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Exact Value of pi π (17 - 8√3)

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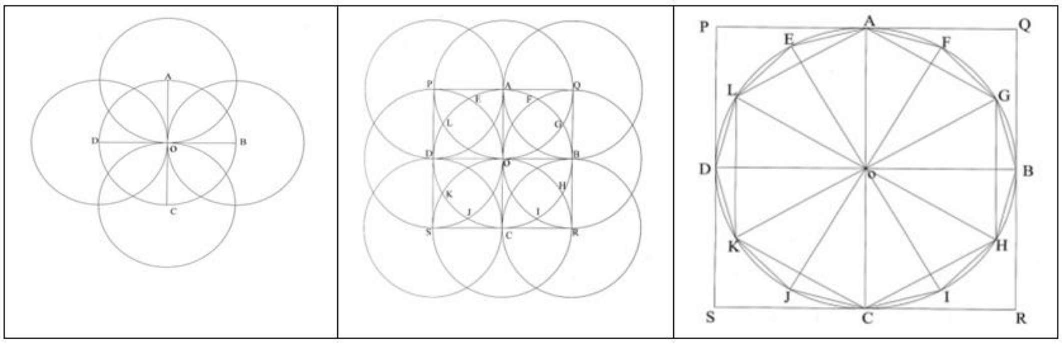
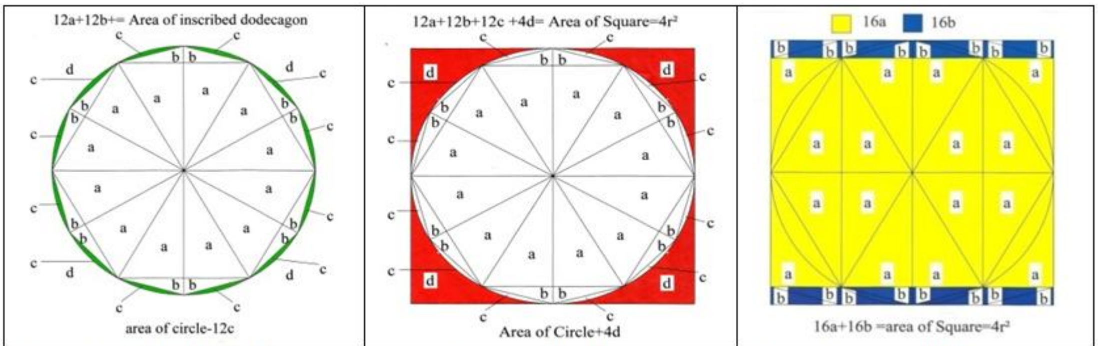
Abstract: *It is believed that pi (π) is a transcendental number. In author's opinion, it is not the fact. The paper aims at showing that pi (π) is an algebraic number with exact value 17-8√3. The derivation of this value is supported by several geometrical constructions, arithmetic calculations and use of some simple algebraic formulae.*

I. Introduction

Determination of the ratio of circumference of circle with its diameter has been a problem of much more interest in the field of mathematics since thousands of years. The ratio of circumference of circle/diameter is denoted by a Greek letter π. It is believed that the constant pi (π) is a transcendental number. Its value has been computed correct up to several trillions of digits. For practical purpose the value of pi is taken approximately equal to 22/7, 355/113 etc. However, the eminent mathematicians of the world suffer from a serious drawbacks while using the methods such as use of infinite series, trigonometry, division of area of circle into infinite symmetric parts n sided polygon and many others, as we cannot measure the end point quantities exactly. This causes an obvious limitation on obtaining exact value of pi.
The sections which follow, describe the derivation for "Exact" value of pi which is a creation of the author. In author's opinion the exact value of pi is 17-8√3.
I have made many proofs & here I am giving one of them.
Basic figures



Basic information



Area of inscribed dodecagon $= (12a + 12b) = 3r^2$	**Area of circumscribed square** $= (12a + 12b + 12c + 4d) = 4r^2$	**Area of circumscribed square** $= (16a + 16b) = 4r^2$

Note: let a, b, c & d each part shows area
Area of inscribed hexagon $= 12a = (1.5\sqrt{3})\, r^2$ $\quad a = (0.125\sqrt{3})\, r^2$
Area of inscribed dodecagon $= (12a + 12b) = 3r^2 = (\text{area of circle} - 12c)$
$(12a + 12b) - 12a = 12b = (3 - 1.5\sqrt{3})\, r^2 \quad b = (0.25 - 0.125\sqrt{3})\, r^2$

Area of circumscribed square **= (12a + 12b + 12c + 4d)** **= $4r^2$** **= (area of circle + 4d)**
Area of circle **= (12a + 12b + 12c)** **= ($3r^2$ + 12c)** **= ($4r^2$ - 4d)** **= πr^2**
π = (3 + 12c) **= (4 – 4d)**

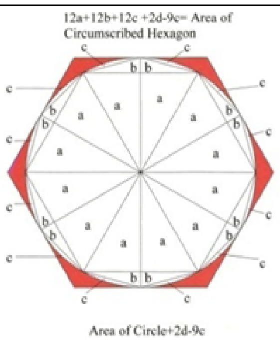
As we know, the exact area of inscribed dodecagon = $3r^2$. In order to calculate exact area of circle, we have to calculate exact area of 12c. Hence there is no need to divide whole circle into infinite number of parts to calculate its accurate area. How to estimate the exact values of part 12c & part 4d?

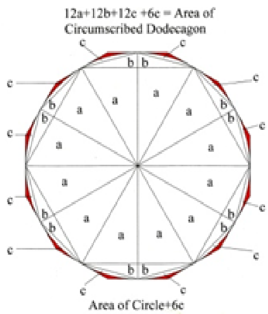
Area of circumscribed square = (12a + 12b + 12c + 4d) = (16a + 16b)
(12a + 12b + 12c + 4d) - (16a + 16b) = 0
= (- 4a – 4b + 12c + 4d) = 0
i.e. **(4a + 4b = 12c + 4d)** **(a + b = 3c + d)**
(a + b – 3c – d) = 0
I found that: **(14b – 2a – 3c) = 0** **(13b + d – 3a) = 0**
(14b – 2a – 3c) – (13b + d – 3a) = (a + b – 3c – d) = 0
i.e. **(14b – 2a – 3c) = (13b + d – 3a)**
From previous information: **(14b – 2a – 3c) = 0,**
Therefore: x (14b – 2a – 3c) = 0 **(Note: x is any number)**
For example: Area of circle + x (14b – 2a – 3c) **= area of circle + 0**
= (12a + 12b + 12c) + 4(14b – 2a – 3c)
= (12a + 12b + 12c) + (56b – 8a – 12c) **= 4a + 68b** **= area of circle**
= [4(0.125$\sqrt{3}$) r^2 + 68(0.25 – 0.125$\sqrt{3}$) r^2] **= [(0.5$\sqrt{3}$) r^2 + (17 – 8.5$\sqrt{3}$) r^2]**
= (17 – 8$\sqrt{3}$) r^2

= area of circumscribed square = $4r^2$ **= (12a + 12b + 12c) + 4d** **= area of circle + 4d**
= (4a + 68b) + 4d
4d = $4r^2$ - (17 – 8$\sqrt{3}$) r^2 **= (8$\sqrt{3}$ - 13) r^2**
d = (2$\sqrt{3}$ – 3.25) r^2

Area of inscribed dodecagon = $3r^2$ = (12a + 12b + 12c) – 12c **= area of circle – 12c**
= (4a + 68b) – 12c
12c **= (17 – 8$\sqrt{3}$) r^2 - $3r^2$** **= (14 – 8$\sqrt{3}$) r^2**
3c **= (3.5 – 2$\sqrt{3}$) r^2**

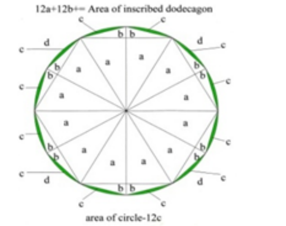
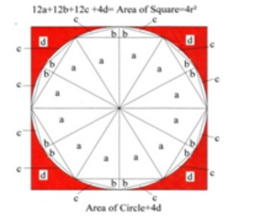
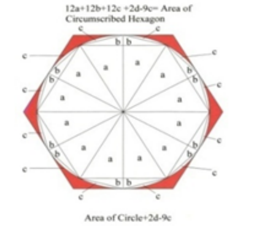
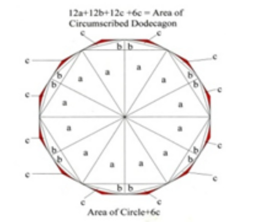
Supported work:





Area of circumscribed hexagon = (2$\sqrt{3}$) r^2 = 16a **= (12a + 12b + 12c) + x = (4a + 68b) + x** **= (17 – 8$\sqrt{3}$) r^2 + x** **x = [(2$\sqrt{3}$) r^2 - (17 – 8$\sqrt{3}$) r^2]** **= (10$\sqrt{3}$ - 17) r^2 = (2d – 9c)** **2d = 2(2$\sqrt{3}$ – 3.25) r^2 - 9c = - 3(3.5 – 2$\sqrt{3}$) r^2** **= (4$\sqrt{3}$ – 6.5) r^2 - (10.5 – 6$\sqrt{3}$) r^2** **= (10$\sqrt{3}$ - 17) r^2** **i.e. area of circumscribed hexagon** **= area of circle + 2d – 9c**	**Area of circumscribed dodecagon = 12(2 -$\sqrt{3}$) r^2** **= 96b = (12a + 12b + 12c) + y** **= (4a + 68b) + y** **= (17 – 8$\sqrt{3}$) r^2 + y** **y =(24 – 12$\sqrt{3}$) r^2 - (17 – 8$\sqrt{3}$) r^2** **= (7 – 4$\sqrt{3}$) r^2 = 6c** **= 2(3.5 – 2$\sqrt{3}$) r^2** **i.e. area of circumscribed dodecagon** **= area of circle + 6c**

Proof of previous equations:

Exact area		Derived area	
			
Area of inscribed dodecagon = $3r^2$=12a+12b = area of circle (– 12c)	**Area of circumscribed square = $4r^2$ = 16a + 16b = area of circle + (4d)**	**Area of circumscribed hexagon = $(2\sqrt{3})\ r^2$ = 16a = area of circle + (2d – 9c)**	**Area of circumscribed dodecagon = $12(2 - \sqrt{3})\ r^2$ = 96b = area of circle + (6c)**

Proof of exact area equal to derived area

[Area of circumscribed 3 square + area of 4 inscribed dodecagon) $= [3(4r^2) + 4(3r^2)]$ $= 24r^2$

= [area of circumscribed (6 hexagon + 1 dodecagon)] $= \{6(2\sqrt{3})\ r^2 + 1[12(2 - \sqrt{3})]\}\ r^2$ $= 24r^2$

Exact area	= Derived area
[Area of circumscribed 3 square	**= [area of circumscribed 6 hexagon**
= area of 3 circle + 3(4d)	**= area of 6 circle + 6(2d – 9c)**
+ area of inscribed 4 dodecagon	**+ area of circumscribed 1 dodecagon**
= area of 4 circle + 4(- 12c)]	**= area of 1 circle + 1(6c)]**
= area of (3 + 4) circle + (12d – 48c)	**= area of (6 + 1) circle + (12d – 54c) + 6c**
= area of 7 circle + (12d – 48c)	**= area of 7 circle + (12d – 48c)**

I have tried to verify the above equations by using other value of pi π but answer doesn't match.
If pi is an approximate value or transcendental number then exact area cannot be equal to derived area.
I got method by using it we can solve infinite examples similar to above

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Area of 1 inscribed dodecagon + area of 2 circumscribed dodecagon
= (area of 1 circle – 12c) + (area of 2 circle + 12c) = area of 3 circle
$= 3r^2 + 2[12(2 - \sqrt{3})\ r^2]$ $= [3r^2 + (48 - 24\sqrt{3})\ r^2]$ $= (51 - 24\sqrt{3})\ r^2]$ $= 3(17 - 8\sqrt{3})\ r^2$

Area of circumscribed (2 hexagon + 3 dodecagon) = [area of 2 circle + 2(2d – 9c) + area of 3 circle + 3(6c)]
= area of (2 + 3) circle + (4d – 18c) + 18c = (area of 5 circle + 4d)
= area of 4 circle + (area of 1 circle + 4d) = area of (4 circle + 1 circumscribed square)
Area of 4 circle $= 2(2\sqrt{3})\ r^2 + 3[12(2 - \sqrt{3})\ r^2] - 1(4r^2)$ $= [(4\sqrt{3})\ r^2 + (72 - 36\sqrt{3})\ r^2 - 1(4r^2)] = 4(17 - 8\sqrt{3})\ r^2$

II. Supported work

We know that:
(Area of circle + 4d) = area of circumscribed square
= (12a + 12b + 12c + 4d) = (16a + 16b) exact equations

Area of circumscribed hexagon = 16a = area of circle + (2d – 9c)
i.e. Area of circle = (16a + 9c – 2d)
Area of circumscribed square = (16a + 9c – 2d) + 4d = (16a + 9c + 2d) derived equation

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Area of circumscribed dodecagon = 96b = area of circle + 6c
i.e. Area of circle = (96b – 6c)
Area of circumscribed square = (96b – 6c) + 4d derived equation

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Area of circle = (4a + 68b)
Area of circumscribed square = (4a + 68b) + 4d derived equation

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Proof of derived equations equal to exact equations: Area of circumscribed square

Derived equations	= exact equations
(16a + 9c + 2d) = (96b – 6c + 4d) = (4a + 68b + 4d)	= (12a + 12b + 12c + 4d) = (16a + 16b)
Area of (4 + 1) circumscribed square = 4(16a + 9c + 2d)] + 1(4a + 68b + 4d) = (64a + 36c + 8d) + (4a + 68b + 4d) = (68a + 68b + 36c + 12d) = $5(4r^2)$	= area of (3 + 2) circumscribed square = 3(12a + 12b + 12c + 4d) + 2(16a + 16b) = (36a + 36b + 36c + 12d) + (32a + 32b) = (68a + 68b + 36c + 12d)
Area of (10 + 1 + 1) circumscribed square = 10(16a + 9c + 2d)] + 1(96b – 6c + 4d) + 1(4a + 68b + 4d) = (160a + 90c + 20d) + (96b – 6c + 4d) + (4a + 68b + 4d) = (164a + 164b + 84c + 28d) = $12(4r^2)$	= area of (7 + 5) circumscribed square = 7(12a + 12b + 12c + 4d) + 5(16a + 16b) = (84a + 84b + 84c + 28d) + (80a + 80b) = (164a + 164b + 84c + 28d)
Area of (22 + 1 + 4) circumscribed square	= area of (16 + 11) circumscribed square

= 22(16a + 9c + 2d)] + 1(96b – 6c + 4d) + 4(4a + 68b + 4d) = (352a + 198c + 44d) + (96b – 6c + 4d) + (16a + 272b + 16d) = (368a + 368b + 192c + 64d) = 27(4r²)	= 16(12a + 12b + 12c + 4d) + 11(16a + 16b) = (192a + 192b + 192c + 64d) + (176a + 176b) = (368a + 368b + 192c + 64d)
Area of (50 + 1 + 11) circumscribed square = 50(16a + 9c + 2d)] + 1(96b – 6c + 4d) + 11(4a + 68b + 4d) = (800a + 450c + 100d) + (96b – 6c + 4d) + (44a + 748b + 44d) = (844a + 844b + 444c + 148d) = 62(4r²)	= area of (37 + 25) circumscribed square = 37(12a + 12b + 12c + 4d) + 25(16a + 16b) = (444a + 444b + 444c + 148d) + (400a + 400b) = (844a + 844b + 444c + 148d)
Area of (86 + 3 + 17) circumscribed square = 86(16a + 9c + 2d)] + 3(96b – 6c + 4d) + 17(4a + 68b + 4d) = (1376a + 774c + 172d) + (288b – 18c + 12d) + (68a + 1156b + 68d) = (1444a + 1444b + 756c + 252d) = 106(4r²)	= area of (63 + 43) circumscribed square = 63(12a + 12b + 12c + 4d) + 43(16a + 16b) = (756a + 756b + 756c + 252d) + (688a + 688b) = (1444a + 1444b + 756c + 252d)

Algebraic method: note x is any number

s. r. no.	Derived equations			= Exact equations	
	(16a + 9c + 2d)	= (96b – 6c + 4d)	= (4a + 68b + 4d)	= (12a + 12b + 12c + 4d)	= (16a + 16b)
1	4x		x	3x	2x
2	4x + 2	1	x – 1	3x + 1	2x + 1
3	4x + 2	-1	x + 2	3x + 2	2x + 1

For example no. 1 x = 5
5[4(16a + 9c + 2d)] + 5(4a + 68b + 4d) = 5[3(12a + 12b + 12c + 4d)] + 5[2(16a + 16b)]
= (320a + 180c + 40d) + (20a + 340b + 20d) = (180a + 180b + 180c + 60d) + (160a + 160b)
= (340a + 340b + 180c + 60d) = (340a + 340b + 180c + 60d)

Using derived equations & found area of circle:
Area of 3 circumscribed square = (16a + 9c + 2d) + (96b – 6c + 4d) + (4a + 68b + 4d)
= 20a + 164b + 3c + 10d
= 20a + 164b + (3c + d) + (10d – d)
= 20a + 164b + (a + b) + 9d
= 21a + 165b + 9d
(Area of circumscribed square – 4d = area of circle)
(21a + 165b + 9d) – 9d
= (Area of 2.25 circumscribed square – 9d) + area of (3 – 2.25) circumscribed square
= (21a + 165b) = area of 2.25 circle + 0.75 circumscribed square
= 21(0.125√3) r² + 165(0.25 – 0.125√3) r²
= (2.625√3) r² + (41.25 – 20.625√3) r²
= (41.25 – 18√3) r²
Area of 2.25 circle = (41.25 – 18√3) r² - 0.75(4r²)
= (38.25 – 18√3) r²
Area of circle = (38.25 – 18√3) r² /2.25
= (17 – 8√3) r²

One more example:
Area of (2 + 3) circumscribed square = 2(16a + 9c + 2d) + 3(96b – 6c + 4d)
= (32a + 18c + 4d) + (288b – 18c + 12d) = (32a + 288b + 16d)
(32a + 288b + 16d) – 16d (area of circumscribed square – 4d = area of circle)
= (Area of 4 circumscribed square – 16d) + area of (5 – 4) circumscribed square
= (32a + 288b) = area of 4 circle + 1 circumscribed square
= 32(0.125√3) r² + 288(0.25 – 0.125√3) r²
= (4√3) r² + (72 – 36√3) r²
= (72 – 32√3) r²
Area of 4 circle = (72 – 32√3) r² - 1(4r²)
= (68 – 32√3) r²
Area of circle = (68 – 32√3) r² /4 = (17 – 8√3) r²

III. Conclusions

Exact Area of circle = (17 - 8√3) r²

References

[1]. Exact area of equilateral triangle formula = (√3 ÷ 4) × side², Area of circumscribed hexagon = (2√3) r², Area of circumscribed dodecagon = 12(2 - √3) r²
[2]. Basic Algebra & Geometry concept, History of pi (π) Complete thesis of my research titled as ‘‘Exact value of pi ’’ has being published in following journals:
[3]. IOSR(international organization scientific research) journal of mathematics in May-June 2012.
[4]. IJERA (international journal of Engineering research and applications) in July-August 2013.

[5]. IJMSI (international journal of mathematics and Statistics Invention) in Feb. 2015
[6]. Soft copy of my thesis is now also available on internet and one can get it by making search with following keywords: “Gogawale pi”

3.143593539448981651780429267953021064457557969516954975553544164384535864729599703350830505942011394594990868676746375760240439240008380169506755903245010811775254144781555343456201439191322600613879877501253864773416906794621962648625296230183622946717057107295847600107700961124884258797219947391593947053214963151192419182502477399334833363808321336017681403930393…

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