(m,n)$ power commutative, if $xy^mx^n = xz^my^n$ for all $x,y,z \in N$.

3.7 Definition: Let $N$ be a near-ring and $m \geq 1$ and $n \geq 1$ be fixed integers. $N$ is said to be weak $(m,n)$ power anti-commutative, if $xy^mx^n = -xz^my^n$ for all $x,y,z \in N$.

3.8 Lemma: Let $N$ be a near–ring (not necessarily associative) satisfying $(x-y)^k = x^k - y^k$ for $k = m,n$ where $m \geq 1$ and $n \geq 1$ are fixed integers. If $xy^mx^n = \pm xz^my^n$ for all $x,y,z \in N$, then $N$ is either weak $(m,n)$ power commutative or weak $(m,n)$ power anti-commutative.

Proof: For each $a \in N$, let $C_a = \{ x \in N : xa^mz^n = yz^ma^n \forall z \in N \}$

$A_a = \{ x \in N : xa^mz^n = -yz^ma^n \forall z \in N \}$

By the hypothesis of the lemma, $N = C_a \cup A_a$

We note that, if $x,y \in C_a$ then $x - y \in C_a$

For $x,y \in C_a$ implies $xa^mz^n = yz^ma^n \forall z \in N$ (1)

and $ya^mz^n = yz^ma^n \forall z \in N$ (2)

Equation (1) – (2) gives,

$(x - y)a^mz^n = (x - y)z^ma^n \forall z \in N.$

$\Rightarrow (x - y) \in C_{a^m}$

Similarly $x,y \in A_a$ implies $x - y \in A_a$

We claim that either $N = C_a$ or $N = A_a$.

Suppose $N \neq C_a$ and $N \neq A_a$, there are elements $b \in C_a - A_a$ and $d \in A_a - C_a$.

Now, $b + d \in N = C_a \cup A_a$.

If $b + d \in C_a$ then $d = (b + d) - b \in C_a$, a contradiction.

Similarly, if $b + d \in A_a$, then $b = (b + d) - d \in A_a$, again a contradiction.

Hence either $N = C_a$ or $N = A_a$.

Let $A = \{ a \in N : C_a = N \}$

and $B = \{ a \in N : A_a = N \}$

Clearly $N = A \cup B$.

We note that if $x,y \in A$ implies $x - y \in A$.

For if $x,y \in A$ implies $C_x = N$ and $C_y = N$.

This implies $xz^ma^n = xa^mz^n$ and $yz^ma^n = ya^mz^n$ for all $a,z \in N$.

So, $(x - y)z^ma^n = (x - y)a^mz^n$ for all $a,z \in N$, which proves that $x - y \in A$.

Similarly $x,y \in B$ implies $x - y \in B$.

We claim that either $N = A$ or $N = B$.

Suppose $N \neq A$ and $N \neq B$, there are elements $u \in A - B$ and $v \in B - A$.

Now, $u + v \in N = A \cup B$.

If $u + v \in A$, then $v = (u + v) - u \in A$, a contradiction.

If $u + v \in B$, then $u = (u + v) - v \in B$, again a contradiction.

Hence either $N = A$ or $N = B$.

This proves that $N$ is either weak $(m,n)$– power commutative or weak $(m,n)$– power anti–commutative.

3.9 Note: When $m = n = 1$, we get Lemma 3.3. When $n = 1$, we get Lemma 3.4.

References


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