Effect of Thermal Radiation on Steady MHD Convective Flow Past A Continuously Moving Vertical Plate in A Porous Medium

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Abstract: This paper focuses on the effect of thermal radiation on a steady two dimensional MHD convective flow of an incompressible viscous electrically conducting fluid past a continuously moving porous plate embedded in a porous medium. A uniform magnetic field is assumed to be applied transversely to the direction of the main flow. The expressions for the velocity field, temperature field, and concentration field, skin friction at the plate, Nusselt number and Sherwood number are obtained in non dimensional form for a wide range of the governing flow parameters. The effects of the flow parameters on the velocity, temperature, concentration skin friction coefficient, Nusselt number and Sherwood number are discussed graphically. It is observed that the radiation parameter N decelerates the velocity profile and decreases the magnitude of the temperature distribution and concentration in the boundary layer.

Keywords: MHD, Electrically conducting fluid, viscous fluid, free convection, skin friction.

I. Introduction

Free convection in electrically conducting fluids through an external magnetic field has been a subject of considerable research interest of a large number of scholars for a long time due to its diverse applications in the fields such as nuclear reactions, geothermal engineering, liquid metals and plasma flows, among others. MHD convection flow problems are very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Models studies of above phenomena of MHD convection have been made by many. Bejan and Khair [1], Trevisian and Bejan [2], Acharya et. al. [3], Raptis and Konfousias [4], and Ahmed et. al. [5] are some of them. Steady and transient free convection of an electrically conducting fluid from a vertical plate in the presence of magnetic field was studied by Gupta [6]. Lykoudis [7] investigated natural convections of an electrically conducting fluid with a magnetic field. Takhar et. al. [8] computed flow and mass transfer on a stretching sheet under the consideration of magnetic field and chemically reactive species. Sakiadis [9] investigated the growth of the two dimensional velocity boundary layer over a continuously moving flat plate. Vajravelu [10] studied the exact solutions for hydrodynamic boundary layer flow and heat transfer over a continuously moving horizontal flat surface with uniform section and internal heat generation/absorption. The energy flux caused by concentration gradient is called Dufour effect and the same by temperature gradient is called the Soret effect. These effects have a vital role in the high temperature and high concentration gradient. Dursunkaya and work [11] studied diffusion thermo and thermal diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafousias and Williams [12] presented the same effects on mixed-convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. The effect of Soret and Dufour parameters on free s medium has been reported by Lakshmi Narayan and Murthy [13]. Recently Vajravelu et. al. [14] presented a mathematical model to study the influence of heat transfer on Jeffrey fluid in a porous stratum.

When technological processes take place at higher temperatures thermal radiation heat transfer has become very important and its effects cannot be neglected. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes a very important for the design of the pertinent equipment. Hossain and Takhar [15] studied the effect of radiation on mixed convection along a vertical plate with uniform surface temperature. Different researches have been forwarded to analyze the effect of thermal radiation on different flows (Aliakbar, et. al. [16], Hayat [17], Cortell [18] among other researchers).

Here our main objectives are to study the effect of radiation parameter on steady MHD convective flow past a continuously moving porous vertical plate. The work is an extension of the work done by Sarma et. al.[19].

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II. Mathematical Analysis

The equations governing the steady motion of an incompressible viscous and electrically conducting fluid in presence of a magnetic field are

The equation of continuity: \( \text{div} \ \vec{q} = 0 \)  

The Gauss law of magnetism: \( \text{div} \ \vec{B} = 0 \)

The modified Navier–Stokes equation:  
\[
\rho \left( \frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q} \right) = -\nabla \rho + \vec{J} \times \vec{B} + \nu \nabla^2 \vec{q} + \vec{g}
\]  

The energy equation:  
\[
\rho C_p \left( \frac{\partial \vec{q}}{\partial t} + \vec{q} \cdot \nabla \vec{q} \right) = k \nabla^2 \vec{T} + \phi + \frac{\vec{J}^2}{\sigma}
\]  

The species continuity equation:  
\[
\rho M \nabla \cdot \vec{C} = 0
\]

The Ohm’s law:  
\[
\vec{J} = \sigma \left[ \vec{E} + \vec{q} \times \vec{B} \right]
\]

The equation of state according to classical Boussinesq approximation is

\[
\rho = \rho_o \left[ 1 - \beta(T - T_o) - \beta(C - C_o) \right]
\]  

where \( \vec{q} \) is the velocity vector , \( p \) is the pressure , \( \vec{g} \) is the acceleration due to gravity , \( \nu \) is the kinematic viscosity , \( T \) is the temperature , \( \vec{B} \) is the magnetic induction vector , \( \vec{J} \) is the current density , \( \vec{E} \) is the electric field (here assumed to be zero) , \( \sigma \) is the electrical conductivity , \( k \) is the thermal conductivity , \( \phi \) is the viscous dissipation of energy per unit volume , \( C_p \) is the specific heat at constant pressure , \( \rho \) is the density of fluid , \( \vec{J} \times \vec{B} \) is the Lorentz force per unit volume , \( C_o \) is the species concentration , \( D_M \) is the coefficient of chemical molecular diffusivity , \( \rho_o \) is the fluid density far away from the plate , \( T_o \) is the fluid temperature far away from the plate , \( C_o \) is the species concentration far away from plate , \( \beta \) is the coefficient of volume expansion for heat transfer , \( \beta \) is the coefficient of volume expansion for mass transfer and the other symbols have their usual meanings.

We now consider the steady two dimensional forced convective and mass transfer flow of an incompressible viscous and electrically conducting fluid past a continuously moving porous vertical plate with radiation effect under the following assumptions:

i. All the fluid properties except the density in the Buoyancy force term are constants.

ii. The magnetic Reynolds number is so small that the induced magnetic field can be neglected .

iii. The viscous dissipation and Ohmic dissipation of energy are negligible.

iv. The plate is electrically non-conducting.

We introduce a coordinate system \((\vec{x}, \vec{y}, \vec{z})\) with the \( \vec{x} \) axis parallel to the plate in the vertically upward direction and \( \vec{y} \) axis perpendicular to it directed into the fluid region .With the usual boundary layer approximation and the Boussinesq approximation ,the equation (1),(2),(3),(4) and (5) reduce to

\[
\frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{v}}{\partial y} = 0
\]

\[
\rho \left( \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = g \rho \beta (T - T_o) + \rho G \beta (C - C_o) + \mu \frac{\partial^2 \vec{u}}{\partial y^2} - \sigma B_o^2 \vec{u} - \rho \frac{\partial \phi}{\partial y} - \rho u \nabla^2 \vec{u} = 0
\]
\[ \rho C_p \left( \bar{u} \frac{\partial T}{\partial x} + \bar{v} \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \rho D_m K_T \frac{\partial^2 C}{\partial y^2} - \frac{\partial q_r}{\partial y} \]  

(8)

\[ \bar{u} \frac{\partial C}{\partial x} + \bar{v} \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \]  

(9)

where \( \bar{u} \) and \( \bar{v} \) are the velocity components in \( \bar{x} \) and \( \bar{y} \) direction respectively, \( B_0 \) is the strength of the applied magnetic field, \( K \) is the permeability, \( q_r \) is the radiative heat flux, \( K_T \) is the thermal diffusion ratio, \( T_m \) is the mean fluid temperature and \( C_s \) is the concentration susceptibility.

As the plate is infinite in length in \( \bar{x} \) direction, therefore all the quantities are assumed to be independent of \( \bar{x} \) except possibly the pressure. Moreover as the motion is steady and two dimensional with the \( \bar{z} \) plane as the plate of the motion, therefore all the flow variables excluding pressure \( p \) are independent of \( \bar{z} \) and time. This phenomenon indicates that \( \bar{u}, \bar{T} \) and \( \bar{C} \) are functions of \( \bar{y} \) alone.

On the basis of the above assumption, the equation (6) transforms to

\[ \frac{\partial \bar{y}}{\partial y} = 0 \Rightarrow \bar{v} = \text{constant} = v_0 \text{ (say)} \]  

(6a)

By virtue of equation (6a) and under the infinite plate assumption, equations (7), (8) and (9)

Reduce to the following sets of differential equations.

\[- \rho v_0 \frac{d \bar{u}}{d \bar{y}} = g \beta \rho \left( \bar{T} - T_x \right) + g \beta \bar{C} \left( \bar{C} - \bar{C}_\infty \right) + \mu \frac{d^2 \bar{u}}{d \bar{y}^2} - \sigma B_0^2 \bar{u} - \rho \bar{u} \frac{v_0}{K} \]  

(7a)

\[- \rho C_p v_0 \frac{d \bar{T}}{d \bar{y}} = k \frac{d^2 \bar{T}}{d \bar{y}^2} + \rho D_m K_T \frac{d^2 \bar{C}}{d \bar{y}^2} - \frac{dq_r}{d \bar{y}} \]  

(8a)

\[- v_0 \frac{d \bar{C}}{d \bar{y}} = \frac{D_m}{T_m} \frac{d^2 \bar{C}}{d \bar{y}^2} + \frac{D_m K_T}{T_m} \frac{d^2 \bar{T}}{d \bar{y}^2} \]  

(9a)

The boundary conditions are

\[ \bar{y} = 0, \bar{u} = u_w, \bar{v} = -v_0, \frac{\partial \bar{T}}{\partial \bar{y}} = -\frac{q}{k}, \bar{C} = \bar{C}_\infty \]  

\[ \bar{y} \to \infty, \bar{u} = 0 \quad \bar{T} \to \bar{T}_x, \bar{C} \to \bar{C}_\infty \]  

(10)

We introduce the following non-dimensional quantities to normalize the flow model:

\[ y = \frac{y v_0}{u_w}, u = \frac{\bar{u}}{u_w}, x = \frac{x v_0}{u_w}, \theta = \frac{\bar{T} - T_x}{T_f - T_x}, Gr = \frac{\rho C_p \left( \bar{C} \right)}{u_w v_0^2}, Pr = \frac{\mu C_p}{k}, M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, \]

\[ Gm = \frac{\nu g \beta \bar{C}_w \left( \bar{C} - \bar{C}_\infty \right)}{u_w v_0^2}, Sc = \frac{v}{D_m}, Sr = \frac{D_m K_T q}{T_n v_0 \left( \bar{C}_w - \bar{C}_\infty \right)}, Du = \frac{D_m K_T \left( \bar{C}_w - \bar{C}_\infty \right) v_0 k}{C_s C_p q v_0^3}, \]

\[ q_r = -\frac{4 \sigma C_p \bar{T}^4}{3 K \bar{C}_w^3}, N = \frac{K k}{4 \sigma C_p T_n^3}, S = \frac{v_0^2 K}{\nu^2} \]

where \( \bar{C}_w \) the species concentration of the fluid at the plate, \( Gm \) the Grashof number for mass transfer, \( Gr \) the Grashof number for heat transfer, \( M \) the Hartmann number, \( Pr \) the Prandtl number, \( Sc \) the Schmidt number, \( Sr \) the Soret number, \( Du \) the Dufour number, \( T_n \) the temperature of the fluid at the plate, \( q \)
the heat flux, \( \phi \) the species concentration , \( \sigma_s \) the Stephen Boltzmann constant, \( K_c \) the mean absorption coefficient, \( N \) the radiation parameter, \( S \) the permeability of the medium.

The non – dimensional form of the equations (7a), (8a) and (9a) are as follows:

\[
\frac{d^2 u}{dy^2} + \frac{du}{dy} + Gr \theta + Gm \phi - \left( M + \frac{1}{S} \right) u = 0 \\
\left(1 + \frac{4}{3N} \right) \frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} + Pr Du \frac{d^2 \phi}{dy^2} = 0 \\
\frac{d^2 \phi}{dy^2} + Sc \frac{d\phi}{dy} + ScSr \frac{d^2 \theta}{dy^2} = 0
\]  

Subject To The Boundary Conditions:

\[
y = 0, u = 1, \frac{d\theta}{dy} = -1, \phi = 1 \\
y \to \infty, u = 0, \theta = 0, \phi = 0
\]  

3. Solution Of The Problems:

The solutions of the equation (12) and (13) and (11) subject to the boundary conditions (14) are respectively:

\[
\theta = B_2 e^{-A_y} + B_1 e^{-A_y} \\
\phi = L_5 e^{-A_y} + L_3 e^{-A_y} \\
U = (1 - A_0 - A_{10}) e^{-A_y} + A_0 e^{-A_y} + A_{10} e^{-A_y}
\]  

The Coefficient Of Skin Friction Is Defined In Dimensionless Form As:

\[
\tau = \left[ \frac{du}{dy} \right]_{y=0} = -A_9 (1 - A_9 - A_{10}) - A_3 A_4 - A_2 A_{10}
\]  

The non – dimensional heat flux at the plate \( y = 0 \) in terms of Nusselt number \( Nu \) is given by:

\[
Nu = \left[ \frac{d\theta}{dy} \right]_{y=0} = -A_4 B_2 - A_5 B_1
\]  

The non-dimensional rate of mass transfer at the plate \( y=0 \) in terms of Sherwood number \( Sh \) is given by:

\[
Sh = \left[ \frac{d\phi}{dy} \right]_{y=0} = -A_4 L_1 - A_3 L_2
\]  

APPENDIX:

\[
B_1 = \frac{A_6 - A_4}{A_4 A_6 - A_7 A_4}, B_2 = \frac{1 - B_1 A_5}{A_4}, A_6 = \frac{1}{Du} \left\{ 1 - \frac{1}{Pr} \left( 1 + \frac{4}{3N} \right) \right\},
\]

\[
A_7 = \frac{1}{Du} \left\{ 1 - \frac{1}{Pr} \left( 1 + \frac{4}{3N} \right) \right\}, A_1 = 1 + \frac{4}{3N} - Pr Du Sc Sr, A_2 = Sc \left( 1 + \frac{4}{3N} \right) + Pr, A_3 = Pr Sc,
\]

\[
A_9 = -\left( \frac{Gr + Gm A_5}{A_4 ^2 - A_5} \right) B_2 \sqrt{A_4 ^2 - \left( M + \frac{1}{S} \right)}, A_{10} = -\left( \frac{Gr + Gr A_5}{A_5 ^2 - A_5 - \left( M + \frac{1}{S} \right)} \right), L_4 = B_2 A_6 = \frac{B_2}{Du} \left\{ 1 - \frac{1}{Pr} \left( 1 + \frac{4}{3N} \right) \right\},
\]

\[
L_2 = B_4 A_7 = \frac{B_4}{Du} \left\{ 1 - \frac{1}{Pr} \left( 1 + \frac{4}{3N} \right) \right\}, A_4 = \frac{A_2 + \sqrt{A_2 ^2 - 4 A_4 A_1}}{2 A_1}
\]
III. Results And Discussion

In order to get physical insight into the problem, the numerical calculations are carried out for \( u, \theta, \phi, \tau \) which are respectively the non-dimensional velocity field, temperature field, concentration field, and skin friction at the plate by assigning some arbitrary chosen specific values to the physical parameters like \( S, N, Pr, Du, Sc \).

Figures 1 to 5 represent the variations of non-dimensional velocity profile \( u \) against \( y \) for different values of governing parameters like \( S, N, Pr, Du, Sc \).

Figure 1 shows that the velocity profile increases with the increase of the permeability parameter \( S \).

From figure 2 we observe that the velocity decreases with the increase of the radiation parameter \( N \).

Figure 3 depicts that velocity decreases with the increase of Prandtl number \( Pr \).

Figure 4 displays velocity profile for different values of Dufour number \( Du \).\( Du \) decelerates the velocity as observed from this figure.

Figure 5 shows that the velocity profile decreases with the increase of the Schmidt number \( Sc \).

Figures 6 to 9 demonstrate the variations of temperature distribution against \( y \) under the influence of the parameters \( N, Pr, Du, Sc \).

Figure 6 depicts the effect of radiation parameter \( N \) over the temperature distribution. Temperature distribution increases with the radiation parameters.

Increasing the Prandtl number \( Pr \) decreases the temperature of the flow field as seen in fig 7. Figure 8 shows that the temperature distribution increases with the increasing value of \( Du \). From figure 9 we observed that the Schmidt number increases the temperature distribution.

The influence of \( N, Pr, Du, Sc \) on the concentration profiles are plotted in figures 10 to 13. From these figures we observed that concentration profile decreases with the increase \( N, Pr, Du \).

The variation of the skin friction \( \tau \) at the plate against \( M \) under the influence of \( N, Sc, S, Du \) is shown in figures 14, 15, 16 and 17.

From the three figures 14, 15, 17, it is clear that \( \tau \) at the plate decreases due to the effect of \( N, Sc \) and \( Du \). Figure 16 displays the influence of \( S \) on the skin friction. This figure depicts that skin friction \( \tau \) at the plate increases with the increasing value of \( S \).

IV. Conclusions

1. The velocity increases with the increase of permeability parameter \( S \) whereas it decreases with radiation parameter \( N \).
2. Temperature profile rises with the radiation parameter \( N \).
3. Concentration profile decreases with the radiation parameter \( N \).
4. Skin-friction decreases with the increasing value of the radiation parameter \( S \).
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Fig5: Velocity profiles for different Sc when 
$Gr=5.0, Pr=0.7, M=2.0, Gm=10.0, S=0.1, Du=0.22, S=1.0, N=1.0$

Fig6: Temperature profiles for different N when 
$Gr=5.0, Pr=0.7, M=2.0, Gm=10.0, Sc=0.2, S=0.1, Du=0.45, S=1.0$

Fig7: Temperature profile for different Pr when 
$Gr=5.0, M=2.0, Gm=10.0, Sc=0.2, S=0.1, Du=0.22, S=1.0, N=1.0$

Fig8: Temperature profiles for different Du when 
$Gr=5.0, Pr=0.7, M=2.0, Gm=10.0, Sc=0.2, S=0.1, Du=1.0, N=1.0$

Fig9: Temperature profiles for different Sc when 
$Gr=5.0, Pr=0.7, M=2.0, Gm=10.0, S=0.1, Du=0.22, S=1.0, N=1.0$

Fig10: Concentration profiles for different N when 
$Gr=5.0, Pr=0.7, M=2.0, Gm=10.0, Sc=0.2, S=0.1, Du=0.45, S=1.0$

Fig11: Concentration profiles for different Pr when 
$Gr=5.0, M=2.0, Gm=10.0, Sc=0.2, S=0.1, Du=0.22, S=1.0, N=1.0$

Fig12: Concentration profiles for different Du when 
$Gr=5.0, Pr=0.7, M=2.0, Gm=10.0, Sc=0.2, S=0.1, Du=1.0, N=1.0$
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