Some Results on Associative Ring with Unity

B.Sridevi 1, Dr. D.V.Rami Reddy2 & G.Rambupal Reddy3
Asst. Professor in Mathematics, Ravindra College of Engineering for Women, Karnool
Professor in Mathematics, K.L University, Vijayawada, Andhra Pradesh, India.
Associate Professor in Mathematics, National P.G College, Nandyal, A.P, India.

Abstract: In this paper we have mainly obtained some theorems related to Associative ring with unity.

Key words: Associative ring, ring with unity

I. Introduction

Quadri Ashraf (5) generalized some results on Associative Rings. They proved that $R$ is an associative semi prime ring in which $(xy)^2-yx'y$ is centre, then $R$ is commutative ring, then $R$ is Commutative. In this paper, we show that a Associative Ring with unity such that $(yx)x=(xy)x$, $(xy)^2=x^2+y$, for all $x,y$ in $R$. Then $R$ is commutative. Throughout the paper $\mathbb{Z}(R)$ denotes the centre of non Associative ring $R$ and $(x,y)=xy-yx$ for all $x, y$ in $R$

Main Results: we prove the following theorems

Theorem 1: Let $R$ be a associate ring with unity 1 such that $(xy)x=(xy)x$ for all $x,y$ in $R$ then $R$ is commutative.

Proof: Given condition is $(xy)x=(xy)x$\[\text{Replacing } x \text{ by } x+1 \text{ in the above condition , then } [y(x+1)](x+1) = [(x+1)y](x+1) \]
\[(xy)(x+1) = (xy)(x+1) \]
\[(yx+y)(x+1) = (xy)x+y+yx+y \text{ (by the given condition, cancellation law)} \]
\[yx = xy \quad \forall x,y \in R \]
Hence $R$ is a commutative ring for all $x,y$.

Theorem 2: Let $R$ be a associate ring with unity 1 such that $(xy)^2 = x^2+y$ for all $x,y$ in $R$, then $R$ is commutative.

Proof: Given condition is $(xy)^2 = x^2+y$\[\text{Replacing } x \text{ by } x+1 \text{ in the given identity, } [(x+1)y]^2 = (x+1)y^2(x+1) \]
\[(xy+y)(x+1) = (xy+xy+y)x^2+y+2x \]
\[y^2x + y^2y + y^2x + y^2y \]
by the given condition and cancellation law
\[yxy = y^2x \]

Replacing $y$ by $y+1$ in the above result
\[(y+1)x(y+1) = (y+1)^2x \]
\[(y+1)x(y+1) = (y^2+2y+1)x \]
\[(yx+x)(y+1) = y^2x+2yx+x \]
\[yxy + yxy + x = y^2x+2yx+x \quad \text{by the given condition, cancellation law} \]
\[xy = yx \quad \forall x,y \in R \]
Hence $R$ is commutative ring for all $x,y$.

Theorem 3: If $R$ is a Ring with unity satisfying $(xy-x^my^n) = 0$ for all $x,y \in R$ and fixed integers $m>1, n\geq 1$ then $R$ is commutative

Proof: Given condition $(xy-x^my^n) = 0$ for all $x,y \in R$ and also given that $m>1, n\geq 1$

Let $m=2, n=1$. then given condition $[xy-x^2y, x]=0$ for all $x,y \in R$, That is $(xy-x^2y) = x(x-y)$. Replacing $x$ by $x+1$ in the above result,
\[[y(x+1)](x+1) = [(x+1)y](x+1) \]
\[[xy+yx+y]x^2 = (x+1)y^2(x+1) \]
\[xy + xy + yx = y^2x+2yx+x \quad \text{by the given condition, cancellation law} \]
\[xy - y^2x = x(xy-xy^2)+xy-xy^2 \quad \text{ (From the given condition)} \]
Some Results on Associative Ring with Unity

\[ yx = xy \quad \forall \ x, y \in R \quad \text{(by the theorem } y^2 x = xy^2) \]

Hence R is a commutative ring for all x, y

References

[5]. M. Ashraf, M.A. Quadri and D. Zelinsky, Some polynomial identities that imply commutative