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On Semi*δ- Open Sets in Topological Spaces

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Abstract: *In this paper, we introduce a new class of sets, namely semi*δ-open sets, using δ-open sets and the generalized closure operator. We find characterizations of semi*δ-open sets. We also define the semi*δ-interior of a subset. Further, we study some fundamental properties of semi*δ-open sets and semi*δ-interior.*
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I. Introduction

Norman Levine [3] introduced semi-open sets in topological spaces in 1963. Since the introduction of semi-open sets, many generalizations of various concepts in topology were made by considering semi-open sets instead of open sets. N.V. Velicko[15] introduced the concept of δ-open sets in 1968. Levine [4] also defined and studied generalized closed sets in 1970. Dunham [2] introduced the concept of generalized closure using Levine's generalized closed sets and studied some of its properties. In 1997, Park, Lee and Son [17] have introduced and studied δ-semi-open sets in topological spaces.

In this paper, analogous to Park, Lee and Son's δ-semi-open sets, we define a new class of sets, namely semi*δ-open sets, using the generalized closure operator due to Dunham instead of the closure operator in the definition of δ-semi-open sets. We further show that the concept of semi*δ-open sets is weaker than the concept of δ-open sets but stronger than the concept of δ-semi-open sets. We find characterizations of semi*δ-open sets. We investigate fundamental properties of semi*δ-open sets. We also define the semi*δ-interior of a subset and study some of its basic properties.

II. Preliminaries

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of a space (X, τ), $Cl(A)$ and $Int(A)$ denote the closure and the interior of A respectively.

Definition 2.1.A subset A of a space X is **generalized closed** (briefly g-closed) [4] if $Cl(A)\subseteq U$ whenever U is an open set in X containing A.

Definition 2.2. If A is a subset of a space X, the **generalized closure** [2] of A is defined as the intersection of all g-closed sets in X containing A and is denoted by $\boldsymbol{Cl^*(A)}$.

Definition 2.3.A subset A of a topological space (X, τ) is **semi-open** [3](respectively **semi*-open** [12]) if there is an open set U in X such that $U\subseteq A\subseteq Cl(U)$ (respectively $U\subseteq A\subseteq Cl^*(U)$) or equivalently if $A\subseteq Cl(Int(A))$ (respectively $A\subseteq Cl^*(Int(A))$).

Definition 2.4.A subset A of a topological space (X, τ) is **pre-open** [5](respectively **pre*-open** [14]) if $A\subseteq Int(Cl(A))$ (respectively $A\subseteq Int^*(Cl(A))$).

Definition 2.5.A subset A of a topological space (X, τ) is **□-open** [7](respectively **□*-open** [10]) if $A\subseteq Int(Cl(Int(A)))$, (respectively $A\subseteq Int^*(Cl(Int^*(A)))$).

Definition 2.6.A subset A of a topological space (X, τ) is **semi-preopen**[1]= β - open (respectively **semi*-preopen**[9]) if $A\subseteq Cl(Int(Cl(A)))$ (respectively $A\subseteq Cl^*(pInt(A))$).

Definition 2.7.A subset A of a topological space (X, τ) is **regular-open**[6] if $A=Int(Cl(A))$.

Definition 2.8. The ***δ-interior***[15]of A is defined as the union of all regular-open sets of X contained in A. It is denoted by **□*Int*(*A*).**

Definition 2.9.A subset A of a topological space (X, τ) is **□-open**[11] if $A=\delta Int(A)$.

Definition 2.10.A subset A of a topological space (X, τ) is **semi □-open** [6](respectively **semi* □-open** [13]) if there is a α-open set U in X such that $U\subseteq A\subseteq Cl(U)$ (respectively $U\subseteq A\subseteq Cl^*(U)$) or equivalently if $A\subseteq Cl(\alpha Int(A))$. (respectively$A\subseteq Cl^*(\alpha Int(A))$).

Definition 2.11.A subset A is **□-semi-open** [17] if $A\subseteq Cl(\delta Int(A))$.

The class of all semi-open (respectively semi*-open ,pre-open, pre*-open, α-open, α*-open, semi-preopen, semi*-preopen, semi α-open, semi* α-open, regular-open, δ-open and δ-semi-open) sets in (X, τ) is denoted by SO(X) (respectively S*O(X), PO(X). P*O(X),αO(X) , α*O(X) SPO(X), S*PO(X) ,SαO(X), S*αO(X), RO(X), δ O(X) and δSO(X).

Definition 2.12. The semi-interior (respectively semi*-interior[12], pre-interior[6], pre*-interior, α-interior, α*-interior,semipre-interior[1], semi*-pre-interior, semi α-interior, semi* α-interior, δ-interior and δ-semi-interior) of a subset *A* is defined to be the union of all semi-open (respectively semi*-open , pre-open, pre*-open, α-open, α*-open, semi-preopen, semi*-preopen, semi α-open, semi* α-open, regular-open and δ-semi-open) subsets of *A*. It is denoted by *sInt*(*A*) (respectively *s*Int*(*A*), *pInt(A), p*Int(A), αInt(A), α*Int(A), spInt(A), s*pInt(A),sαInt*(*A*), *s*αInt*(*A*),*δInt*(*A*)and δ*sInt*(*A*)).

Definition 2.13.A topological space *X* is T1/2[4] if every g-closed set in *X* is closed.

Theorem 2.14.[2] *Cl** is a Kuratowski closure operator in *X*.

Definition 2.15.[2] If τ* is the topology on *X* defined by the Kuratowski closure operator *Cl**, then (*X*, τ*) is T1/2.

Definition 2.16.[16] A space *X* is locally indiscrete if every open set in *X* is closed.

III. Semi* Δ -Open Sets

Definition 3.1: A subset *A* of a topological space (*X*, τ) is called a ***semi*δ-open set*** if there exists a δ-open set *U* in *X* such that *U*⊆*A*⊆*Cl**(*U*).

The class of all semi*δ-open sets in (*X*, τ) is denoted by S*δO(*X*, τ) or simply S*δO(*X*).

Theorem 3.2. For a subset *A* of a topological space (*X*, τ) the following statements are equivalent:

(i) *A* is semi*δ-open.

(ii) *A*⊆*Cl**(δ*Int*(*A*)).

(iii) *Cl**(δ*Int*(*A*))=*Cl**(*A*).

Proof: (i)⟹(ii): If *A* is semi*δ-open, then there is a δ-open set *U* in X such that *U*⊆*A*⊆*Cl**(*U*).

Now *U*⊆*A* ⟹*U*=δ*Int*(*U*)⊆δ*Int*(*A*) ⟹*A*⊆*Cl**(*U*)⊆*Cl**(δ*Int*(*A*)).

(ii)⟹(iii): By assumption, *A*⊆*Cl**(δ*Int*(*A*)). Since *Cl** is a Kuratowski operator, we have *Cl**(*A*)⊆*Cl**(*Cl**(δ*Int*(*A*)))=*Cl**(δ*Int*(*A*)). Now δ*Int*(*A*)⊆*A* implies that *Cl**(δ*Int*(*A*))⊆*Cl**(*A*). Therefore, *Cl**(δ*Int*(*A*))=*Cl**(*A*).

(iii)⟹(i): Take *U*=δ*Int*(*A*). Then *U* is aδ-open set in X such that *U*⊆*A*⊆*Cl**(*A*)=*Cl**(δ*Int*(*A*))=*Cl**(*U*). Therefore by Definition 3.1, *A* is semi*δ-open.

Remark 3.3. In any topological space (*X*, τ), ϕ and *X* are semi*δ-open sets. Every nonempty semi*δ-open set must contain a nonempty open set and therefore cannot be nowhere dense.

Theorem 3.4. Arbitrary union of semi*δ-open sets in *X* is also semi*δ-open in *X*.

Proof: Let {*A*i} be a collection of semi*δ-open sets in X. Since each *A*i is semi*δ-open, there is a δ-open set *U*i in *X* such that *U*i⊆*A*i⊆*Cl**(*U*i). Then ∪*U*i⊆∪*A*i⊆∪*Cl**(*U*i)⊆*Cl**(∪*U*i). Since ∪*U*i is δ-open, by Definition 3.1, ∪*A*i is semi*δ-open.

Remark 3.5. The intersection of two semi*δ-open sets need not be semi*δ-open as seen from the following examples. However the intersection of a semi*δ-open set and an open set is semi*δ-open as shown in Theorem 3.8.

Example 3.6: Let *X* = {*a*, *b*, *c*} and τ ={ϕ, {*a*}, {*b*}, {*a*, *b*}, *X*}. In the space (*X*, τ), the subsets {*a*, c} and {*b*, c} are semi*δ-open but their intersection {c} is not semi*δ-open.

Example 3.7: Let *X* = {*a*, *b*, *c, d*} and τ ={ϕ, {*a*}, {*b*},{c},{*a*, *b*},{*a*, c}, {*b*,*c*}, {*a*, *b*, *c* }, *X*}. In the space (*X*, τ), the subsets {b, *d*} and {c, *d*} are semi*δ-open but their intersection {*d*} is not semi*δ-open.

Theorem 3.8. If *A* is semi*δ-open in *X* and *B* is open in *X*, then *A*∩*B* is semi*δ-open in *X*.

Proof: Since *A* is semi*δ-open in *X*, there is a δ-open set *U* such that *U*⊆*A*⊆*Cl**(*U*).

Since *B* is open, we have *U*∩*B*⊆*A*∩*B*⊆*Cl**(*U*)∩*B*⊆*Cl**(*U*∩*B*). Since *U*∩*B* is δ-open, by Definition 3.1, *A*∩*B* is semi*δ-open in *X*.

Theorem 3.9. S*δO(*X*, τ) forms a topology on *X* if and only if it is closed under finite intersection.

Proof: Follows from Remark 3.3 and Theorem 3.4.

Theorem 3.10. Every δ-open set is semi*δ-open.

Let *U* be δ-open in *X*. Then by Definition 3.1, *U* is semi*δ-open.

Remark 3.11. The converse of the above theorem is not true as shown in the following examples.

Example 3.12. In the space (*X*, τ) where *X*={*a*, *b*, *c*} and τ={ϕ, {*a*}, {*b*}, {*a*, *b*}, *X*}, the subsets {*a*, *c*} and{b, *c*} are semi*δ-open but not δ-open.

Example 3.13. In the space (*X*, τ) where *X*={*a*, *b*, *c*, *d*} and τ={ϕ, {*a*}, {*b*}, {*a*, *b*},{*a*, *b*, *c*}, *X*}, the subsets {*a*, *d*}, {b, *d*}and {*a*, *b*,*d*} are semi*δ-open but not δ-open.

Theorem 3.14. In any topological space,

(i) Every semi*δ-open set is δ-semi-open.

(ii) Every semi*δ-open set is semi - open.

(iii) Every semi*δ-open set issemi* - open.

(iv) Every semi*δ-open set is semi*-preopen.

(v) Every semi*δ-open set is semi-preopen.

(vi) Every semi*δ-open set is semi*α-open
(vii) Every semi*δ-open set is semiα-open.

Proof:(i) Let A be a semi*δ-open set. Then from Theorem 3.2, $A\subseteq Cl^*(\delta Int(A))$. Since $Cl^*(\delta Int(A)) \subseteq Cl(\delta Int(A))$, we have $A\subseteq Cl(\delta Int(A))$. Hence A is δ-semi-open. Suppose A is a semi*δ-open set. Then from Theorem 3.2, $A\subseteq Cl^*(\delta Int(A))$. Since $Cl^*(\delta Int(A)) \subseteq Cl(\delta Int(A))$ and $\delta Int(A) \subseteq Int(A)$, we have $A\subseteq Cl(Int(A))$. Hence, A is semi-open. This proves (ii). Suppose A is a semi*δ-open set. Then from Theorem 3.2, $A\subseteq Cl^*(\delta Int(A)) \subseteq Cl^*(Int(A))$. Hence, A is semi*-open. Thus (iii) is proved. Let A be a semi*δ-open set. Then there is a δ-open set U in X such that $U\subseteq A\subseteq Cl^*(U)$. Since every δ-open set is preopen, by Definition 2.6, A is semi*-preopen. This proves (iv). The statement (v) follows from (iv) and the fact that every semi*-preopen set is semi-preopen. Let A be a semi*δ-open set. Then there is a δ-open set U in X such that $U\subseteq A\subseteq Cl^*(U)$. Since every δ-open set is α-open, by Definition 2.10, A is semi*α-open. This proves (vi). The statement (vii) follows from (vi) and the fact that every semi*α-open set is semiα-open.

Remark 3.15. The converse of each of the statements in Theorem 3.11 is not true as shown in the following examples.

Example 3.16. In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\},\{a, b, c\}, X\}$, the subsets $\{a, c\}$, {b, c},$\{a, b,c\}$, $\{a, c,d\}$ and $\{b,c,d\}$are semiδ-open but not semi*δ-open.

Example 3.17. In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, X\}$, the subsets$\{a\}$,$\{a, b\}$, {a,c},$\{a, d\}$,$\{a, b,c\}$,$\{a, b,d\}$and $\{a, c,d\}$ are semi-open but not semi*δ-open.

Example 3.18. In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a,b,c\}, X\}$, the subset $\{a, b,c\}$ is semi*-open but not semi*δ-open.

Example 3.19. In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a, b\},\{a, b, c\}, X\}$, the subsets$\{a\}$, {b}, $\{a, b\}$, {a, c},$\{a, d\}$,$\{b, c\}$, $\{ b, d\}$, $\{a, b, c\}$,$\{a, b, d\}$,$\{a, c, d\}$ and {b, c, d} are semi*-preopen but not semi*δ-open.

Example 3.20. In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\},\{ b, c,d\}, X\}$, the subsets{b}, {c},{d},$\{a, b\}$, {a, c},$\{a, d\}$,$\{b, c\}$, $\{ b, d\}$,{c,d},$\{a, b, c\}$,$\{a, b, d\}$ and $\{a, c, d\}$ are semi-preopen but not semi*δ-open.

Example 3.21. In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}$,{a, b, c}, $X\}$, the subsets{a}, $\{a, b\}$, {a, c},$\{a, d\}$,$\{a, b, c\}$,$\{a, b, d\}$ and $\{a, c, d\}$ are semi*α-open but not semi*δ-open.

Example 3.22. In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\},\{a, b\},\{a, b, c\}, X\}$, the subsets{a}, $\{a, b\}$, {a, c},$\{a, d\}$,$\{a, b, c\}$,$\{a, b, d\}$ and $\{a, c, d\}$ are semiα-open but not semi*δ-open.

Theorem 3.23. In any topological space (X,τ), $\delta O(X, \tau)\subseteq S^*\delta O(X, \tau)\subseteq \delta SO(X, \tau)$ .That is the class of semi*δ-open set is placed between the class of δ-open sets and the class of δ-semi-open sets.

Proof: Follows from Theorem 3.10 and Theorem 3.14.

Remark 3.24.

(i)If (X, τ) is a locally indiscrete space,
$\tau=\delta O(X, \tau)=S^*\delta O(X, \tau)=\delta SO(X, \tau)=S^*O(X,\tau)=SO(X, \tau)=\alpha O(X,\tau)= S^*\alpha O(X, \tau) = S\alpha O(X, \tau)=RO(X, \tau)$.
(ii) The inclusions in Theorem 3.23 may be strict and equality may also hold. This can be seen from the following examples.

Example 3.25. In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b, c, d\}, X\}$
$\delta O(X, \tau)=S^*\delta O(X, \tau)=\delta SO(X, \tau)$.

Example 3.26. In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau =\{\phi, \{a\}, \{b\},\{c\},\{a, b\},\{a, c\}, \{b,c\}, \{a, b, c \}, X\}$, $\delta O(X, \tau) \subsetneq S^*\delta O(X, \tau) = \delta SO(X, \tau)$.

Example 3.27. In the topological space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\},\{a, b, c\}, X\}$, $\delta O(X, \tau) \subsetneq S^*\delta O(X, \tau) \subsetneq \delta SO(X, \tau)$.

Remark 3.28: If X is a T1/2 space, the g-closed sets and the closed sets coincide and hence $Cl^*(U) = Cl(U)$. Therefore the class of semi*δ-open sets and the class of δ-semi-open sets coincide. In particular, in the real line with usual topology, the semi*δ-open sets and the δ-semi-open sets coincide. But the converse is not true. That is, a space, in which the class of semi*δ-open sets and the class of δ-semi-open sets coincide, need not be T1/2 and this can be seen from the following Example. In these spaces the class of semi*δ- open sets and the class of δ-semi-open sets coincide but they are not T1/2.

Example 3.29: Let X = {a, b, c, d} and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\},\{b ,c\},\{a, b, c\}, X\}$,GC(X, τ) = {ϕ, {d}, {a, d}, {b, d}, {c, d}, {a, c, d}, {a, b, d}, {b, c, d}, X}. In the space (X, τ), S*δO(X, τ) = δSO(X, τ) = {ϕ, {a}, {a, d}, {b, c},{a, b, c},{b, c, d}, X} but the g-closed sets and the closed sets are not coincide. Therefore, the space is not T1/2.

Theorem 3.30: If (X, τ) is any topological space, then S*δO(X, τ*) =δSO(X, τ*).

Proof: Follows from the fact that the space (X, τ*) is T1/2 [Theorem 2.15] and Remark 3.28.

Theorem 3.31. Let A be semi*δ-open and $B\subseteq X$ such that$\delta Int(A)\subseteq B\subseteq Cl^*(A)$. Then B is semi*δ-open.

Proof: Since A is semi*δ-open, by Theorem 3.2, we have $Cl^*(A)=Cl^*(\delta Int(A))$. Since $\delta Int(A)\subseteq B$, $\delta Int(A)\subseteq \delta Int(B)$ and hence $Cl^*(\delta Int(A))\subseteq Cl^*(\delta Int(B))$. Therefore by the assumption, we have

B⊆*Cl**(*A*)=*Cl**(δ*Int*(*A*))⊆*Cl**(δ*Int*(*B*)). Hence *B*⊆*Cl**(δ*Int*(*B*)). Again by invoking Theorem 3.2, *B* is semi*δ-open.

Remark 3.32. The concept of semi*δ-open sets and open sets are independent as seen from the following example:

Example 3.33. In the topological space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ, {*a*}, {*b*}, {*a, b*}, {*a, b, c*}, *X*}, the subsets {*a, d*},{b, *d*}and {*a, b, d*} are semi*δ-open but not open and {a, *b, c*} is open but not semi*δ-open.

Remark 3.34. The concept of semi*δ-open sets and g-open sets are independent as seen from the following example:

Example 3.35. In the topological space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ, {*a*}, {*b*}, {*a, b*}, {*a, b, c*}, *X*}, the subsets {*a, d*},{b, *d*}and {*a, b, d*} are semi*δ-open but not g-open and {c},{*a, c*},{b, c}and{a, *b, c*} are g-open but not semi*δ-open.

Remark 3.36. The concept of semi*δ-open sets and α-open sets are independent as seen from the following examples:

Example 3.37. In the topological space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ,{*a, b*}, {*a, b, c*}, *X*}, the subsets {*a, b*},{a, b, *c*}and {*a, b, d*} are α-open but not semi*δ-open.

Example 3.38. In the topological space (*X*, τ) where *X*={*a, b, c, d*} and τ ={ϕ, {*a*}, {*b*},{c},{*a, b*},{*a*, c}, {*b,c*}, {*a, b, c* }, *X*},the subsets {*a, d*},{*b, d*}, { *c, d*},{*a, b, d*},{*a, c, d*} and {b, *c, d*} are semi*δ-open but not α-open.

Remark 3.39. The concept of semi*δ-open sets and pre-open sets are independent as seen from the following examples:

Example 3.40. In the topological space (*X*, τ) where *X*={*a, b, c*} and τ={ϕ,{*a*},{ *b*},{*a, b*}, *X*}, the subsets {*a, c*}and{b, *c*} are semi*δ-open but not pre-open.

Example 3.41. In the topological space (*X*, τ) where *X*={*a, b, c*} and τ ={ϕ, {*a, b*}, *X*},the subsets {*a*},{*b*}, {*a, b*},{*a, c*}and{*b, c*} are pre-open but not semi*δ-open.

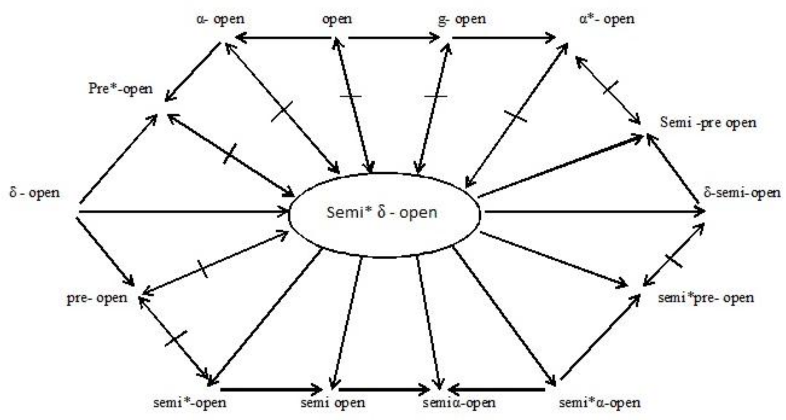
Remark 3.42. The concept of semi*δ-open sets and α*-open sets are independent as seen from the following examples:

Example 3.43. In the topological space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ, {*a*}, {*b*}, {*a, b*}, {*a, b, c*}, *X*}, the subsets {*a, d*}and{b, *d*} are semi*δ-open but not α*-open and { *c*},{ *a, c*},{b, *c*}and {a, *b, c*} areα*-open but not semi*δ-open.

Remark 3.44. The concept of semi*δ-open sets and pre*-open sets are independent as seen from the following examples:

Example 3.45. In the topological space (*X*, τ) where *X*={*a, b, c, d*} and τ={ϕ, {*a*}, {*b*}, {*a, b*}, {*a, b, c*}, *X*}, the subsets {*a, d*}and{b, *d*} are semi*δ-open but not pre*-open and { *c*},{ *a, c*},{b, *c*}and {a, *b, c*} arepre*-open but not semi*δ-open.

From the above discussions we have the following diagram:



IV. Semi* Δ -Interior Of A Set

Definition 4.1. The ***semi*δ-interior*** of *A* is defined as the union of all semi*δ-open sets of *X* contained in *A*. It is denoted by ***s*δInt*(*A*).**

Definition 4.2. Let *A* be a subset of a topological space (*X*, τ). A point *x* in *X* is called a ***semi*δ-interior point*** of *A* if there is a semi*δ-open subset of *A* that contains *x*.

Theorem 4.3. If *A* is any subset of a topological space (*X*, τ), then

(i) *s*δInt*(*A*) is the largest semi*δ-open set contained in *A*.

(ii) *A* is semi*δ-open if and only if s**δInt*(*A*)=*A*.

(iii) *s*δInt*(*A*) is the set of all semi*δ-interior points of *A*.

(iv) *A*is semi*δ-open if and only if every point of *A* is a semi*δ-interior point of *A*.

Proof:(i) Being the union of all semi*δ-open subsets of A, by Theorem 3.4, $s^*\delta Int(A)$ is semi*δ-open and contains every semi*δ-open subset of A. This proves (i).
(ii) A is semi*δ-open implies $s^*\delta Int(A)=A$ is obvious from Definition 4.1. On the other hand, suppose $s^*\delta Int(A)=A$. By (i), $s^*\delta Int(A)$ is semi*δ-open and hence A is semi*δ-open.
(iii) By Definition 4.1, $x\in s^*\delta Int(A)$ if and only if x belongs to some semi*δ-open subset U of A. That is, if and only if x is a semi*δ-interior point of A.
(iv) follows from (ii) and (iii).

Theorem 4.4. (Properties of Semi* δ-Interior)
In any topological space (X, τ) the following statements hold:
(i) $s^*\delta Int(\phi)=\phi$.
(ii) $s^*\delta Int(X)=X$.
If A and B are subsets of X,
(iii) $s^*\delta Int(A)\subseteq A$.
(iv) $A\subseteq B \Longrightarrow s^*\delta Int(A)\subseteq s^*\delta Int(B)$.
(v) $s^*\delta Int(s^*\delta Int(A))=s^*\delta Int(A)$.
(vi) $\delta Int(A)\subseteq s^*\delta Int(A)\subseteq \delta sInt(A)\subseteq A$.
(vii) $s^*\delta Int(A\cup B)\supseteq s^*\delta Int(A)\cup s^*\delta Int(B)$.
(viii) $s^*\delta Int(A\cap B)\subseteq s^*\delta Int(A) \cap s^*\delta Int(B)$.

Proof: (i), (ii), (iii) and (iv) follow from Definition 4.1. By Theorem 4.3(i), $s^*\delta Int(A)$ is semi*δ-open and by Theorem 4.3(ii), $s^*\delta Int(s^*\delta Int(A))=s^*\delta Int(A)$. Thus (v) is proved. The statements (vi) follows from Theorem 3.10and Theorem 3.14(i). Since $A\subseteq A\cup B$, from statement (iv) we have $s^*\delta Int(A)\subseteq s^*\delta Int(A\cup B)$. Similarly, $s^*\delta Int(B)\subseteq s^*\delta Int(A\cup B)$. This proves (vii). The proof for (viii) is similar.

Remark 4.5. In (vi) of Theorem 4.4,each of the inclusions may be strict and equality may also hold. This can be seen from the following examples:

Example 4.6: In the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi,\{a\},\{b\},\{a, b\}, \{a, b, c\}, X\}$.
Let $A= \{a, b\}$. Then $\delta Int(A)=s^*\delta Int(A)=\delta sInt(A)=\{a, b\}=A$.
Let $B=\{a, b, d\}$.Then $\delta Int(B)=\{a, b\}$; $s^*\delta Int(B)=\delta sInt(B)=\{a, b, d\}$.
Here$\delta Int(B)\subsetneq s^*\delta Int(B)=\delta sInt(B)=B$.
Let $C= \{ b, c\}$.Then $\delta Int(C)=s^*\delta Int(C)=\{b\}$; $\delta sInt(C)=\{ b, c\}$.
Here $\delta Int(C) =s^*\delta Int(C) \subsetneq \delta sInt(C) =C$.
Let $D= \{c, d\}$.Then $\delta Int(D)=s^*\delta Int(D)=\delta sInt(D)=\phi$.
Here $\delta Int(D)=s^*\delta Int(D)=\delta sInt(D)\subsetneq D$.
Let $E=\{b, c, d\}$.Then $\delta Int(E)=\{b\}$; $s^*\delta Int(E)=\{b, d\}$; $\delta sInt(E)=\{b, c, d\}$.
Here $\delta Int(E)\subsetneq s^*\delta Int(E)\subsetneq \delta sInt(E)=E$.

Remark 4.7: The inclusions in (vii) and (viii) of Theorem 4.4 may be strict and equality may also hold. This can be seen from the following examples.

Example 4.8: Consider the space (X, τ) in Example 4.6
Let $A= \{a, b\}$ and $B=\{b, d\}$ then $A\cup B=\{a, b, d\}$;
$s^*\delta Int(A)=\{a, b\}$; $s^*\delta Int(B)=\{b, d\}$; $s^*\delta Int(A\cup B)=\{a, b, d\}$;
Here $s^*\delta Int(A\cup B)= s^*\delta Int(A) \cup s^*\delta Int(B)$
Let $C= \{a, b\}$ and $D =\{b, c\}$ then $C\cap D=\{b\}$;
$s^*\delta Int(C)=\{a, b\}$; $s^*\delta Int(D)=\{b\}$; $s^*\delta Int(C\cap D)=\{b\}$;
Here $s^*\delta Int(C\cap D)= s^*\delta Int(C) \cap s^*\delta Int(D)$
Let $E= \{a, c, d\}$ and $F =\{b, c, d\}$ then $E\cap F =\{c, d\}$;
$s^*\delta Int(E)=\{a, d\}$; $s^*\delta Int(F)=\{ b, d\}$; $s^*\delta Int(E\cap F)=\phi$; $s^*\delta Int(E) \cap s^*\delta Int(F)=\{d\}$
Here $s^*\delta Int(E\cap F)\subsetneq s^*\delta Int(E) \cap s^*\delta Int(F)$
Let $G=\{a, b\}$ and $H=\{ c, d\}$ then $G\cup H=\{a, b, c, d\}=X$;
$s^*\delta Int(G)=\{a, b\}$; $s^*\delta Int(H)= \phi$;$s^*\delta Int(G\cup H)=\{a, b, c, d\}$; $s^*\delta Int(G) \cup s^*\delta Int(H)=\{a, b\}$;
Here $s^*\delta Int(G) \cup s^*\delta Int(H)\subsetneq s^*\delta Int(G\cup H)$.

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