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On Some Bilateral Generating Relations Involving I-Function

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Abstract: *The aim of this research paper is to establish some bilateral generating relations involving I-function of two variables.*

I. Introduction

The I–function of two variables introduced by Sharma & Mishra [2], will be defined and represented as follows:

$$I\begin{bmatrix}x\\y\end{bmatrix} = I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_i',q_i':r':p_i'',q_i'':r''}\left[\begin{matrix}x\\y\end{matrix}\Big|^{[(a_j:\alpha_j,A_j)_{1,n}],[(a_{ji}:\alpha_{ji},A_{ji})_{n+1,p_i}]}_{[(b_{ji}:\beta_{ji},B_{ji})_{1,q_i}]}\right.$$
$$\left.\begin{matrix}:[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_i'}];[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_i''}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta\gamma_{ji'})_{m_1+1,q_i'}];[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_i''}]\end{matrix}\right]$$
$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\,x^\xi y^\eta\,d\xi d\eta, \qquad (1)$$

where

$$\phi_1(\xi,\eta) = \frac{\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)}{\sum_{i=1}^{r}[\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi-A_{ji}\eta)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi+B_{ji}\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_1}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_1}\Gamma(1-c_j+\gamma_j\xi)}{\sum_{i'=1}^{r'}[\prod_{j=m_1+1}^{q_i'}\Gamma(1-d_{ji'}+\delta_{ji'}\xi)\prod_{j=n_1+1}^{p_i'}\Gamma(c_{ji'}-\gamma_{ji'}\xi)},$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{m_2}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_2}\Gamma(1-e_j+E_j\eta)}{\sum_{i''=1}^{r''}[\prod_{j=m_2+1}^{q_i''}\Gamma(1-f_{ji''}+F_{ji''}\eta)\prod_{j=n_2+1}^{p_i''}\Gamma(e_{ji''}-E_{ji''}\eta)},$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, $p_{i'}$, $p_{i''}$, q_i, $q_{i'}$, $q_{i''}$, n, n_1, n_2, n_j and m_k are non negative integers such that $p_i \geq n \geq 0$, $p_{i'} \geq n_1 \geq 0$, $p_{i''} \geq n_2 \geq 0$, $q_i > 0$, $q_{i'} \geq 0$, $q_{i''} \geq 0$, (i = 1, …, r; i′ = 1, …, r′; i′′ = 1, …, r′′; k = 1, 2) also all the A's, α's, B's, β's, γ's, δ's, E's and F's are assumed to be positive quantities for standardization purpose; the definition of I-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the ξ–plane and runs from $-\omega\infty$ to $+\omega\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(d_j-\delta_j\xi)$ (j = 1, ..., m_1) lie to the right, and the poles of $\Gamma(1-c_j+\gamma_j\xi)$ (j = 1, ..., n_1), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour.

The contour L_2 is in the η–plane and runs from $-\omega\infty$ to $+\omega\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(f_j-F_j\eta)$ (j=1,....., n_2) lie to the right, and the poles of $\Gamma(1-e_j+E_j\eta)$ (j = 1, ..., m_2), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour. Also

$$R' = \sum_{j=1}^{p_i}\alpha_{ji} + \sum_{j=1}^{p_i'}\gamma_{ji'} - \sum_{j=1}^{q_i}\beta_{ji} - \sum_{j=1}^{q_i'}\delta_{ji'} < 0,$$

$$S' = \sum_{j=1}^{p_i}A_{ji} + \sum_{j=1}^{p_i''}E_{ji''} - \sum_{j=1}^{q_i}B_{ji} - \sum_{j=1}^{q_i''}F\delta_{ji'} < 0,$$

$$U' = \sum_{j=n+1}^{p_i}\alpha_{ji} - \sum_{j=1}^{q_i}\beta_{ji} + \sum_{j=1}^{m_1}\delta_j - \sum_{j=m_1+1}^{q_i'}\delta_{ji'} + \sum_{j=1}^{n_1}\gamma_j - \sum_{j=n_1+1}^{p_i'}\gamma_{ji'} > 0, \qquad (2)$$

$$V' = -\sum_{j=n+1}^{p_i}A_{ji} - \sum_{j=1}^{q_i}B_{ji} - \sum_{j=1}^{m_2}F_j - \sum_{j=m_{21}+1}^{q_i''}F_{ji''} + \sum_{j=1}^{n_2}E_j - \sum_{j=n_2+1}^{p_i''}E_{ji''} > 0, \qquad (3)$$

and $|\arg x| < \frac{1}{2}U'\pi$, $|\arg y| < \frac{1}{2}V'\pi$.

In the present investigation we require the following formulae:

From Rainvile [1, p.93]:

$$_2F_1\left[\begin{matrix}-n, & a;\\ 1+a+n;\end{matrix} -1\right] = \frac{(1+a)_n}{(1+a/2)_n}, \qquad (4)$$

From Shrivastava and Manocha [3, p.37 (10), 34, 44],

$$(\alpha)_n = (\alpha, n) = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}, \tag{5}$$

$$(1-z)^{-a} = \sum_{n=0}^{\infty} (a)_n \frac{z^n}{n!}, \tag{6}$$

II. Bilateral Generating Relations

In this section we establish the following bilateral Generating Relations:

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} \; {}_2F_1\left[{-n,\; a; \atop 1+a+n;} -1\right]$$

$$. I^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_i,q_i:r;p_i'+1,q_i':r';p_i'',q_i'':r''}\left[{x \atop y}\middle| {\ldots,\ldots:(-a/2-n,0),\ldots:\ldots,\ldots \atop \ldots,\ldots:\ldots,\ldots:\ldots,\ldots}\right]$$

$$= (1-t)^{-(a+1)} I^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_i,q_i:r;p_i'+1,q_i':r';p_i'',q_i'':r''}\left[{x \atop y}\middle| {\ldots,\ldots:(-a/2,0),\ldots:\ldots,\ldots \atop \ldots,\ldots:\ldots,\ldots:\ldots,\ldots}\right], \tag{7}$$

provided that $U' > 0, V' > 0, |\arg x| < \frac{1}{2}U'\pi, |\arg y| < \frac{1}{2}V'\pi$ where U' and V' are given in (2) and (3).

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} \; {}_2F_1\left[{-n,\; a; \atop 1+a+n;} -1\right]$$

$$. I^{0,n_1;m_2+1,n_2;m_3,n_3}_{p_i,q_i:r;p_i',q_i'+1:r';p_i'',q_i'':r''}\left[{x \atop y}\middle| {\ldots,\ldots:\ldots:\ldots,\ldots \atop \ldots,\ldots:(1+a/2+n,0),\ldots:\ldots,\ldots}\right]$$

$$= (1-t)^{-(a+1)} I^{0,n_1;m_2+1,n_2;m_3,n_3}_{p_i,q_i:r;p_i',q_i'+1:r';p_i'',q_i'':r''}\left[{x \atop y}\middle| {\ldots,\ldots:\ldots:\ldots,\ldots \atop \ldots,\ldots:(1+a/2,0),\ldots:\ldots,\ldots}\right], \tag{8}$$

provided that $U' > 0, V' > 0, |\arg x| < \frac{1}{2}U'\pi, |\arg y| < \frac{1}{2}V'\pi$ where U' and V' are given in (2) and (3).

Proof:

To prove (7), consider

$$\Delta = \sum_{n=0}^{\infty} \frac{t^n}{n!} \; {}_2F_1\left[{-n,\; a; \atop 1+a+n;} -1\right]$$

$$. I^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_i,q_i:r;p_i'+1,q_i':r';p_i'',q_i'':r''}\left[{x \atop y}\middle| {\ldots,\ldots:(-a/2-n,0),\ldots:\ldots,\ldots \atop \ldots,\ldots:\ldots,\ldots:\ldots,\ldots}\right]$$

On expressing I-function in contour integral form as given in (1) and using (4), we get

$$\Delta = \sum_{n=0}^{\infty} \frac{t^n}{n!} \frac{(1+a)_n}{(1+a/2)_n}$$

$$. \frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\Gamma\{1-(-\tfrac{a}{2}-n)+0\xi\} x^{\xi} y^{\eta}\, d\xi d\eta$$

In the view of (5) and (6), we arrive at R.H.S. of (7) as follows:

$$\Delta = \sum_{n=0}^{\infty} \frac{t^n}{n!} \frac{(1+a)_n}{(1+a/2)_n}$$

$$. \frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta) \left(1+\frac{a}{2}\right)_n \Gamma(1+a/2) x^{\xi} y^{\eta}\, d\xi d\eta$$

$$= \frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)\Gamma(1+a/2)$$

$$. \left[\sum_{n=0}^{\infty} \frac{t^n}{n!} (1+a)_n\right] x^{\xi} y^{\eta}\, d\xi d\eta$$

$$= \frac{1}{(2\pi\omega)^2} \int_{L_1} \int_{L_2} \phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)$$

$$\Gamma(1 + a/2)(1 - t)^{-(a+1)} x^{\xi} y^{\eta} \, d\xi d\eta$$

$$= (1 - t)^{-(a+1)} I^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_i,q_i:r;p_i'+1,q_i':r';p_i'',q_i'':r''} \left[{x \atop y} \middle| {\ldots\ldots,\ldots\ldots:(-a/2,0),\ldots\ldots:\ldots\ldots,\ldots\ldots \atop \ldots\ldots,\ldots\ldots:\ldots\ldots\ldots\ldots,\ldots\ldots\ldots:\ldots\ldots,\ldots\ldots} \right].$$

Proceeding on similar lines as above, the results (8) can be derived easily.

III. Particular Cases

On choosing r = 1, r′ = 1 and r′′ = 1 in main integrals, we get following integrals in terms of H-function of two variables:

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} \; {}_2F_1\left[{-n,\; a; \atop 1+a+n;} - 1\right]$$

$$.H^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_1,q_1;p_2+2,q_2;p_3,q_3} \left[{x \atop y} \middle| {\ldots\ldots,\ldots\ldots:(-a/2-n,0),\ldots\ldots:\ldots\ldots,\ldots\ldots \atop \ldots\ldots,\ldots\ldots:\ldots\ldots\ldots\ldots,\ldots\ldots\ldots\ldots:\ldots\ldots,\ldots\ldots} \right]$$

$$= (1 - t)^{-(a+1)} H^{0,n_1;m_2,n_2+1;m_3,n_3}_{p_1,q_1;p_2+2,q_2;p_3,q_3} \left[{x \atop y} \middle| {\ldots\ldots,\ldots\ldots:(-a/2,0),\ldots\ldots:\ldots\ldots,\ldots\ldots \atop \ldots\ldots,\ldots\ldots:\ldots\ldots\ldots\ldots,\ldots\ldots\ldots:\ldots\ldots,\ldots\ldots} \right], \qquad (9)$$

provided that $U > 0, V > 0, |\arg x| < \frac{1}{2}U\pi, |\arg y| < \frac{1}{2}V\pi$ where U and V are given by:

$$U = -\sum_{j=n_1+1}^{p_1} \alpha_j - \sum_{j=1}^{q_1} \beta_j + \sum_{j=1}^{m_2} \delta_j - \sum_{j=m_2+1}^{q_2} \delta_j + \sum_{j=1}^{n_2} \gamma_j - \sum_{j=n_2+1}^{p_2} \gamma_j > 0, \qquad (10)$$

$$V = -\sum_{j=n_1+1}^{p_1} A_j - \sum_{j=1}^{q_1} B_j + \sum_{j=1}^{m_3} F_j - \sum_{j=m_3+1}^{q_3} F_j + \sum_{j=1}^{n_3} E_j - \sum_{j=n_3+1}^{p_3} E_j > 0, \qquad (11)$$

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} \; {}_2F_1\left[{-n,\; a; \atop 1+a+n;} - 1\right]$$

$$.H^{0,n_1;m_2+1,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3} \left[{x \atop y} \middle| {\ldots\ldots,\ldots\ldots:\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots:\ldots\ldots,\ldots\ldots \atop \ldots\ldots,\ldots\ldots:(1+a/2+n,0),\ldots\ldots:\ldots\ldots,\ldots\ldots} \right]$$

$$= (1 - t)^{-(a+1)} H^{0,n_1;m_2+1,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3} \left[{x \atop y} \middle| {\ldots\ldots,\ldots\ldots:\ldots\ldots\ldots\ldots\ldots\ldots:\ldots\ldots,\ldots\ldots \atop \ldots\ldots,\ldots\ldots:(1+a/2,0),\ldots\ldots:\ldots\ldots,\ldots\ldots} \right], \qquad (12)$$

provided that $U > 0, V > 0, |\arg x| < \frac{1}{2}U\pi, |\arg y| < \frac{1}{2}V\pi$ where U and V are given in (10) and (11).

References

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