Regular Weakly Generalized Locally Closed Sets in Ideal Topological Spaces

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Abstract: In this paper we have introduced the concept of regular weakly generalized locally closed sets in ideal topological spaces. Properties and characterizations are discussed.

Keywords: \( I_{rwg}lc \) set, \( I_{rwg}lc^+ \) set, \( I_{rwg}lc^{**} \) set.

I. Introduction

A nonempty collection \( I \) of subsets on a topological space \( (X, \tau) \) is called a topological ideal [3] if it satisfies the following two conditions:

(i) If \( A \in I \) and \( B \subseteq A \) implies \( B \in I \) (heredity)
(ii) If \( A \in I \) and \( B \in I \), then \( A \cup B \in I \) (finite additivity)

Local function in topological spaces using ideals was introduced by Kuratowski [3]. Donchev [2] introduced the concept of I-locally closed sets. After that Navaneetha Krishnan and Sivaraj [4] introduced \( I \)-locally \(*\) closed sets and \( I_e \)-locally \(*\) closed sets.

II. Preliminaries

Definition 2.1.: A subset \( A \) of a topological space \( (X, \tau, I) \) is called

(i) \( I \)-locally \(*\) closed [4] if there exist an open set \( U \) and a \(*\) closed set \( F \) such that \( A = U \cap F \),
(ii) \( I_e \)-locally \(*\) closed [4] if there exist an \( I_e \)-open set \( U \) and a \(*\) closed set \( F \) such that \( A = U \cap F \).

Definition 2.2: For a subset \( A \) of a topological space \( (X, \tau) \) is said to be

(i) \( GLC^*(X, \tau) \) if there exist a \( g \)-open set \( U \) and a closed set \( F \) of \( (X, \tau) \) such that \( A = U \cap F \),
(ii) \( GLC^{**}(X, \tau) \) if there exist a \( g \)-open set \( U \) and a \( g \)-closed set \( F \) of \( (X, \tau) \) such that \( A = U \cap F \).

Definition 2.3: A subset \( A \) of an ideal topological space \( (X, \tau, I) \) is called a

(i) \( rpsIlc\)-set [5] if there exists a \( rpsI \)-open set \( U \) and a \( rpsI \)-closed set \( F \) of \( (X, \tau, I) \) such that \( A = U \cap F \),
(ii) \( rpsIlc^+\)-set [5] if there exists a \( rpsI \)-open set \( U \) and a closed set \( F \) of \( (X, \tau, I) \) such that \( A = U \cap F \),
(iii) \( rpsIlc^{**}\)-set [5] if there exists a \( rpsI \)-open set \( U \) and a \( rpsI \)-closed set \( F \) of \( (X, \tau, I) \) such that \( A = U \cap F \).

III. \( I_{rwg}LC \) SETS AND \( I_{rwg}LC^+ \) SETS

In this section, regular weakly generalized locally closed sets are and introduced.

Definition 3.1: A subset \( A \) of an ideal topological spaces \( (X, \tau, I) \) is said to be a regular weakly generalized locally closed \( (I_{rwg}LC) \) set if \( A = U \cap F \) where \( U \) is \( I_{rwg}\)-open and \( F \) is \( I_{rwg}\)-closed in \( X \).

Definition 3.2: A subset \( A \) of an ideal topological space \( (X, \tau, I) \) is said to be \( I_{rwg}LC^+ \) if there exist an \( I_{rwg}\)-open set \( U \) and a \( I_e \)-closed set \( F \) of \( X \) such that \( A = U \cap F \).

Definition 3.3: A subset \( A \) of an ideal topological spaces \( (X, \tau, I) \) is said to be \( I_{rwg}LC^{**} \) if there exist a \( I_{rwg}\)-open set \( U \) and a \( I_e \)-closed set \( F \) of \( X \) such that \( A = U \cap F \).

The collection of all \( I_{rwg}LC \) - sets ( resp. \( I_{rwg}LC^+ \) and \( I_{rwg}LC^{**} \) ) of \( (X, \tau, I) \) is denoted by \( I_{rwg}LC \) in \( (X, \tau) \) (resp. \( I_{rwg}LC^+ \) in \( (X, \tau) \) and \( I_{rwg}LC^{**} \) in \( (X, \tau) \)).

Theorem 3.4: For a ideal topological space \( (X, \tau, I) \) the following implications hold.

(i) \( ILC(X, \tau) \subseteq IRWGLC(X, \tau) \subseteq IRWGLC^+(X, \tau) \subseteq IRWGLC^{**}(X, \tau) \)
(ii) \( ILC(X, \tau) \subseteq IRWGLC^{**}(X, \tau) \subseteq IRWGLC(X, \tau) \)

The reverse implications need not be true as seen from the following example.

Example 3.5: Let \( X = \{ a, b, c \} \), \( \tau = \{ \emptyset, \{a\}, \{b\}, \{a, b\}, X \} \), \( I = \{ \emptyset, \{a\} \} \), then \( Ilc \) closed sets are \( \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, X \} \) and the \( I_{rwg}LC \) sets are \( \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, X \} \). Hence \( \{a,c\} \) is an \( I_{rwg}LC \) sets but not \( Ilc \) set.

Example 3.6: Let \( X = \{ a, b, c, d \} \), \( \tau = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, X \} \), \( I = \{ \emptyset, \{a\} \} \), then \( Ilc \) closed sets are \( \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, X \} \) and the \( I_{rwg}LC \) sets are \( \{ \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, X \} \). Hence \( \{a,d\} \) are \( I_{rwg}LC \) sets but not \( Ilc \) set.

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Theorem 3.7: Let A be any subset of X, then
(i) A is inweg-closed in X if and only if A = inweg cl(A)
(ii) inweg cl(A) is inweg-closed in X
(iii) x ∈ inweg cl(A) if and only if A ∩ U ≠ ∅ for every inweg-open set U containing x.

Proof: (i) and (ii) are trivially true.
(iii) Suppose that there exists an inweg-open set U containing x such that A ∩ U ≠ ∅. Since X − U is inweg-closed and A ⊆ X − U, inweg cl(A) ⊆ X − U. Therefore x ∈ inweg cl(A). Conversely suppose that x ∉ inweg cl(A). Then U = X − inweg cl(A) is inweg-open set containing x and A ∩ U = ∅.

Theorem 3.8: For a subset A of (X, τ, I), the following statements are equivalent.
(i) A ∈ inweg LC(X, τ)
(ii) A = U ∩ inweg cl(A) for some inweg-open set U.
(iii) inweg cl(A) − A is inweg-closed.
(iv) AU (X − inweg cl(A)) is inweg-open.

Proof: (i) ⇒ (ii) Suppose A ∈ inweg LC(X, τ). Then there exists an inweg-open subset U and inweg-closed subset F such that A = U ∩ F. Since A ⊆ U and A ⊆ inweg cl(A), A ⊆ U ∩ inweg cl(A). Also by Theorem 3.7, inweg cl(A) is inweg-closed in X. Hence inweg cl(A) ⊆ F and U ∩ inweg cl(A) ⊆ U ∩ F = A. Therefore A = U ∩ inweg cl(A).
(ii) ⇒ (i) By Theorem 3.7, inweg cl(A) is inweg-closed and hence A = U ∩ inweg cl(A) ∈ inweg LC(X, τ).

Theorem 3.9: For a subset A of (X, τ, I), the following statements are equivalent.
(i) A ∈ inweg LC′(X, τ)
(ii) A = U ∩ cl(A) for some inweg-open set U.
(iii) cl'(int(A)) − A is inweg-closed.
(iv) AU (X − cl'(int(A))) is inweg-open.

Proof: The proof is similar to that of above theorem.

Theorem 3.10: Let A be a subset of (X, τ, I). If A ∈ inweg LC''(X, τ) then inweg cl(A) − A is inweg-closed and A ∪ (X − inweg cl(A)) is inweg-open.

Proof: Let A ∈ inweg LC''(X, τ). Then there exists an open set U such that A = U ∩ inweg cl(A). A U (X − inweg cl(A)) = (U ∩ inweg cl(A)) U (X − inweg cl(A)) = U ∩ inweg cl(A) U (X − inweg cl(A)) = U ∩ A = open. Since every open set is inweg-open, A U (X − inweg cl(A)) is inweg-open. Let W = A U (X − inweg cl(A)). Then W is inweg-open implies W = X − inweg cl(A) closed and W = X − (A U (X − inweg cl(A))) = inweg cl(A) ∩ A = inweg cl(A) − A. Thus inweg cl(A) − A is inweg-closed.

Theorem 3.11: Let A and B be subsets of (X, τ, I). If A ∈ inweg LC(X, τ) and B is inweg-open, then A ∩ B ∈ inweg LC(X, τ).

Proof: Let A ∈ inweg LC(X, τ). Then A = U ∩ F where U is inweg-open and F is inweg-closed. So A ∩ B = U ∩ F ∩ B = U ∩ B = F. This implies that A ∩ B ∈ inweg LC(X, τ).

Theorem 3.12: Let A and B be subsets of (X, τ, I). If A ∈ inweg LC′(X, τ) and B ∈ inweg LC′(X, τ) then A ∩ B ∈ inweg LC′(X, τ).

Proof: Let A and B ∈ inweg LC′(X, τ). Then there exists inweg-open sets P and Q such that A = P ∩ cl(A) and B = Q ∩ cl(B). Therefore A ∩ B = P ∩ cl(A) ∩ Q ∩ cl(B) = P ∩ Q ∩ cl(A) ∩ cl(B) where P ∩ Q is inweg-open and cl(A) and cl(B) is closed. This shows that A ∩ B ∈ inweg LC′(X, τ).

Theorem 3.13: If A ∈ inweg LC''(X, τ) and B is inweg-open, then A ∩ B ∈ inweg LC''(X, τ).

Proof: Let A ∈ inweg LC''(X, τ). Then there exists an open set U and an inweg-closed set F such that A = U ∩ F. So A ∩ B = U ∩ F ∩ B = U ∩ B = F. This proves that A ∩ B ∈ inweg LC''(X, τ).

Theorem 3.14: Let A and B be subsets of (X, τ, I). If a and b are inweg-open in (X, τ, I) and A ∈ inweg LC' (Z, τZ), then A ∈ inweg LC' (X, τ).

Proof: Suppose that A is inweg cl', then there exists an inweg-open set U of (Z, τZ) such that A = U ∩ cl(A). But cl(A) = Z ∩ cl(A). Therefore, A = U ∩ Z ∩ cl(A) where U ∩ Z is inweg-open. Thus A ∈ inweg LC' (X, τ).
References


