Inventory Model with Demand Rate Is 3-Variables Weibull Function Constant Deterioration, Constant Holding Cost and Inflation without Shortages

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Abstract: An inventory model with 3-variable Weibull distribution demand rate and constant deterioration is developed. The holding cost is constant. Shortages are not allowed. The salient feature of this inventory model is for non-perishable products or deteriorating items. A numerical example has been illustrated using MATLAB to describe the model and the sensitivity analysis of various parameters is carried out.

Keywords - Constant deterioration; 3-variables Weibull demand; Inflation; Inventory; Constant holding cost

I. Introduction

rate and we position our model in relative to previous work. Section 2 details the model assumptions. In Section 3, we have given notation used in model. In Section 4, we formulate the model as a cost minimization problem. In Section 5, a mathematical solution is given to illustrate the model. Section 6 details an algorithm for optimizing the solution. In Section 7, we sum with a conclusion of this work and suggest directions for future research. Numerical Solution and analysis of various parameters is taken in Section 8.

II. Assumptions

The following assumptions are made in developing the model.

➢ The inventory system considers a single item only.
➢ The demand rate is 3-variable Weibull distribution of time.
➢ The deterioration rate is constant.
➢ The inventory system is considered over a finite time horizon.
➢ Lead time is zero.
➢ Shortages are not allowed.

III. Notations

The following notations use for inventory model.

$A$: Setup cost.
$D(t)$: 3-variable Weibull distribution demand rate in a period $[0, T]$
$\theta(t)$: Constant Deterioration rate
$Q_0$: Initial ordering quantity
$TD$: Total demand in a cycle period $[0, T]$
$DU$: Deteriorating unit in a cycle period $[0, T]$
$C_d$: Deteriorating cost per unit
$DC$: Deteriorating cost
$HC$: Holding cost
$TC(T)$: Total inventory cost
$T^*$: Optimal length size
$Q^*_0$: Optimal initial order quantity
$TC^*(T)$: Optimal total cost in the period $[0, T]$

IV. Mathematical Formulation

Consider the inventory model of 3-variable Weibull distribution demand rate. As the inventory reduces due to demand rate as well as deterioration rate during the interval, the differential representing the inventory status is governed by $[0, T]$

$$\frac{dI(t)}{dt} + \theta(t) = -D(t), \quad 0 \leq t \leq T$$

Where

$$D(t) = \frac{\alpha}{\beta} \left( \frac{t-\mu}{\beta} \right)^{\alpha-1} e^{-\left( \frac{t-\mu}{\beta} \right)^\alpha}, \quad D(t) \geq 0, \quad \alpha, \beta \geq 0$$

V. Mathematical Solution

The solution with boundary condition $I(T) = 0$, of the Equation

$$\frac{dI(t)}{dt} + \theta(t) = -\frac{\alpha}{\beta} \left( \frac{t-\mu}{\beta} \right)^{\alpha-1} e^{-\left( \frac{t-\mu}{\beta} \right)^\alpha}, \quad 0 \leq t \leq T$$
Inventory Model with Demand Rate Is 3-Variables Weibull Function Constant Deterioration....

\[ I(t) = -(1 + \theta) \left[ \left( \frac{t - \mu}{\beta} \right)^{a} - \frac{1}{2} \left( \frac{t - \mu}{\beta} \right)^{2a} \right] + \theta \left[ \frac{\beta}{\alpha + 1} \left( \frac{t - \mu}{\beta} \right)^{\alpha+1} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{t - \mu}{\beta} \right)^{2\alpha+1} \right] \]

\[ + (1 + \theta^T) \left[ \left( \frac{T - \mu}{\beta} \right)^{a} - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2a} \right] - \theta \left[ \frac{\beta}{\alpha + 1} \left( \frac{T - \mu}{\beta} \right)^{\alpha+1} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \] \]

\[ e^{-\alpha} \]

(3)

Where use the expansion \( e^{-x} \approx 1 - x + \frac{x^2}{2} \) .... \( x \) is small and positive.

So the initial order quantity is obtained by putting the boundary condition in Equation (3) \( I(0) = Q_0 \). Therefore,

\[ Q_0 = -(\left( \frac{-\mu}{\beta} \right)^{a} - \frac{1}{2} \left( \frac{-\mu}{\beta} \right)^{2a}) + \theta \left[ \frac{\beta}{\alpha + 1} \left( \frac{-\mu}{\beta} \right)^{\alpha+1} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{-\mu}{\beta} \right)^{2\alpha+1} \right] \]

\[ + (1 + \theta^T) \left[ \left( \frac{T - \mu}{\beta} \right)^{a} - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2a} \right] - \theta \left[ \frac{\beta}{\alpha + 1} \left( \frac{T - \mu}{\beta} \right)^{\alpha+1} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \] \]

(4)

The total demand during the cycle period \([0, T]\) is

\[ TD = \int_0^T D(t) dt \]

\[ = \int_0^T \frac{\alpha}{\beta} \left( \frac{t - \mu}{\beta} \right)^{\alpha+1} e^{\left( \frac{t - \mu}{\beta} \right)^{\alpha}} dt \]

\[ = \left[ \left( \frac{T - \mu}{\beta} \right)^{a} - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2a+1} \right] - \left( \frac{-\mu}{\beta} \right)^{a} - \frac{1}{2} \left( \frac{-\mu}{\beta} \right)^{2a+1} \] \]

(5)

Then the number of deterioration units is

\[ DU = Q_0 - TD \]

\[ DU = \left[ \frac{1}{2} \left( \frac{-\mu}{\beta} \right)^{2a} + \theta \left[ \frac{\beta}{\alpha + 1} \left( \frac{-\mu}{\beta} \right)^{\alpha+1} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{-\mu}{\beta} \right)^{2\alpha+1} \right] \]

\[ + (1 + \theta^T) \left[ \left( \frac{T - \mu}{\beta} \right)^{a} - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2a} \right] - \theta \left[ \frac{\beta}{\alpha + 1} \left( \frac{T - \mu}{\beta} \right)^{\alpha+1} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \]

\[ - \left[ \left( \frac{T - \mu}{\beta} \right)^{a} - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2a+1} \right] \] \]

(6)

The deterioration cost for the cycle \([0, T]\)

\[ DC = C_d \times \text{(Number of deterioration units)} \]
Inventory Model with Demand Rate Is 3-Variables Weibull Function Constant Deterioration....

\[
DC = C_a^* + T \left\{ \left[ \frac{T - \mu}{\beta} \right]^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right\} - \theta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]
\]

Holding Cost for the cycle \([0, T]\) is

\[
HC = \int_0^T he^{-\alpha t} I(t) dt
\]

\[
HC = \left\{ \left[ \frac{T - \mu}{\beta} \right]^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right\} h^* \]

\[
\begin{align*}
&+ T(1 + \theta T) \left[ \frac{T - \mu}{\beta} \right]^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]\ - \theta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \\
&+ (rT + \theta rT + \theta T^2) \left[ \frac{T - \mu}{\beta} \right]^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]\ - \theta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \\
&- (r + \theta + r\theta + 2\theta T) \left[ \frac{T - \mu}{\beta} \right]^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]\ - \theta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \\
&- T(r + \theta) \left[ \frac{T - \mu}{\beta} \right]^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]\ - \theta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \\
&+ T^2 \left[ \frac{T - \mu}{\beta} \right]^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]\ - \theta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \\
&- \beta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]\ + \theta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \\
&+ T^2 \left[ \frac{T - \mu}{\beta} \right]^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]\ - \theta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \\
&\frac{\beta \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]}{\left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right]
\end{align*}
\]
The total inventory cost $TC(T) = \frac{1}{T}[A + DC + HC]$

Therefore the total variable cost per unit time

$TC(T) = \frac{1}{T}[A + C_r * \left \{ \frac{1}{2} (\frac{-\mu}{\beta})^{2 \alpha} + \theta \left \{ \frac{\beta}{\alpha + 1} \left( \frac{-\mu}{\beta} \right)^{\alpha+1} \right \} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{-\mu}{\beta} \right)^{2\alpha+1} \right \} + (1 + \theta T) \left \{ \frac{1}{2} (\frac{T - \mu}{\beta})^{2 \alpha} - \theta \left \{ \frac{\beta}{\alpha + 1} \left( \frac{T - \mu}{\beta} \right)^{\alpha+1} \right \} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right \} \right]$

$- \left[ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right] \right) + h^* \{(1 + \theta T) \left \{ \frac{\beta}{\alpha + 1} \left( \frac{T - \mu}{\beta} \right)^{\alpha+1} \right \} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right \}$

$+ T(1 + \theta T) \left \{ \frac{\beta}{\alpha + 1} \left( \frac{T - \mu}{\beta} \right)^{\alpha+1} \right \} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right \}$

$+ (rT + \theta T + \theta T^2) \left \{ \frac{\beta}{\alpha + 1} \left( \frac{T - \mu}{\beta} \right)^{\alpha+1} \right \} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right \}$

$- 2\theta^2 \left[ \frac{\beta^3}{(\alpha + 1)(\alpha + 2)(\alpha + 3)} \left( \frac{T - \mu}{\beta} \right)^{\alpha+3} \right] - \frac{\beta^3}{2(2\alpha + 1)(\alpha + 1)(2\alpha + 3)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+3} \right \}$

$- \theta(\alpha + 1) \left[ \frac{\beta^3}{(\alpha + 1)(\alpha + 2)(\alpha + 3)} \left( \frac{T - \mu}{\beta} \right)^{\alpha+3} \right] - \frac{\beta^3}{2(2\alpha + 1)(\alpha + 1)(2\alpha + 3)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+3} \right \}$

$+ (T - (\alpha + 1)\theta T) \left \{ \left( \frac{T - \mu}{\beta} \right)^\alpha - \frac{1}{2} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right \} - \theta \left \{ \frac{\beta}{\alpha + 1} \left( \frac{T - \mu}{\beta} \right)^{\alpha+1} \right \} - \frac{\beta}{2(2\alpha + 1)} \left( \frac{T - \mu}{\beta} \right)^{2\alpha+1} \right \}$

$- \left[ \frac{\beta}{\alpha + 1} \left( \frac{-\mu}{\beta} \right)^{\alpha+1} \right] + \theta \left \{ \frac{\beta^2}{(\alpha + 1)(\alpha + 2)} \left( \frac{-\mu}{\beta} \right)^{\alpha+2} \right \} - \frac{\beta^2}{2(2\alpha + 1)(\alpha + 1)} \left( \frac{-\mu}{\beta} \right)^{2\alpha+2} \right \}$

$+ (r + \theta r) \left \{ \frac{\beta^2}{(\alpha + 1)(\alpha + 2)} \left( \frac{-\mu}{\beta} \right)^{\alpha+2} \right \} - \frac{\beta^2}{2(2\alpha + 1)(\alpha + 1)} \left( \frac{-\mu}{\beta} \right)^{2\alpha+2} \right \}$

$+ 2\theta^2 \left[ \frac{\beta^3}{(\alpha + 1)(\alpha + 2)(\alpha + 5)} \left( \frac{-\mu}{\beta} \right)^{\alpha+3} \right] - \frac{\beta^3}{2(2\alpha + 1)(\alpha + 1)(2\alpha + 3)} \left( \frac{-\mu}{\beta} \right)^{2\alpha+3} \right \}$

$- \theta(\alpha + 1) \left \{ \frac{\beta^3}{(\alpha + 1)(\alpha + 2)(\alpha + 3)} \left( \frac{-\mu}{\beta} \right)^{\alpha+3} \right \} - \frac{\beta^3}{2(2\alpha + 1)(\alpha + 1)(2\alpha + 3)} \left( \frac{-\mu}{\beta} \right)^{2\alpha+3} \right \}$

$\text{(9)}$
The necessary and sufficient conditions for minimize cost a given value \( T \) are
\[
\frac{dTC(T)}{dT} = 0 \quad \text{And} \quad \frac{d^2TC(T)}{dT^2} > 0 \]
then differentiation with respect to \( T \) of (9), we get

VI. Algorithm

To find out the solution following algorithm used

Step1: Find derivative \( \frac{dTC(T)}{dT} \) and put \( \frac{dTC(T)}{dT} = 0 \)

Step2: Solve equation for \( T \)

Step3: Find the derivative \( \frac{d^2TC(T)}{dT^2} \) and check \( \frac{d^2TC(T)}{dT^2} > 0 \) for \( T^* \) optimal length

Step4: Find optimal total cost and initial order quantity

VII. Conclusion

In the present paper, we developed an inventory model for variable deteriorating item with inflation and the model with 3-variable Weibull distribution demand rate is developed. The deterioration cost is constant. Shortages are not allowed. The model is very practical for the industries in which the demand rate is depending upon the time and holding cost is constant with inflation. This model can further be extended by taking more realistic assumptions such as finite replenishment rate, variable deterioration rate, variable holding etc.

VIII. Numerical Solution

Consider an inventory system with the following parameter in proper units \( A = 800 \), \( \alpha = 8 \), \( \mu = 0.6 \), \( \beta = 3 \), \( \theta = 0.02 \), \( r = 0.04 \), \( h = 10 \) and \( C_d = 12 \). The computer output of the program by using Mat lab software is \( T^* = 4.2013 \), \( Q^* = 182.95 \) and \( TC^* = 343.708 \). The effect of changes in the parameter of the inventory model also can be study by using Mat lab. If \( \alpha < \beta \) the total cost decreases and if \( \mu < \beta \) the total cost is also decreases. If \( \alpha \) increases the total cost also decreases but if \( r \) increases then total cost increases.

References

Inventory Model with Demand Rate Is 3-Variables Weibull Function Constant Deterioration....


