IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 4 Ver. II (Jul. - Aug.2016), PP 87-90
www.iosrjournals.org

On Quasi-weak m-power commutative Near - rings and Quasi - weak (m,n) power commutative Near – rings

G.Gopalakrishmoorthy[1], S.Geetha[2] and S.Anitha[3]
[1]*Principal, Sri krishnasamy Arts and Science College , Sattur – 626203, Tamilnadu.*
[2]*Dept. of Mathematics, Pannai College of Engineering and Technology, Keelakkandani, Sivagangai - 630561.*
[3]*Lecturer,Raja Doraisingam Government Arts College, Sivagangai – 630 561, Tamil Nadu.*

Abstract: *A right near – ring N is called weak commutative if xyz = xzy for every x,y,z ϵ N (Definition 9.4 [10]). A right near – ring N is called pseudo commutative (Definition 2.1 [11]) if xyz = zyx for all x,y,z ϵ N.A right near – ring N is called quasi – weak commutative (Definition 2.1 [7]) if xyz = yxz for all x,y,z ϵ N. We call a right near – ring N to be quasi – weak m – power commutative if* $x^m y z = y^m x z$ *for all x,y,z ϵ N. N is said to be Quasi – weak (m,n) power commutative near – ring if* $x^m y^n z = y^m x^n z$ *for all x,y,z ϵ N.In this paper we study and establish various results of Quasi – weak m – power commutative near – ring and Quasi – weak (m,n) power commutative near -- ring.*

I. Introduction

S.Uma,R.Balakrishnan and T.Tamizhchelvam [11] called a near- ring N to be pseudo commutative if xyz = zyx for every x,y,z ϵ N.G.Gopalakrishnamoorthy and S.Geetha [4] called a ring R to be m power commutative if $x^m y = y^m x$ for all x,y ϵ R where m ≥ 1 is a fixed integer.They also called a ring R to be (m,n) power commutative if $x^m y^n = y^m x^n$ for all x,y ϵ R where m ≥ 1 and n ≥ 1 are fixed integers. G.Gopalakrishnamoorthy and R.Veega [6] called a near – ring N to be pseudo m- power commutative if $x^m y z = z y^m x$ for all x,y,z ϵ N where m ≥ 1 is a fixed integer. G.Gopalakrishnamoorthy, N.Kamaraj and S.Geetha [7] defined a near – ring N to be Quasi – weak commutative if xyz = yxz for all x,y,z ϵ N.

In this paper we define quasi-weak m – power commutative near – ring and quasi – weak (m,n) power commutative near – ring and establish some results.

II. Preliminaries

Throughout this paper N denotes a right near – ring with atleast two elements. For any non-empty set A ⊆ N , we denote A- {0} by A* .In this section we present some known definitions and results which are useful in the development of this paper.

2.1 Definition [10]
A near – ring N is called weak-commutative if xyz = xzy for every x,y,z ϵ N.

2.2 Definition
A right near- ring N is said to be distributive near – ring if a.(b + c) = a . b + a . c for all a,b,c ϵ N.

2.3 Definition [11]
A near – ring N is called pseudo commutative if xyz = zyx for every x,y,z ϵ N.

2.4 Definition
A near – ring N is said to be pseudo anti- commutative if xyz = - zyx for every x,y,z ϵ N.

2.5 Definition [6]
A near – ring N is said to be pseudo m- power commutative if $x^m yz = zy^m x$ for all x,y,z ϵ N.

2.6 Definition [6]
A near – ring N is said to be pseudo m- power anti - commutative if $x^m yz = - zy^m x$ for all x,y,z ϵ N.

2.7 Lemma [6]
Let N be a near-ring. If xyz = ± zyx for all x,y,z ϵ N, then N is either pseudo Commutative or pseudo anti-commutative.

2.8 Lemma [6]
Let N be a near-ring. If $x^m yz = \pm z y^m x$ for all x,y,z ϵ N, then a N is either pseudo m – power Commutative or pseudo m – power anti- commutative.

III. Quasi- weak m- power commutative near - rings

3.1 Definition[7]
A near – ring N is said to be quasi-weak commutative if xyz = yxz for all x,y,z ϵ N.

3.2 Definition[7]

A near – ring N is said to be quasi-weak anti - commutative if $xyz = -yxz$ for all $x,y,z \in N$.

3.3 Definition

Let N be a near – ring. N is said to be quasi-weak m – power commutative if $x^m yz = y^m xz$ for all $x,y,z \in N$, where $m \geq 1$ is a fixed integer.

3.4 Definition

Let N be a near – ring. N is said to be quasi-weak m – power anti - commutative if $x^m y z = -y^m x z$ for all $x,y,z \in N$, where $m \geq 1$ is a fixed integer.

3.5 Lemma

Let N be a distributive near – ring.If $xyz = \pm yxz$ for all $x,y,z \in N$ then N is either quasi – weak commutative or quasi – weak anti - commutative.

Proof:

For each $a \in N$, let

$$C_a = \{ x \in N / xaz = axz \ \forall z \in N \}$$

$$A_a = \{ x \in N / xaz = -axz \ \forall z \in N \}$$

By the hypothesis of the lemma,

$$N = C_a \cup A_a$$

We note that if $x,y \in C_a$,then $x - y \in C_a$.

For $x,y \in C_a$ implies $xaz = +axz \ \forall z \in N$ → (1)

and $yaz = +ayz \ \forall z \in N$ → (2)

(1) – (2) gives

$(x - y)az = a(x - y)z \ \forall z \in N$

which implies $(x - y) \in C_a$.

Similarly, if $x,y \in A_a$,then $x - y \in A_a$.

We claim that either $N = C_a$ or $N = A_a$.

Suppose $N \neq C_a$ and $N \neq A_a$, then there are elements $b \in C_a - A_a$ and $d \in A_a - C_a$.

Now $b + d \in N = C_a \cup A_a$

.

If $b + d \in C_a$ then $d = (b + d) - b \in C_a$, a contradiction.

If $b + d \in A_a$ then $b = (b + d) - d \in A_a$, again a contradiction.

Hence either $N = C_a$ or $N = A_a$.

Let $A = \{ a \in N / C_a = N \}$

and $B = \{ a \in N / A_a = N \}$

Clearly $N = A \cup B$.

We note that that if $x,y \in A$,then $x - y \in A$.

For if $x,y \in A \Rightarrow C_x = N$ and $C_y = N$.

This implies $axz = xaz$ and $ayz = yaz$ for all $a,z \in N$,

So $a(x - y)z = (x - y)az$ for all $a,z \in N$,which proves that $x - y \in A$.

Similarly, if $x,y \in B$,then $x - y \in B$.

We claim that either $N = A$ or $N = B$.

Suppose $N \neq A$ and $N \neq B$,there are elements $u \in A - B$ and $v \in B - A$.

Now, $u + v \in N = A \cup B$.

If $u + v \in A$, then $v = (u + v) - u \in A$, a contradiction.

If $u + v \in B$, then $u = (u + v) - v \in B$, again a contradiction.

Hence either $N = A$ or $N = B$.

This proves that N is either quasi – weak commutative or quasi – weak anti – commutative.

3.6 Lemma :

Let N be a near – ring (not necessarily associative) satisfying $(x - y)^m = x^m - y^m$ for all $x,y \in N$,where $m \geq 1$ is a fixed integer.If $x^m y z = \pm y^m x z$ for all $x, y,z \in N$,then N is either quasi – weak m – power commutative or quasi – weak m – power anti – commutative.

Proof:

For each $a \in N$, let

$$C_a = \{ x \in N / x^m a z = a^m x z \ \forall z \in N \}$$

$$A_a = \{ x \in N / x^m a z = -a^m x z \ \forall z \in N \}$$

By the hypothesis of the lemma,

$N = C_a \cup A_a$

We note that, if $x,y \in C_a$ then $x - y \in C_a$

For $x,y \in C_a$ implies $x^m a z = a^m x z \ \forall \ z \in N$ $\longrightarrow (1)$

and $y^m a z = a^m y z \ \forall \ z \in N$ $\longrightarrow (2)$

Equation (1) – (2) gives,

$(x^m - y^m) a z = a^m (x - y) z \ \forall \ z \in N.$

$\Longrightarrow (x - y)^m a z = a^m (x - y) z \ \forall \ z \in N.$

$\Longrightarrow (x - y) \in C_a.$

Similarly $x, y \in A_a$ implies $x - y \in A_a$.

We claim that either $N = C_a$ or $N = A_a$. .

Suppose $N \neq C_a$ and $N \neq A_a$,there are elements $b \in C_a - A_a$ and $d \in A_a - C_a$.

Now, $b + d \in N = C_a \cup A_a$.

If $b + d \in C_a$ then $d = (b + d) - b \in C_a$, a contradiction.

Similarly, if $b + d \in A_a$,then $b = (b + d) - d \in A_a$, again a contradiction.

Hence either $N = C_a$ or $N = A_a$.

Let $A = \{ a \in N / C_a = N \}$

and $B = \{ a \in N / A_a = N \}$

Clearly $N = A \cup B$.

We note that if $x,y \in A$ implies $x - y \in A$.

For if $x,y \in A$ implies $C_x = N$ and $C_y = N$.

This implies $a^m x z = x^m a z$ and $a^m y z = y^m a z$ for all $a,z \in N$.

So, $a^m (x - y) z = (x^m - y^m) a z$ for all $a,z \in N$,

(i.e.,) $a^m (x - y) z = (x - y)^m a z$ for all $a,z \in N$, which proves that $x - y \in A$.

Similarly $x,y \in B$ implies $x - y \in B$.

We claim that either $N = A$ or $N = B$.

Suppose $N \neq A$ and $N \neq B$, there are elements $u \in A - B$ and $v \in B - A$.

Now, $u + v \in N = A \cup B$.

If $u + v \in A$, then $v = (u + v) - u \in A$, a contradiction.

If $u + v \in B$, then $u = (u + v) - v \in B$, again a contradiction.

Hence either $N = A$ or $N = B$.

This proves that N is either quasi-weak m – power commutative or quasi- weak m – power anti – commutative.

3.7 Note :

When m = 1,we get Lemma 3.5.

3.8 Definition :

Let N be a near-ring and $m \geq 1$ and $n \geq 1$ be fixed integers.N is said to be quasi- weak (m,n) Power commutative,if $x^m y^n z = y^m x^n z$ for all $x,y,z \in N$.

3.9 Definition:

Let N be a near-ring and $m \geq 1$ and $n \geq 1$ be fixed integers.N is said to be quasi-weak (m,n) Power anti - commutative,if $x^m y^n z = - y^m x^n z$ for all $x,y,z \in N$.

3.10 Lemma:

Let N be a near – ring (not necessarily associative) satisfying $(x-y)^k = x^k - y^k$ for $k = m,n$ where $m \geq 1$ and $n \geq 1$ are fixed integers.If $x^m y^n z = \pm y^m x^n z$ for all $x,y,z \in N$, then N is either quasi- weak (m.n)- power Commutative or quasi- weak (m,n) – power anti-commutative.

Proof:

For each $a \in N$, let

$C_a = \{ x \in N / x^m a^n z = a^m x^n z \ \forall \ z \in N \}$

$A_a = \{ x \in N / x^m a^n z = - a^m x^n z \ \forall \ z \in N \}$

By the hypothesis of the lemma,

$N = C_a \cup A_a$

We note that, if $x,y \in C_a$ then $x - y \in C_a$

For $x,y \in C_a$ implies $x^m a^n z = a^m x^n z \ \forall \ z \in N$ $\longrightarrow (1)$

and $y^m a^n z = a^m y^n z \ \forall \ z \in N$ $\longrightarrow$ (2)

Equation (1) – (2) gives,

$(x^m - y^m) a^n z = a^m (x^n - y^n) z \ \forall \ z \in N.$

$\Rightarrow (x - y)^m a^n z = a^m (x - y)^n z \ \forall \ z \in N.$

$\Rightarrow (x - y) \in C_a.$

Similarly x, y $\in A_a$ implies $x - y \in A_a$.

We claim that either $N = C_a$ or $N = A_a$. .

Suppose $N \neq C_a$ and $N \neq A_a$,there are elements $b \in C_a - A_a$ and $d \in A_a - C_a$.

Now, $b + d \in N = C_a \cup A_a$.

If $b + d \in C_a$ then $d = (b + d) - b \in C_a$, a contradiction.

Similarly, if $b + d \in A_a$,then $b = (b + d) - d \in A_a$, again a contradiction.

Hence either $N = C_a$ or $N = A_a$.

Let $A = \{ a \in N / C_a = N \}$

and $B = \{ a \in N / A_a = N \}$

Clearly $N = A \cup B$.

We note that if $x,y \in A$ implies $x- y \in A$.

For if $x,y \in A$ implies $C_x = N$ and $C_y = N$.

This implies $a^m x^n z = x^m a^n z$ and $a^m y^n z = y^m a^n z$ for all $a,z \in N$.

$\Rightarrow a^m (x^n - y^n) z = (x^m - y^m) a^n z$ for all $a,z \in N$.

So, $a^m (x - y)^n z = (x - y)^m a^n z$ for all $a,z \in N$, which proves that $x - y \in A$.

Similarly $x,y \in B$ implies $x - y \in B$.

We claim that either $N = A$ or $N = B$.

Suppose $N \neq A$ and $N \neq B$, there are elements $u \in A - B$ and $v \in B - A$.

Now, $u + v \in N = A \cup B$.

If $u + v \in A$, then $v = (u + v) - u \in A$, a contradiction.

If $u + v \in B$, then $u = (u + v) - v \in B$, again a contradiction.

Hence either $N = A$ or $N = B$.

This proves that N is either quasi-weak (m,n) – power commutative or quasi-weak(m,n) – power anti – commutative.

3.11 Note:

When m = n = 1,we get Lemma 3.5.

When n = 1, we get Lemma 3.6.

References

[1]. H.E.Bell,Quasi Centres,Quasi Commutators, and Ring Commutativity,Acta Maths,Hungary 4(1 – 2)(1983), 127 – 136.

[2]. L.O.Chung and Jiang Luh,Scalar Central elements in an algebra over a Principal ideal domain,Acta Sci. Maths 41,(1979),289 – 293.

[3]. .G.Gopalakrishnamoorthy and R.Veega, On Quasi – Periodic,Generalised Quasi-Periodic Algebras,Jour.of Inst. of Mathematics and Computer Sciences,Vol 23,No 2 (2010).

[4]. G.Gopalakrishnamoorthy and S.Geetha,On (m,n) – Power Commutativity of rings and Scalar (m,n) – Power Commutativity of Algebras,Jour. of Mathermatical Sciences ,Vol 24(3),2013, 97-110.

[5]. G.Gopalakrishnamoorthy and R.Veega,On Scalar Power Central Elements in an Algrbra over a Principal ideal domain,Jour. of Mathematical Sciences,Vol 24(3),2013,111-128.

[6]. G.Gopalakrishnamoorthy and R.Veega,On pseudo m – Power Commutative near – rings and Pseudo (m,n) PowerCommutative near - rings,International Journal of Mathematical Research and Science,Vol (4), 2013,71 – 80.

[7]. G.Gopalakrishnamoorthy,Kamaraj and S.Geetha,On quasi – weak Commutative Near – rings,International Journal of Mathamatics Research,Vol 5(5),2013,431- 440.

[8]. G.Gopalakrishnamoorthy,S.Geetha and S.Anitha, On Quasi – weak Commutative Boolean – like Near - Rings,Malaya Journal of Matematik,accepted.

[9]. G.Gopalakrishnamoorthy,S.Geetha and S.Anitha, On Quasi Weak Commutative Near – Rings II, Malaya Journal of Matematik ,accepted.

[10]. PilzGiinter,Near – rings,North Holland,Amerterdam,1983.

[11]. S.Uma,R.Balakrishnan and T.Tamizh Chelvam,Pseudo Commutative Near- rings,Scientia Magna, Vol (2010),No 2,75-85.