Effect of Uniform Horizontal Magnetic Field on Thermal Instability in A Rotating Micropolar Fluid Saturating A Porous Medium

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Abstract: This paper deals with the theoretical investigation of thermal instability of a thin layer of electrically non-conducting, incompressible, rotating micropolar fluid saturating a porous medium in the presence of horizontal magnetic heated from below. A dispersion relation is obtained for a flat fluid layer contained between two free boundaries using a linear stability analysis theory and normal mode analysis method. The influence of various parameters like medium permeability, rotation, coupling parameter, micropolar coefficient , magnetic field and micropolar heat conduction parameter has been analyzed on the onset of stationary convection and results are depicted graphically. The principle of exchange (PES) is found valid. The sufficient condition for non-existence of overstability are also obtained.

Keywords: Thermal Instability; Micropolar Fluid; Horizontal Magnetic Fluid; Rotation; Porous Medium

I. Introduction

Eringen[2] was the first who introduced a continuum theory of micro-fluids in 1964 which takes into account the local motions and deformations of the substructure of the fluid. Compared to the classical Newtonian fluids, micropolar fluids are characterized by two supplementary variables such as the spin and micro-inertia tensor. Microrotation takes place due to spin whereas the micro-inertia tensor describes the distributions of atoms and molecules inside the fluid elements. Moreover, the stress tensor in no longer symmetric. Liquid crystals, colloidal fluids, blood etc. are examples of micropolar fluids. Thermal effects in microfluids have also been discussed by Eringen[1] and Kazakia et. al[10]. Sastry and Rao[9] discussed the stability of a hot rotating micropolar fluid layer between two rigid boundaries without taking into account the rotation effect in the angular momentum equation. On the other hand Bhattacharyya and Abbas[8] took into account the rotation effect in both momentum equations with free boundaries. Sastry and Rao[9] noticed that when the fluid is heated from below, the rotation of the system delays the onset of stability. Further, for small or moderate values of Taylor number T, the system is more stable as the micropolar parameter increases. Y. Oin and P.N. Kaloni[11] studied the effect of rotation on thermal convection in micropolar fluid and found that the effect of rotation in a stationary convection is stabilizing and that higher the values of the micropolar parameters, the more the stabilizing effect. For over-stability to be possible, P_r (Prandtl number) must be less than 1/(1+k), where k is a coupling parameter. Sharma and Kumar[3] studied the effect of rotation on thermal convection in micropolar fluid in non-porous medium and found that the presence of coupling between thermal and micropolar effects and uniform rotation may introduce oscillatory modes in the system. Sharma and Kumar[4] studied the effect of rotation on thermal convection in micropolar fluid in porous medium and found that the presence of rotation and coupling between spin and heat fluxes may bring over-stability in the system and the medium permeability also brings in over-stability in the system. The permeability has a stabilizing effect on stationary convection. Sharma and Kumar[5, 6] also studied the effect of magnetic field on the micropolar fluids heated from below in a non-porous and porous medium, they found that in the presence of various coupling parameters, the magnetic field has a stabilizing effect whereas the medium permeability has destabilizing effect on stationary convection. Reena and Rana[7] have studied the thermal convection of rotating micropolar fluid in hydromagnetic saturating a porous medium, they took the vertical magnetic field and found that the oscillatory modes are introduced due to the presence of rotation and magnetic field. For stationary convection, the medium permeability has destabilizing effect under the condition $H^2 > \frac{8\Omega\sqrt{b}}{P_2}$ and the

magnetic field has a stabilizing effect under the condition $\overline{\delta} < \frac{\epsilon}{A}$ and $H^2 > \frac{8\Omega\sqrt{b}}{P_2}$, where P_2 is the magnetic

Prandtl number.

II. **Mathematical Formulation**

In this problem, we consider an infinite, horizontal, electrically non-conducting, incompressible micropolar fluid layer of thickness d. This layer is heated from below such that the lower boundary is held at constant temperature $T = T_0$ and the upper boundary is held at fixed temperature $T = T_1$ so that $T_0 > T_1$, therefore a uniform temperature gradient $\beta = \left| \frac{dT}{dz} \right|$ is maintained. The physical structure of the problem is one of infinite

extent in x and y directions bounded by the planes z = 0 and z = d. The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of poricity \in and the medium permeability κ , which is acted upon by a uniform rotation $\Omega(0, 0, \Omega_0)$ and gravity field $\vec{\mathbf{g}} = (0, 0, -g)$. A uniform magnetic field $\vec{\mathbf{H}} = (H_0, 0, 0)$ is applied along x-axis. The magnetic Raynold number is assumed to very small so that the induced magnetic field can be neglected in comparison to the applied field. We also assumed that both the boundaries are free and no external couples and the heat sources are present.



Fig. 1

The equations governing the motion of micropolar rotating fluids saturating a porous medium following Boussinesq approximation are as follows :

The equation of continuity for an incompressible fluid is ...(1)

$$\nabla . \vec{\mathbf{q}} = 0$$

The equation of momentum, following Darcy law, is given by

$$\frac{\rho_o}{\epsilon} \left[\frac{\partial \vec{\mathbf{q}}}{\partial t} + \frac{1}{\epsilon} (\vec{\mathbf{q}} \cdot \nabla) \vec{\mathbf{q}} \right] = -\nabla p - \rho g \, \hat{e}_z - \left(\frac{\zeta + \mu}{\kappa} \right) \vec{\mathbf{q}} + \zeta \nabla \times \vec{\mathbf{N}} \qquad + \frac{2\rho_o}{\epsilon} (\vec{\mathbf{q}} \times \vec{\mathbf{\Omega}}) + \frac{\mu_e}{4\pi} (\nabla \times \vec{\mathbf{H}}) \times \vec{\mathbf{H}} \qquad \dots (2)$$

The equation of internal momentum is given by

$$\rho_{o} i \left[\frac{\partial \vec{\mathbf{N}}}{\partial t} + \frac{1}{\epsilon} (\vec{\mathbf{q}} \cdot \nabla) \vec{\mathbf{N}} \right] = (\alpha' + \beta') \nabla (\nabla \cdot \vec{\mathbf{N}}) + \gamma' \nabla^{2} \vec{\mathbf{N}} + \zeta \left(\frac{1}{\epsilon} \nabla \times \vec{\mathbf{q}} - 2\vec{\mathbf{N}} \right) \qquad \dots (3)$$

The equation of energy is given by

$$\left[\rho_o C_v \in +\rho_s C_s (1-\epsilon)\right] \frac{\partial T}{\partial t} + \rho_o C_v (\vec{\mathbf{q}} \cdot \nabla) T = \chi_T \nabla^2 T + \delta(\nabla \times \vec{\mathbf{N}}) \cdot \nabla T \qquad \dots (4)$$

and the equation of state is ρ

$$= \rho_o [1 - \alpha (T - T_0)] \qquad ...(5)$$

Where $\vec{q}, \vec{N}, p, \rho, \rho_o, \rho_s, \mu, \zeta, \mu_e, \kappa, j, \alpha', \beta', \gamma', T, t, \chi_T, \delta, \alpha, T_o, C_v, C_s$ and \hat{e}_z denote respectively filter velocity, microrotation, pressure, fluid density, reference density, density of solid matrix, fluid viscosity, coupling viscosity coefficient, magnetic permeability, medium permeability, micro-inertia coefficient, micropolar viscosity coefficients, temperature, time, thermal conductivity, micropolar heat conduction coefficient, coefficient of thermal expansion, reference temperature, specific heat at constant volume, specific heat of solid matrix and unit vector along z-direction.

The Maxwell's equations become

$$=\frac{\partial H}{\partial t} = \nabla \times (\vec{\mathbf{q}} \times \vec{\mathbf{H}}) + \in \gamma_m \nabla^2 \vec{\mathbf{H}} \qquad \dots (6)$$

and $\nabla \cdot \vec{\mathbf{H}} = 0$

where γ_m is the magnetic viscosity.

III. **Basic State Of The Problem**

 $\vec{\mathbf{q}} = \vec{\mathbf{q}}_b = (0, 0, 0), \ \vec{\mathbf{N}} = \vec{\mathbf{N}}_b = (0, 0, 0), \ \vec{\mathbf{\Omega}} = \vec{\mathbf{\Omega}}_b = (0, 0, \Omega_0), \ p = p_b(z), \ \rho = \rho_b(z) \quad \text{and} \quad \vec{\mathbf{H}} = \vec{\mathbf{H}}_b = (H_0, 0, 0)$ 0.9790/5728-12135265 www.iosrjournals.org 53 | Pa

...(7)

The basic state of the problem is taken as

Using this basic state, equations (1) to (7) yield

$\frac{dp_b}{dz} + \rho_b g = 0$	(8)
$T = -\beta z + T_0$	(9)
$\rho = \rho_o (1 + \alpha \beta z)$	(10)

IV. Dispersion Relation

Making use of the following perturbations

and

 $\vec{\mathbf{q}} = \vec{\mathbf{q}}_b + \vec{\mathbf{q}}', \ \vec{\mathbf{N}} = \vec{\mathbf{N}}_b + \vec{\mathbf{N}}', \ \rho = \rho_b + \rho', \ T = T_b + \theta, \ \vec{\mathbf{H}} = \vec{\mathbf{H}}_b + \vec{\mathbf{h}}$ and following non dimensional variables and dropping the stars,

$$x = x^* d, \ y = y^* d, \ z = z^* d, \ \vec{\mathbf{q}}' = \frac{\kappa_T}{d} \vec{\mathbf{q}}^*, \ \vec{\mathbf{N}}' = \frac{\kappa_T}{d^2} \vec{\mathbf{N}}, \ t = \frac{\rho_o d^2}{\mu} t^*, \ \theta = \beta d\theta^*, \ p' = \frac{\mu \kappa_T}{d^2} p^*, \ \vec{\mathbf{\Omega}} = \frac{\mu}{\rho_o d^2} \vec{\mathbf{\Omega}}^*$$

 $\vec{\mathbf{h}} = H_o \vec{\mathbf{h}}^*$, where $\kappa_T = \frac{\chi_T}{\rho_o C_v}$ is the thermal diffusivity, and using normal mode analysis $[w, \zeta_z, \xi_z, \theta, m_z, h_z] = [W(z), X(z), G(z), \Theta(z), M(z), B(z)] \exp[ik_x + ik_y + \sigma t]$

Where σ is the stability parameter which is, in general, a complex constant and $a = \sqrt{k_x^2 + k_y^2}$ is the wave number and eliminating *X*, *G*, Θ , *M* and *B* we have

$$\begin{bmatrix} \frac{\sigma}{\epsilon} + \frac{1}{K_1} (1+K) \end{bmatrix} (D^2 - a^2) W = -Ra^2 \Big[EP_r \, \sigma - (D^2 - a^2) \Big]^{-1} \Big\{ W + \overline{\delta} \frac{K}{\epsilon} \Big[\overline{\delta} \, \sigma - C_2 (D^2 - a^2) + 2K \Big]^{-1} (D^2 - a^2) W \Big\}$$

$$- \frac{K^2}{\epsilon} (D^2 - a^2)^2 \Big[\overline{\mathbf{j}} \, \sigma - C_2 (D^2 - a^2) + 2K \Big]^{-1} W - \frac{2\Omega_0}{\epsilon} \Big\{ \frac{\sigma}{\epsilon} + \frac{1}{K_1} (1+K) + \frac{K^2}{\epsilon} \Big[\overline{\mathbf{j}} \, \sigma - C_2 (D^2 - a^2) + 2K \Big]^{-1} (D^2 - a^2) \Big\}$$

$$+ k_x^2 Q \Big[\epsilon P_r \, \sigma - \frac{\epsilon P_r}{P_m} (D^2 - a^2) \Big]^{-1} \Big\}^{-1} \frac{2\Omega_0}{\epsilon} D^2 W - k_x^2 Q \Big[\epsilon P_r \, \sigma - \frac{\epsilon P_r}{P_m} (D^2 - a^2) \Big]^{-1} (D^2 - a^2) W$$

$$(11)$$

V. Boundary Conditions

Here, we consider the case when both boundaries are free as well as being perfect conductor of heat, while the adjoining medium is perfectly conducting, then we have

$$w = \frac{\partial^2 w}{\partial z^2} = 0 = \theta, \quad \vec{\mathbf{N}} = \vec{\mathbf{0}} \text{ at } z = 0 \text{ and } z = 1 \text{ and } (\nabla \times \vec{\mathbf{N}})_z = 0 \text{ at } z = 0, 1 \qquad \dots (12)$$

The boundary conditions (12) now become

$$W = D^{2}W = 0 = DX = M, \Theta = 0, DB = 0 \text{ at } z = 0 \text{ and } 1$$
Also $DM = 0, B = 0$ on a perfectly conducting
boundary also $D^{2}\Theta = 0, D^{2}G = 0, D^{2}M = 0, D^{3}B = 0,$

$$D^{3}X = 0 \text{ at } z = 0 \text{ and } z = 1$$

From equation (13), we observe that $D^{(2n)}W = 0$, *n* is a positive integer at z = 0, 1

Therefore the proper solution *W* characterizing the lowest mode is $W = W_0 \sin \pi z$, Where W_0 is a constant. Substituting for *W* in equation (11), we have

$$\begin{split} & \left[EP_{r}\,\sigma+b\right] \left\{ b\left[\frac{\sigma}{\epsilon}+\frac{1}{K_{1}}\left(1+K\right)\right] \left[\overline{\mathbf{j}}\,\sigma+C_{2}\,b+2K\right] \left[\epsilon P_{r}\,\sigma+\frac{\epsilon P_{r}b}{P_{m}}\right] \right. \\ & \left. -\frac{K^{2}}{\epsilon}b^{2}\left[\epsilon P_{r}\,\sigma+\frac{\epsilon P_{r}b}{P_{m}}\right] + bk_{x}^{2}\,\mathcal{Q}\left[\overline{\mathbf{j}}\,\sigma+C_{2}b+2K\right] \right\} \times \left\{ \left[\frac{\sigma}{\epsilon}+\frac{1}{K_{1}}\left(1+K\right)\right] \left[\overline{\mathbf{j}}\,\sigma+C_{2}b+2K\right] \left[\epsilon P_{r}\,\sigma+\frac{\epsilon P_{r}b}{P_{m}}\right] \right. \\ & \left. -\frac{K^{2}}{\epsilon}b\left[\epsilon P_{r}\,\sigma+\frac{\epsilon P_{r}b}{P_{m}}\right] + k_{x}^{2}\,\mathcal{Q}\left[\overline{\mathbf{j}}\,\sigma+C_{2}b+2K\right] \right\} \\ & = Ra^{2}\left[\epsilon P_{r}\,\sigma+\frac{\epsilon P_{r}b}{P_{m}}\right] \left[\overline{\mathbf{j}}\,\sigma+C_{2}b+2K-\frac{\overline{\mathbf{\delta}}\,Kb}{\epsilon}\right] \times \left\{ \left[\frac{\sigma}{\epsilon}+\frac{1}{K_{1}}\left(1+K\right)\right] \left[\overline{\mathbf{j}}\,\sigma+C_{2}b+2K\right] \left[\epsilon P_{r}\,\sigma+\frac{\epsilon P_{r}b}{P_{m}}\right] \right. \\ & \left. -\frac{K^{2}}{\epsilon}b\left[\epsilon P_{r}\,\sigma+\frac{\epsilon P_{r}b}{P_{m}}\right] + k_{x}^{2}\,\mathcal{Q}\left[\overline{\mathbf{j}}\,\sigma+C_{2}b+2K\right] \right\} \quad \frac{-4\pi^{2}\,\Omega_{0}^{2}}{\epsilon^{2}} \left[EP_{r}\,\sigma+b\right] \left[\overline{\mathbf{j}}\,\sigma+C_{2}b+2K\right]^{2} \left[EP_{r}\,\sigma+\frac{\epsilon P_{r}b}{P_{m}}\right]^{2} \qquad \dots (14) \end{split}$$
Where

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VI. Stationary Convection

For stationary marginal state we put
$$\sigma = 0$$
 in (14), we get

$$b\left\{b\left[\frac{1}{K_{1}}(1+K)\right]\left[C_{2}b+2K\right]\left[\frac{\in P_{r}b}{P_{m}}\right] - \frac{K^{2}}{\epsilon}b^{2}\left[\frac{\in P_{r}b}{P_{m}}\right] + bk_{x}^{2}Q[C_{2}b+2K]\right\}$$

$$\times \left\{\left[\frac{1}{K_{1}}(1+K)\right]\left[C_{2}b+2K\right]\left[\frac{\in P_{r}b}{P_{m}}\right] - \frac{K^{2}}{\epsilon}b\left[\frac{\in P_{r}b}{P_{m}}\right] + k_{x}^{2}Q[C_{2}b+2K]\right\}$$

$$= Ra^{2}\left[\frac{\in P_{r}b}{P_{m}}\right]\left[C_{2}b+2K-\frac{\tilde{\delta}Kb}{\epsilon}\right]\left\{\left[\frac{1}{K_{1}}(1+K)\right]\left[C_{2}b+2k\right]\left[\frac{\in P_{r}b}{P_{m}}\right] - \frac{K^{2}}{\epsilon}b\left[\frac{\in P_{r}b}{P_{m}}\right] - \frac{K^{2}}{\epsilon}b\left[\frac{\in P_{r}b}{P_{m}}\right] + k_{x}^{2}Q[C_{2}b+2K]\right\} - \frac{4\pi^{2}\Omega_{0}^{2}}{\epsilon^{2}}(b)[C_{2}b+2K]^{2}\left[\frac{\in P_{r}b}{P_{m}}\right]^{2} \dots (15)$$

When Q = 0 (in the absence of magnetic field), we have

$$R = \frac{b}{a^{2}} \left[\frac{b \left\{ \frac{1}{K_{1}} (1+K) (C_{2}b+2K) - \frac{bK^{2}}{\epsilon} \right\} \left\{ \frac{1}{K_{1}} (1+K) (C_{2}b+2K) - b\frac{K^{2}}{\epsilon} \right\}}{\frac{+\frac{4\pi^{2}\Omega_{0}^{2}}{\epsilon^{2}} (C_{2}b+2K)^{2}}{\left\{ C_{2}b+2K - \frac{\overline{\delta}Kb}{\epsilon} \right\} \left\{ \frac{1}{K_{1}} (1+K) (C_{2}b+2K) - \frac{bK^{2}}{\epsilon} \right\}} \right]} \qquad ...(16)$$

When $\Omega_0 = 0$ (In the absence of rotation) we have

$$R = \frac{b^2}{a^2} \left[\frac{\frac{1}{K_1} (1+K)(C_2b+2K) - \frac{bK^2}{\epsilon}}{\left[(C_2b+2K) - \frac{\bar{\mathbf{\delta}}Kb}{\epsilon} \right]} \right] ...(17)$$

When $\bar{\delta} = 0$ (In the absence of coupling between spin and heat fluxes), equation (17) reduces to the expression obtained by **Sharma** and **Kumar (1997)**.

From
$$(15)$$
, we get

$$R = \frac{b\left\{\frac{\epsilon}{K_{1}}(1+K)(b^{2}+2Ab)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}^{2}+4\pi^{2}\Omega_{0}^{2}(b^{2}+2Ab)^{2}}{a^{2}(b+2A-\bar{\delta}bA\epsilon^{-1})\left\{\frac{\epsilon}{K_{1}}(1+K)(b^{2}+2bA)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}} \qquad \dots(18)$$
Where $A = \frac{K}{2}$

 C_2 To investigate the effects of medium permeability K_1 , rotation Ω_0 , coupling parameter K, micro coefficient A, magnetic field Q, and micropolar heat conduction parameter $\overline{\delta}$, we examine the behaviour of $\frac{dR}{dK_1}, \frac{dR}{d\Omega_0}, \frac{dR}{dK}, \frac{dR}{dA}, \frac{dR}{dQ}, \frac{dR}{d\overline{\delta}}$ analytically.

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From (18), we obtain

$$2\left[\frac{\epsilon}{K_{1}}(1+K)(b^{2}+2bA)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right]^{2} \times \left\{\frac{-\epsilon}{K_{1}^{2}}(1+K)(b^{2}+2Ab)\right\} + \left\{\frac{\epsilon}{K_{1}^{2}}(1+K)(b^{2}+2bA)\right\} \times \left[\left\{\frac{\epsilon}{K_{1}}(1+K)(b^{2}+2Ab)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}^{3} + \left\{\frac{\epsilon}{K_{1}}(1+K)(b^{2}+2Ab)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}^{3} + 4\pi^{2}\Omega_{0}^{2}b(b+2A)^{2}\right] \qquad \dots (19)$$

Thus, the medium permeability has a destabilizing effect when $\frac{\epsilon}{A} > \max_{\overline{\delta}} \{\overline{\delta}, K_1\}$ and $Q > \frac{2\pi\Omega_0 P_r \sqrt{b}}{P_m k_x^2}$.

In the absence of micropolar heat conduction parameter ($\overline{\delta} = 0$) and rotation ($\Omega_0 = 0$), equation (19) reduces to

$$\frac{dR}{dK_1} = -\frac{b \in (1+K)(b^2 + 2Ab)}{a^2(b+2A)K_1^2} = -\frac{b^2 \in (1+K)}{a^2K_1^2}$$

Which is always negative, thus in this case the medium permeability always has a destabilizing effect on the system for stationary convection in porous medium whatever the strength of magnetic field is applied. In the absence of rotation $(\Omega_0 = 0)$, equation (19) reduces to

$$\frac{dR}{dK_1} = \frac{-b \in (1+K)(b^2 + 2Ab)}{a^2 K_1^2 (b + 2A - \overline{\delta}bA \in \overline{-1})}$$

$$\Rightarrow \qquad \frac{dR}{dK_1} < 0 \quad \text{if} \quad \overline{\mathbf{\delta}} < \frac{\epsilon}{A} \quad \text{and} \quad \frac{dR}{dK_1} > 0 \quad \text{if} \quad \overline{\mathbf{\delta}} > \frac{\epsilon}{A} + \frac{2\epsilon}{b}$$

Thus, the medium permeability may have dual role in the absence of rotation whatever the strength of magnetic field is applied.

In the absence of magnetic field (Q = 0), equation (19) reduces to

$$\frac{dR}{dK_{1}} = \frac{-b\frac{\epsilon}{K_{1}^{2}}(1+K)(b^{2}+2Ab)}{a^{2}\left[2A+b(1-\bar{\mathbf{\delta}}A\epsilon^{-1})\right]} \left[\frac{\left[\frac{\epsilon}{K_{1}}(1+K)(b^{2}+2bA)-b^{2}KA\right]^{2}-4\pi^{2}\Omega_{0}^{2}b(b+2A)^{2}}{\left[(b^{2}+2bA)\frac{\epsilon}{K_{1}}(1+K)-b^{2}KA\right]^{2}}\right]$$

Thus, in the absence of magnetic field the medium permeability has destabilizing effect when

$$\frac{\epsilon}{A} > \max \{\overline{\mathbf{\delta}}, K_1\} \text{ and } \Omega_0 < \frac{\epsilon A \sqrt{b}}{\pi K_1 (b+2A)}$$

From (18), we have

$$\frac{dR}{d\Omega_0} = \frac{1}{a^2(b+2A-\bar{\mathbf{\delta}}bA\in^{-1})} \left[\frac{8\pi^2 \,\Omega_0 \,(b^2+2bA)^2}{\left\{ \frac{\epsilon}{K_1} (1+K)(b^2+2bA) - b^2 KA + \frac{P_m}{P_r} k_x^2 \,Q(b+2A) \right\}} \right] \dots (20)$$

Thus, the rotation has destabilizing effect if $\frac{\epsilon}{A} > \max\{\overline{\delta}, k_1\}$. Whatever the magnetic field is applied.

In the absence of micropolar heat conduction parameter ($\overline{\delta} = 0$), equation (20) reduces to

$$\frac{dR}{d\Omega_0} = \frac{8\pi^2\Omega_0 b^2(b+2A)}{a^2 \left\{ \frac{\epsilon}{K_1} (1+K)(b^2+2bA) - K^2 bA + \frac{P_m}{P_r} k_x^2 Q(b+2A) \right\}}$$

Thus, the rotation has stabilizing effect when $k_1 < \frac{\epsilon}{A}$, whatever the strength of magnetic field is applied in the absence of micropolar heat conduction parameter. From (18), we have

$$\frac{dR}{dQ} = \frac{bP_m k_x^2 (b+2A)}{a^2 P_r (b+2A-\bar{\mathbf{\delta}}bA \in ^{-1})} \times \left[\frac{\left\{ \frac{\epsilon}{K_1} (1+K) (b^2+2bA) - b^2 KA + \frac{P_m}{P_r} k_x^2 Q (b+2A) \right\}^2 - 4\pi^2 \Omega_0^2 b (b+2A)^2}{\left\{ \frac{\epsilon}{K_1} (1+K) (b^2+2bA) - b^2 KA + \frac{P_m}{P_r} k_x^2 Q (b+2A) \right\}^2} \right] \qquad ..(21)$$

Thus, the magnetic field has a stabilizing effect when $\frac{\epsilon}{A} > \max. \{\overline{\delta}, k_1\}$ and $Q > \frac{2\pi\Omega_0 P_r \sqrt{b}}{P_m k_x^2}$.

In the absence of micropolar heat conduction parameter $(\overline{\delta} = 0)$ and rotation $(\Omega_0 = 0)$, we have

$$\frac{dR}{dQ} = \frac{bP_m k_x^2}{a^2 P_r} > 0 ,$$

which is always positive, therefore, the magnetic field has a stabilizing effect, whatever the strength of magnetic field is applied.

From (18), we have

$$\frac{dR}{dK} = \frac{b\left[\frac{\epsilon}{K_{1}}(b^{2}+2Ab)-b^{2}A\right]}{a^{2}(b+2A-\overline{\delta}Ab\epsilon^{-1})} \left[\frac{\left\{\frac{\epsilon}{K_{1}}(1+K)(b^{2}+2Ab)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}^{2}}{\left\{\frac{-4\pi^{2}\Omega_{0}^{2}b(b+2A)^{2}}{\left\{\frac{\epsilon}{K_{1}}(1+K)(b^{2}+2bA)-b^{2}kA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}^{2}}\right] \qquad \dots (22)$$

Thus, the coupling parameter has stabilizing effect for all K > 0 if $\frac{\epsilon}{A} > \max \cdot \{\overline{\mathbf{\delta}}, K_1\}$, and $Q > \frac{2\pi \Omega_0 P_r \sqrt{b}}{P_m k_x^2}$.

In the absence of rotation $(\Omega_0 = 0)$, equation (22) reduces to

$$\frac{dR}{dK} = \frac{b\left[\frac{\epsilon}{K_1}(b^2 + 2Ab) - b^2A\right]}{a^2(b + 2A - \delta Ab \epsilon^{-1})}$$

Thus the coupling parameter has a stabilizing effect if $\frac{\epsilon}{A} > \max \{\overline{\delta}, K_1\}$. Whatever the strength of magnetic field is applied.

In the absence of magnetic field (Q = 0), the equation (22) reduces to

$$\frac{dR}{dK} = \frac{b\left[\frac{\epsilon}{K_1}(b^2 + 2Ab) - b^2A\right]}{a^2(b + 2A - \bar{\mathbf{\delta}}Ab \epsilon^{-1})} \left[\frac{\left\{\frac{\epsilon}{K_1}(1 + K)(b^2 + 2Ab) - b^2KA\right\}^2 - 4\pi^2 \Omega_0^2 b(b + 2A)^2}{\left\{\frac{\epsilon}{K_1}(1 + K)(b^2 + 2bA) - b^2KA\right\}^2}\right]$$

Thus, in the absence of magnetic field, the coupling parameter has a stabilizing effect if $\frac{\epsilon}{A} > \max\{\overline{\delta}, K_1\}$ and

$$\Omega_0 < \frac{\in A\sqrt{b}}{\pi \, K_1 \, (b+2A)}$$

From (18), we have

$$b(b+2A-\overline{\delta}bA\in^{-1})\left\{\frac{2b\in}{K_{1}}(1+K)-b^{2}K+\frac{2P_{m}}{P_{r}}k_{x}^{2}Q\right\}$$

$$\times\left[\left\{\frac{\in}{K_{1}}(1+K)(b^{2}+2bA)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}^{2}-4\pi^{2}\Omega_{0}^{2}b(b+2A)^{2}\right]$$

$$+4\pi^{2}\Omega_{0}^{2}b^{2}(b+2A)(2b+4A-2\overline{\delta}Ab\in^{-1}+\overline{\delta}b^{2}\in^{-1})$$

$$\times\left\{\frac{\in}{K_{1}}(1+K)(b^{2}+2bA)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}$$

$$\frac{dR}{dA}=\frac{1}{a^{2}}\frac{-b(2-\overline{\delta}b\in^{-1})\left\{\frac{\in}{K_{1}}(1+K)(b^{2}+2Ab)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}^{3}}{(b+2A-\overline{\delta}bA\in^{-1})^{2}\left\{\frac{\in}{K_{1}}(1+K)(b^{2}+2bA)-b^{2}KA+\frac{P_{m}}{P_{r}}k_{x}^{2}Q(b+2A)\right\}^{2}}$$
...(23)

Thus, the micropolar coefficient has a stabilizing effect if $\frac{2}{b} < \overline{\mathbf{\delta}} < \frac{\epsilon}{A}$, $K_1 < \frac{2 \in (1+K)}{bK}$ and $Q > \frac{2\pi\Omega_0\sqrt{b}P_r}{P_m k_x^2}$

In this absence of rotation $(\Omega_0 = 0)$, equation (23) reduces to

$$b(b+2A-\overline{\delta}bA \in {}^{-1})\left\{\frac{2b \in K_1}{K_1}(1+K) - b^2K + \frac{2P_m}{P_r}k_x^2Q\right\}$$
$$\frac{dR}{dA} = \frac{1}{a^2} \frac{-b(2-\overline{\delta}b \in {}^{-1})\left\{\frac{\in}{K_1}(1+K)(b^2+2Ab) - b^2KA + \frac{P_m}{P_r}k_x^2Q(b+2A)\right\}}{(b+2A-\overline{\delta}bA \in {}^{-1})^2}$$

Thus, the micropolar coefficient has a stabilizing effect in the absence of rotation if $\frac{2}{b} < \overline{\mathbf{\delta}} < \frac{\epsilon}{A}$ and $K_1 < \frac{2 \in (1+K)}{bK}$, whatever the strength of magnetic field is applied.

In the absence of micropolar heat conduction parameter ($\overline{\delta} = 0$), equation (23) reduces to

$$\frac{dR}{dA} = \frac{1}{a^2} \left[\frac{-b^4 K \left[\left\{ \frac{\epsilon}{K_1} (1+K) (b^2 + 2bA) - b^2 KA + \frac{P_m}{P_r} k_x^2 Q(b+2A) \right\}^2 - 4\pi^2 \Omega_0^2 b(b+2A)^2 \right]}{(b+2A)^2 \left\{ \frac{\epsilon}{K_1} (1+K) (b^2 + 2Ab) - b^2 KA + \frac{P_m}{P_r} k_x^2 Q(b+2A) \right\}^2} \right]$$

Thus, the micropolar coefficient has destabilizing effect in the absence of micropolar heat conduction parameter $(\overline{\mathbf{\delta}} = 0)$ if $K_1 < \frac{2 \in (1+K)}{bK}$ and $Q > \frac{2\pi\Omega_o \sqrt{b} P_r}{P_m k_x^2}$.

In the absence of rotation $(\Omega_0 = 0)$ and micropolar heat conduction parameter $(\overline{\delta} = 0)$, equation (23) reduces to

$$\frac{dR}{dA} = -\frac{b^4 K}{a^2 (b+2A)^2}$$

Which is always negative, thus in the absence of rotation and micropolar heat conduction parameter, the micropolar coefficient has destabilizing effect whatever the strength of magnetic field is applied. From (18), we have

$$bA \in \left[b\left\{ \frac{\epsilon}{K_1} (1+K)(b^2+2Ab) - b^2KA + \frac{P_m}{P_r} k_x^2 Q(b+2A) \right\}^2 \right]$$
$$\frac{dR}{d\delta} = \frac{+4\pi^2 \Omega_0^2 (b^2+2Ab)^2}{a^2 (b+2A-\overline{\delta}bA \in (-1))^2 \left\{ \frac{\epsilon}{K_1} (1+K)(b^2+2bA) - b^2KA + \frac{P_m}{P_r} k_x^2 Q(b+2A) \right\}}$$

Thus, the microplar heat conduction parameter has a stabilizing effect if $K_1 < \frac{2 \in (1+K)}{bK}$.

VII. Oscillatory Convection

Equation (14) can be rewritten as

$$[A_{1}b\sigma+b^{2}]\{(\sigma+A_{2})(l\sigma+2A+b)(P_{m}\sigma+b)-AKb(P_{m}\sigma+b)+A_{3}(l\sigma+2A+b)\}^{2}$$

$$=Ra^{2} \in (P_{m}\sigma+b)\left(l\sigma+2A+b-\frac{\bar{\delta}Ab}{\epsilon}\right)\{(\sigma+A_{2})(l\sigma+2A+b)(P_{m}\sigma+b)$$

$$-Akb(P_{m}\sigma+b)+A_{3}(l\sigma+2A+b)\}-4\pi^{2}\Omega_{0}^{2}(A_{1}\sigma+b)(l\sigma+2A+b)^{2}(P_{m}\sigma+b)^{2}$$

$$(...(24)$$
Where $EP_{r} = A_{1}, A_{2} = \frac{\epsilon}{K_{1}}(1+K), l = \frac{\bar{\mathbf{j}}}{C_{2}}, A = \frac{K}{C_{2}}, A_{3} = \frac{K_{x}^{2}QP_{m}}{P_{r}}, \frac{K^{2}b}{C_{2}} = AKb$
Put $\sigma = i\sigma_{i}$ in (24) and separating real and imaginary parts, we have

$$R = \frac{b^2(\alpha_1^2 - \alpha_2^2) - 2\alpha_1 \alpha_2 A_1 b \sigma_i + 4\pi^2 \Omega_0^2 \left[b(\alpha_5^2 - \alpha_6^2) - 2\alpha_5 \alpha_6 A_1 \sigma_i \right]}{a^2 \in (\alpha_3 \alpha_1 - \alpha_4 \alpha_2)} \dots (25)$$

and

$$R = \frac{A_{1} b \sigma_{i} (\alpha_{1}^{2} - \alpha_{2}^{2}) + 2\alpha_{1} \alpha_{2} b^{2} + 4\pi^{2} \Omega_{0}^{2} \Big[A_{1} \sigma_{i} (\alpha_{5}^{2} - \alpha_{6}^{2}) + 2\alpha_{5} \alpha_{6} b \Big]}{a^{2} \in (\alpha_{3} \alpha_{2} + \alpha_{4} \alpha_{1})} \qquad \dots (26)$$

Where $\alpha_{1} = (bA_{2} + A_{3})(2A + b) - AKb^{2} - P_{m} \sigma_{i}^{2}(2A + b + A_{2}l) - bl \sigma_{i}^{2}$

$$\begin{aligned} \alpha_2 &= b(2A+b+A_2l)\,\sigma_i + P_m\,A_2\,(2A+b)\,\sigma_i + (A_3l - AKbP_m)\,\sigma_i - lP_m\,\sigma_i^3\\ \alpha_3 &= b\left(2A+b-\frac{\overline{\delta}Ab}{\epsilon}\right) - lP_m\,\sigma_i^2\\ \alpha_4 &= lb\,\sigma_i + P_m\left(2A+b-\frac{\overline{\delta}Ab}{\epsilon}\right)\sigma_i = \left[lb+P_m\left(2A+b-\frac{\overline{\delta}Ab}{\epsilon}\right)\right]\sigma_i\\ \alpha_5 &= b(2A+b) - lP_m\,\sigma_i^2\\ \alpha_6 &= P_m(2A+b)\,\sigma_i + lb\sigma_i = \left[P_m(2A+b) + lb\right]\sigma_i\end{aligned}$$

Eliminating *R* between (25) and (26), we get

$$(\alpha_{3} \alpha_{2} + \alpha_{4} \alpha_{1}) \left[b^{2} (\alpha_{1}^{2} - \alpha_{2}^{2}) - 2\alpha_{1} \alpha_{2} A_{l} b \sigma_{i} + 4\pi^{2} \Omega_{0}^{2} \left\{ b(\alpha_{5}^{2} - \alpha_{6}^{2} - 2\alpha_{5} \alpha_{6} A_{l} \sigma_{i} \right\} \right]$$

$$= (\alpha_{3} \alpha_{1} - \alpha_{4} \alpha_{2}) \left[A_{l} b \sigma_{i} (\alpha_{1}^{2} - \alpha_{2}^{2}) + 2\alpha_{1} \alpha_{2} b^{2} + 4\pi^{2} \Omega_{0}^{2} \left\{ A_{l} \sigma_{i} (\alpha_{5}^{2} - \alpha_{6}^{2}) + 2\alpha_{5} \alpha_{6} b \right\} \right]$$

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Similarly a_2 , a_3 , a_4 , a_5 are defined as the coefficients of σ_i^7 , σ_i^5 , σ_i^3 and σ_i in equation (27). Let $\sigma_i^2 = r$, then equation (27) becomes

$$a_0r^5 + a_1r^4 + a_2r^3 + a_3r^2 + a_4r + a_5 = 0 \qquad \dots (28)$$

Since $r = \sigma_i^2$, which is always positive, so that the sum of roots of equation (28) is positive but this is impossible if $a_0 > 0$ and $a_1 > 0$, because the sum of roots of equation (28) is $\left(-\frac{a_0}{a_1}\right)$. Thus, $a_0 > 0$ and $a_1 > 0$ are the sufficient conditions for the non-existence of overstability. Clearly, $a_0 > 0$ and

$$a_{1} > 0 \text{ if } \frac{2EP_{r}K_{1}}{A(1+K)} < \overline{\delta} < \frac{\epsilon}{A}, \quad \frac{2}{3}EP_{r}, \quad \overline{\delta} + 6 \in \leq K < \frac{4}{EP_{r}}, \quad 0 < P_{m} < 1, \\ 0 < \min\left\{\frac{\pi^{2}\Omega_{0}^{2}l}{2A^{2}}, \left(\frac{4\pi^{2}\Omega_{0}^{2}}{EP_{r}}\right)^{1/3}\right\} < 10 \\ \text{and } \max.\left\{\frac{2\pi^{2}\Omega_{0}^{2}\overline{\delta}P_{r}}{3k_{x}^{2} \in P_{m}}, \quad \frac{\pi^{2}\Omega_{0}^{2}P_{r}(1-P_{m})}{10k_{x}^{2}P_{m}^{2}}\right\} < Q < \min\left\{\frac{4A^{2}P_{r}}{3l^{2}k_{x}^{2}P_{m}}, \frac{2\pi^{2}\Omega_{0}^{2}P_{r}}{3A^{2}k_{x}^{2}P_{m}}\right\}$$

Hence for the conditions given in (29), overstability cannot occur and the principle of exchange of stability (PES) is valid.

VIII. Conclusions:

For stationary Convection :

- The critical Rayleigh number increases as the medium permeability decreases when $\frac{\epsilon}{A} > \max\{\overline{\delta}, K_1\}$ and
 - $Q > \frac{2\pi\Omega_0 P_r \sqrt{b}}{P_m k_x^2}$, thus the medium permeability has a destabilizing effect (see figure 2). In the absence of

rotation and micropolar heat conduction, the medium permeability has destabilizing effect whatever the strength of magnetic field is applied (**see figure 3**). In the absence of rotation, the critical Rayleigh number increases as the medium permeability decreases when $\overline{\delta} < \frac{\epsilon}{A}$ and the critical Rayleigh number increases as the medium permeability increases when $\overline{\delta} < \frac{\epsilon}{A} + \frac{2 \epsilon}{b}$, thus, in the absence of rotation the medium permeability may have dual role, whatever the strength of magnetic field is applied (**see figure 4**). In the absence of magnetic field, the medium permeability has destabilizing effects according to the conditions $\frac{\epsilon}{A} > \max.\{\overline{\delta}, K_1\}, \Omega_0 < \frac{\epsilon A \sqrt{b}}{\pi K_1(b+2A)}$ (**see figure 5**).

- 2. When $\frac{\epsilon}{A} > \max_{\{\overline{\delta}, K_1\}}$, the critical Rayleigh number increases with the decreasing of rotation, thus the rotation has destabilizing effect when $\frac{\epsilon}{A} > \max_{\{\overline{\delta}, K_1\}}$, whatever the magnetic field is applied (see figure 6). In the absence of micropolar heat conduction parameter, the rotation has stabilizing effect when $K_1 < \frac{\epsilon}{A}$ (see figure 7).
- 3. When $\frac{\epsilon}{A} > \max\{\overline{\delta}, K_1\}$ and $Q > \frac{2\pi\Omega_0 P_r \sqrt{b}}{P_m k_x^2}$, the critical Rayleigh number increases with the increasing of
 - Chandrashekhar number, thus the magnetic field has a stabilizing effect when $\frac{\epsilon}{A} > \max_{\overline{\delta}} K_1$ and $2\pi\Omega_0 P_r \sqrt{b}$ (see figure 8) In the absence of micropolar heat conduction parameter and rotation the
 - $Q > \frac{2\pi\Omega_0 P_r \sqrt{b}}{P_m k_x^2}$ (see figure 8). In the absence of micropolar heat conduction parameter and rotation, the

magnetic field also has a stabilizing effect without any condition (see figure 9).

4. When $\frac{\epsilon}{A} > \max \{\overline{\delta}, K_1\}$ and $Q > \frac{2\pi\Omega_0 P_r \sqrt{b}}{P_m k_x^2}$, the coupling parameter has a stabilizing effect (see figure 10). In

the absence of magnetic field, the coupling parameter has a stabilizing effect when $\frac{\epsilon}{A} > \max_{\{\overline{\delta}, K_1\}}$ and

 $\Omega_0 < \frac{\epsilon A \sqrt{b}}{\pi K_1(b+2A)}$ (see figure 12). In the absence of rotation, the coupling parameter has a stabilizing effect

when
$$\frac{\epsilon}{A} > \max_{\overline{\delta}}, K_1$$
 (see figure 11).

- 5. When $\frac{2}{b} < \overline{\delta} < \frac{\epsilon}{A}$, $K_1 < \frac{2\epsilon(1+K)}{bK}$ and $Q > \frac{2\pi\Omega_0 P_r \sqrt{b}}{P_m k_x^2}$, the critical Rayleigh number increases with the
 - increasing of micropolar coefficient, thus, the micropolar coefficient has a stabilizing effect under the above conditions (see figure 13). In the absence of rotation, the micropolar coefficient has a stabilizing effect when $\frac{2 \epsilon}{b} < \overline{\mathbf{\delta}} < \frac{\epsilon}{A}$ and $K_1 < \frac{2 \epsilon (1+K)}{bK}$, whatever the strength of magnetic field is applied. In the absence of micropolar heat conduction parameter, the micropolar coefficient has destabilizing effect when

 $K_1 < \frac{2 \in (1+K)}{bK}$ and $Q > \frac{2\pi\Omega_0 P_r \sqrt{b}}{P_m k_x^2}$. In the absence of rotation and micropolar heat conduction parameter, the

micropolar coefficient has destabilizing effect without any condition.

When $K_1 < \frac{2 \in (1+K)}{bK}$, the critical Rayleigh number increases as the micropolar heat conduction parameter 6.

increases, thus, the micropolar heat conduction parameter has a stabilizing effect if $K_1 < \frac{2 \in (1+K)}{K_1}$.

For oscillatory Convection : The sufficient conditions for the non-existence of overstability are given by conditions (29).

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List of Figures



Fig.2: Marginal instability curve for the variation of R v/s K_1 (Medium permeability) for A=0.5, P₁=2, P_m=4, $k_x=0.05, \ \overline{\delta}=0.05, K=1, \in=0.5, a=1.$



Fig. 3: Marginal instability curve for the variation of R v/s K_1 (Medium permeability) for A=0.5, \in =0.5, K=1, $\overline{\delta} = 0$, $\Omega_0 = 0$, $P_r = 2$, $P_m = 4$, $k_x = 0.05$, a = 1



Fig. 4: Marginal instability curve for the variation of R v/s K₁ (Medium permeability) for $\overline{\delta}$ =0.05, Q=45000, P₁=2, P_m=4, K=1, k_x=0.05, Ω_0 =0.



Fig. 5: Marginal instability curve for the variation of R vs K₁ (Medium permeability) for $\overline{\delta}$ =0.05, Q=0, P_r=2, P_m=4, K=1, k_x=0.05, Ω_0 =0.5, A=0.5, \in =0.5.



Fig. 6: Marginal instability curve for the variation of R vs Ω_0 (Rotation) for A=0.5, \in =0.5, K=1, k_x=0.05, P_r=2, P_m=4, $\overline{\delta}$ =0.05, K₁=0.03, Q=45000.





Fig. 8: Marginal instability curve for the variation of R vs Q for the A=0.05, \in =0.5, P_r=2, P_m=4, K=1, k_x=0.05, K₁=0.03, $\overline{\delta}$ =0.05, Ω_0 =10.



Fig. 9: Marginal instability curve for the variation of R v/s Q for the A=0.5, \in =0.5, P_r=2, P_m=4, K=1, K₁=0.03, $\overline{\delta}$ =0, Ω_0 =0, k_x=0.05.



Fig.10: Marginal instability curve for the variation of R v/s K for A=0.5, \in =0.5, K₁=0.03, k_x=0.05, $\overline{\delta}$ =0.05, P_r=2, P_m=4, Ω_0 =10, Q=45000.



Fig.11: Marginal instability curve for the variation of R v/s K for A=0.5, \in =0.5, K₁=0.03, k_x=0.05, $\overline{\delta}$ =0.05, P_r=2, P_m=4, Ω_0 =0, Q=45000.



Fig. 12: Marginal instability curve for the variation of R vs K for A=0.5, \in =0.5, K₁=0.03, $\overline{\delta}$ =0.05, Ω_0 =0.5, P_r=2, P_m=4, k_x=0.05, Q=0.

Fig. 13: Marginal instability curve for the variation of R v/s $\overline{\delta}$ (Micropolar heat conduction parameter) for A=0.5, \in =0.5, P_r=2, P_m=4, K=1, K₁=0.03, k_x=0.05, Q=45000, Ω_0 =10.