

Application of Lifetime Models in Maintenance (Case Study: Thermal Electricity Generation in Sudan)

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Abstract: The main objective of this paper is that to apply lifetime models on the fault of thermal electricity generation in Sudan to predict faults and failures during period of working and increase its lifetime to insure electricity production sustainability and reducing maintenance cost. Lifetimes data has been taken from Bahri Thermal Station for electricity generation, which is, belong to the National Electricity Authority in Sudan during the period (2011-2015). Through the lifetime models estimation fault distribution, reliability, hazard rate, availability and MTBF have been calculated for the five machines from analysis; it is clear that, fault time for all machines follow Weibull distribution with 2-parameters. The machines no (3, 4 and 6) have high reliability whereas the machines no (1 and 5) have low reliability, when we predict the reliability according to the time we found the that the reliability decrease and hazard rate increase and there is relationship between MTBF and reliability.

Keywords: Lifetime, Reliability, Failure, Fault rate, Hazard rate, Weibull, Maintenance, MTBF

I. Introduction:

The concept of maintenance in the past was limited to reform, which followed the machine was out of work. The maintenance was synonymous with reform that concerned with what has been corrupted when actually there was fault. The causes of the fault were not discovered until it repeated and took a period of time. After the Second World War the tough focused on controlling of maintenance expenses using mathematical models, now the practical application revealed the ineffectiveness of many of these models and they have been changed with more advanced models.

The machines maintenance system considered the safety gateway to authenticate the electricity generation stability; it serves as the guard who prevents the machine from the sudden faults and plays the role on the stability and safety of the machine which it has significant impact on reduction of production and operational costs and it result positively in the economy activity. The research problem is that, there is no stochastic models application for the lifetime, reliability and failure to predict failures that occur for the machines and the consequent of technical malfunctions of the machines, which made its power, run of for the consumer and it raise production and operational costs, which reflect on economic development

The objectives of the study are to apply lifetime models on the electricity generation machines in Sudan, study reliability and failure model, study of probability distributions, which used in the lifetime and compare them in terms of preference.

The data of this study have been collocated for five machines with exceptional to the machine no(2) because it did never got fault in duration of the study. The sample size has been determine according to the method that not tided to the number of time of failure occurs condition for each machine. The technical fault data collected from the efficiency department in the station and it was (failure time) during the period (2011-2015). There are two types of faults; mechanical faults and the faults due to preventive maintenance, in this study, we used the data of mechanical faults.

This study based on the following assumptions:

- Applications of lifetime on machines have a positive impact on the electricity stability.
- Fault times follows Weibull distribution.
- The electricity generating machines have a high reliability.

II. Theoretical Framework:

2.1: Reliability:

Reliability define as the probability of success or the probability that the system will perform it intended function under specified design limits [1].

Reliability that is more specific is the probability that a product are part will operate properly for specified period of time (design life) under the design operating condition without feature. In other words, reliability may be used as measure of the systems success in providing, it is function properly. Reliability is once of quality characteristics that consumer require form the menu facture of products.

Mathematically: reliability $R(t)$ is the probability that a system will be successful in the interval from time 0 to time t :

$$R(t) = P(T > t) \quad t > 0 \dots\dots\dots(1)$$

Where T is a random variable denoting the time -to-failure or failure time.

Unreliability $F(t)$, a measure of failure, is defined as the probability that the system will fail by time t :

$$F(t) = P(T \leq t) \quad \text{for } t \geq 0$$

In the other words, $F(t)$ is the failure distribution function. If the time-To-failure random variable T has a density function $f(t)$, then

$$R(t) = \int_t^{\infty} f(s) ds \dots\dots\dots(2)$$

or, equivalently

$$f(t) = -\frac{d}{dt}[R(t)]$$

The density function can be mathematically described in terms of T as:

$$\lim_{\Delta t \rightarrow 0} p(t < T \leq t + \Delta t) \dots\dots\dots(3)$$

This can be interpreted as the probability that the failure time T will occur between the operating time t and the next interval of operation, $t + \Delta t$.

Consider new and successfully tested system that operates well when put into service at time $t = 0$, the system becomes less likely to remain successful as the time interval of course, is zero $[1]$.

2.2: Fault Rate:

The possibility of fault machine in specific time period t_1 and t_2 can be expressed with following non reliability, equation $[2]$.

$$\int_{t_1}^{t_2} f(t) dt = \int_{\infty}^{t_1} f(t) dt - \int_{-\infty}^{t_2} f(t) dt = F(t_1) - F(t_2)$$

or can be expressed with reliability

$$\int_{t_1}^{t_2} f(t) dt = \int_{t_1}^{\infty} f(t) dt - \int_{t_2}^{\infty} f(t) dt = R(t_1) - R(t_2)$$

The rate that a fault took place with in specific period of time is called as "Fault Rate" throughout the period t_1 indicates for no fault in the beginning of period and therefore the equation can be expressed as follows:

$$\frac{R(t) - R(t_2)}{(t_2 - t_1)R(t_1)} \dots\dots\dots(4)$$

It has been observed that the Fault Rate depend on time if the period t_1 denoted as $t + \Delta t$ the equation (4) stated as follow:

$$\frac{R(t) - R(t + \Delta t)}{\Delta t . R(t)} \dots\dots\dots(5)$$

and means with rate it number of faults in each unit time.

2.3: Hazard Rate:

Define as limits of rate of faults for a period of near-zero equation can be written in the form $[2]$:

$$h(t) = \lim_{\Delta t \rightarrow 0} \frac{R(t) - R(t + \Delta t)}{\Delta t . R(t)} = \frac{1}{R(t)} \left[-\frac{dR(t)}{dt} \right]$$

$$h(t) = \frac{f(t)}{R(t)} \dots\dots\dots(6)$$

To find out possibility of fault machine it have age t in time period [t, t + Δt] written as:

$$f_{pos} = h(t)dt \dots\dots\dots(7)$$

The hazard rate refer to change in rate fault through age of machine. To find out hazard rate for the sample machines N (machine consisting of n element), we will assume N_s(t) is random variable denotes to number of machines working successfully at time t thus, the N_s(t) is binomial distribution.

$$P[N_s(t) = n] = \frac{N!}{n!(N-n)!} [R(t)]^n [1-R(t)]^{N-n}$$

$$n = 0, 1, \dots, N$$

The expected value for N_s(t):

$$E[N_s(t)] = N \cdot [R(t)] = N(t)$$

hence:

$$R(t) = \frac{E(N_s(t))}{N} = \frac{\bar{N}(t)}{N} \dots\dots\dots(8)$$

and reliability in time t, it is arithmetic mean for rate success in t thus:

$$F(t) = 1 - R(t) = 1 - \frac{\bar{N}(t)}{N} = \frac{N - \bar{N}(t)}{N} \dots\dots\dots(9)$$

and rate density fall equal:

$$f(t) = \frac{dF(t)}{dt} = -\frac{1}{N} \cdot \frac{d\bar{N}(t)}{dt}$$

2.4: System Mean Time to Failure:

Suppose that the reliability function for a system is given by R(b), the expected feature time during which a component is expected to perform success fully, or the system mean time to feature (MTTF) [1], given by:

$$MTTF = \int_0^{\infty} t f(t) dt \dots\dots\dots(10)$$

substituting:

$$f(t) = -\frac{d}{dt}[R(t)]$$

From equation (10) and performing integration by part, we obtain:

$$MTTF = -\int_0^{\infty} t dt [R(t)] = [-tR(t)]_0^{\infty} + \int_0^{\infty} R(t) dt \dots\dots\dots(11)$$

The first term on the right hand side of above equation equals zero at both limits, since the system must fail after a finite amount of operating time, therefore, we must have tR(t) → 0 as t → ∞ so equation (11) becomes:

$$MTTF = \int_0^{\infty} R(t) dt \dots\dots\dots(12)$$

Thus, MTTF is the definite integral evolution of the reliability function. In general if λ(t) is defined as the failure rate function, then by definition MTTF is not equal to 1/h(t).

The MTTF should be used when the feature time distribution function is specified because the reliability level implicit by the MTTF depends on the underlying feature time distribution. Although the MTTF measure is one of the most widely used reliability calculation, it also one of most missed calculations, it has been misinterpreted as "guaranteed minimum life time".

2.5: Availability:

Reliability is a measure that requires system success for an entire mission time, no failure or repairs are allowed. The availability of a system is defined as the probability that the system is successful at time t, mathematically:

$$\text{Availability} = \frac{\text{system up time}}{\text{system up time} + \text{system down time}} \dots\dots\dots(13)$$

Availability is a measure of success used primarily for repairable system, for non-repairable system availability A(t) equals reliability R(t). In repairable system A(t) will be equal to or greater than R(t).

The mean time between failures (MTBF) is an important measure in repairable system. This implies that the system has MTBF is an expected value of the random variable time between failures mathematically.

$$\text{MTBF} = \text{MTTF} + \text{MTTR} \dots\dots\dots(14)$$

2.6: Weibull distribution Parameters Estimation:

Let t_1, t_2, \dots, t_n be a random sample from Weibull distribution with p.d.f

$$f(t_1, t_2, \dots, t_n, \alpha, \beta) = \alpha \beta t^{\alpha-1} e^{-\beta t^\alpha} \text{ for } t \geq 0 \dots\dots\dots(15)$$

The likelihood function is

$$L(t_1, t_2, \dots, t_n) = \alpha^n \beta^n \prod_{i=1}^n t_i^{\alpha-1} e^{-\beta t_i^\alpha}$$

$$\ln L(t_1, t_2, \dots, t_n) = n \ln \alpha + n \ln \beta + (\alpha - 1) \sum_{i=1}^n \ln t_i - \beta \sum_{i=1}^n t_i^\alpha$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^n \ln t_i - \beta \sum_{i=1}^n t_i^\alpha \ln t_i = 0$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} - \sum_{i=1}^n t_i^\alpha = 0$$

This, the maximum likelihood estimation of α and β are

$$\hat{\beta} = \frac{n}{\sum_{i=1}^n \ln \left(\frac{T}{t_i} \right)} \dots\dots\dots(16)$$

$$\hat{\alpha} = \frac{n}{t^{\hat{\beta}}} \dots\dots\dots(17)$$

III. Application Aspect:

The application aspect includes to what explained in the theoretical aspect and depending on statistical software "STATGRAPHIC", we would describe the data, test the distribution of the data, estimate lifetime models and comparison between machines in fault distribution, Reliability, hazard rate, availability and MTBF.

3.1: Description of Failure Times:

This study will be described for the failure time, for the five machines with some descriptive measures in order to know the nature of study's data type.

Table (1): Rates of failure times for each machine

Machine	Mean(hr)	Std. (hr)	95% Confidence Interval for Mean	
			Lower Bound	Upper Bound
Machine no(1)	6.764	9.249	9.249	10.217
Machine no(3)	6.442	9.023	9.023	9.1530
Machine no(4)	6.747	11.808	11.808	9.2630
Machine no(5)	8.528	10.719	10.719	13.545
Machine no(6)	15.451	15.174	15.174	22.552
Mean	7.720	11.374	11.374	9.2980

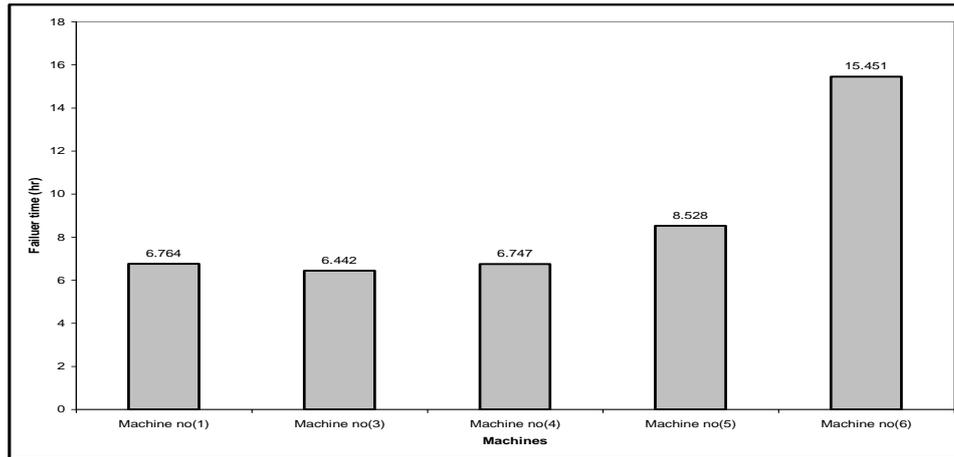


Figure (1): Rates of failure times for each machine

From above table and figure, it has shown that according to the mean values for the five machines, the machine no(6) have the highest mean failure depending on the value of the largest mean (15.45) hours, followed by machine no(5) depending on the value of the second largest mean (8.53) hours, followed by machine no(1) depending on the value of the third largest mean (6.76) hours, lastly machines no(1) and (4) are (6.76) and (6.75) respectively.

3.2: Data distribution Test:

Here we test the following hypothesis:

H_0 : The failure data follow Weibull distribution

H_1 : The failure data not follow Weibull distribution

Table (2): Kolmogorov-Smirnov test for machines

Machine	Statistic	Sample Size	P-value
Machine no(1)	0.23796	30	0.05595
Machine no(3)	0.13482	44	0.47132
Machine no(4)	0.13482	84	0.41859
Machine no(5)	0.27464	23	0.05035
Machine no(6)	0.24790	44	0.14390

From above table, it shows the p-value of Kolmogorov-Smirnov test of all machines is greater than significant level (0.05) that mean the failures time data follow Weibull distribution with 2-parameters, which means the underlying distribution of the lifetime model is Weibull

3.3: Lifetime Models for Machine no(1):

The lifetime test has been conducted for machine no(1) for a period of time (100 hours) and the following measure has been calculated:

Table (3): Results of Lifetime test for machine no(1)

Measure	Value
Distribution of fault $f(t)$	0.51858
Reliability $R(t)$	0.48142
Hazard rate $h(t)$	1.07719
Availability $A(t)$	0.98

From the table (3), it has shown that:

- The probability fault of the machine no(1) is $f(t = 100) = 0.51858$ during (100) hours, this indicate the probability fault of machine no(1) is very high during this period.
- The reliability for machine no(1) is weak since $R(t = 100) = 0.48142$, this mean that the probability for machine to work for (100) hours without fault is (0.48).

- The rate of randomly fault occurred for machine no(1) $h(t = 100) = 1.07719$, this indicate that the rate that occurred fault randomly during (100) hours is very high.
- The probability of available time to repair machine no(1) when it fault is (0.97), this indicate that the machine has high availability.

Table (4): Life table (times) for machine no(1)

Time	Reliability $R(t)$	Cum.Hazard $h(t)$
0	1.00000	0.0000
100	0.48142	0.7310
200	0.24286	1.4153
300	0.12457	2.0829
400	0.06457	2.7400
500	0.03373	3.3893
600	0.01773	4.0326

From the table (4), it has shown that the reliability decreases whenever the working time of the machine increase. When the time (t=100) hours the reliability is about (48%), at time (t=200) hours the reliability is about (24%), at time (t=300) hours the reliability (12%), at time (t=400) hours the reliability is about (6%), at time (t=500) hours the reliability is about (3%), at time (t=600) hours the reliability is about (2%). The hazard rate increases whenever the working time increases too.

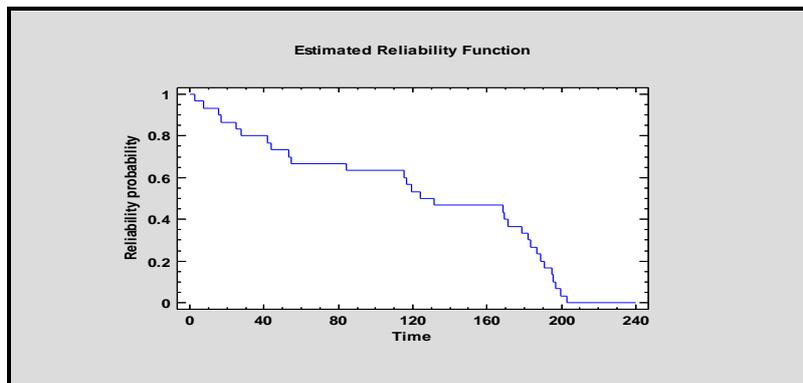


Figure (2): Reliability function vs time for machine no(1)

From the figure (2), it has shown that the reliability decreases whenever the working time of the machine increase in until equal zero.

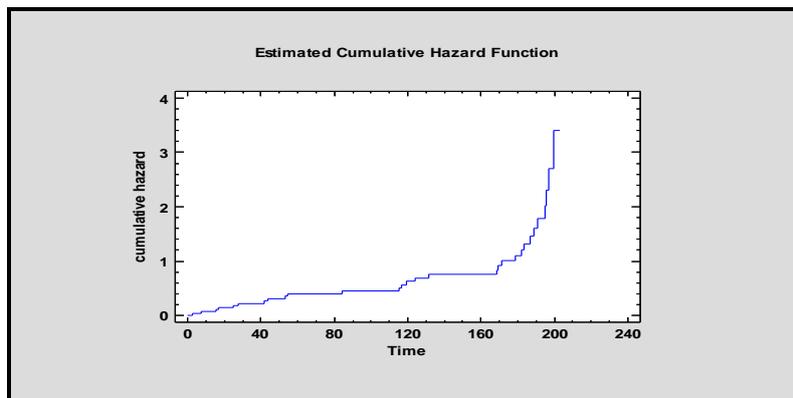


Figure (3): Cumulative hazard function vs time for machine no(1)

From the figure (3), the hazard function increases whenever the working time increases too.

3.4: Lifetime Model for Machine no(3):

The lifetime test has been conducted for machine no(3) for a period of time (100 hours) and the following measure has been calculated:

Table (5): Results of Lifetime test for machine no(3)

Measure	Value
Distribution of fault $f(t)$	0.28492
Reliability $R(t)$	0.71508
Hazard rate $h(t)$	0.39844
Availability $A(t)$	0.97

From the table (5), it has shown that:

- The probability fault of the machine no(3) is $f(t = 100) = 0.28492$ during (100) hours, this indicate that the probability fault of this machine is low during this period.
- The reliability for machine no(3) is $R(t = 100) = 0.71508$ it is weak reliability, this mean that the probability for machine to work for (100) hours without fault is (0.72), the reliability is very high.
- The rate of randomly fault occurred for machine no(3) $h(t = 100) = 0.39844$, this indicate that the rate occurred fault randomly during (100) hours is low.
- The probability of available time to repair machine no(3) when it fault is (0.97), this indicate that this machine has high availability.

Table (6): Life table (times) for machine no(3)

Time	Reliability $R(t)$	Cum.Hazard $h(t)$
0	1.00000	0.0000
100	0.71508	0.3354
200	0.50892	0.6755
300	0.36153	1.0174
400	0.25653	1.3605
500	0.18186	1.7045
600	0.12883	2.0492

From the table (6), it has shown that the reliability decreases whenever the working time of the machine increase. When the time (t=100) hours the reliability is about (72%), at time (t=200) hours the reliability is about (51%), at time (t=300) hours the reliability (36%), at time (t=400) hours the reliability is about (26%), at time (t=500) hours the reliability is about (18%), at time (t=600) hours the reliability is about (13%). The hazard function increases whenever the working time increases too.

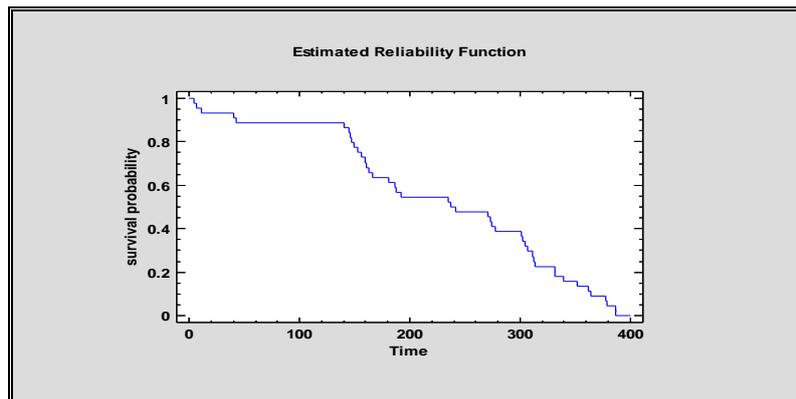


Figure (4): Reliability function vs time for machine no(3)

From the figure (4), it has shown that the reliability decreases whenever the working time of the machine increase in until equal zero.

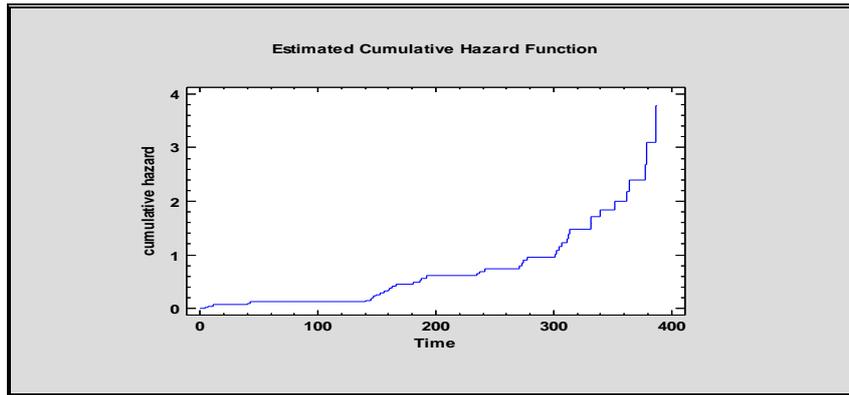


Figure (5): Cumulative hazard function vs time for machine no(3)

From the figure (5), the hazard function increases whenever the working time increases too.

3.5: Lifetime Model for Machine no(4):

The lifetime model has been conducted for machine (4) for a period of time (100 hours) and the following measure has been calculated:

Table (7): Results of Life time test for machine no(4)

Measure	Value
Distribution of fault $f(t)$	0.37294
Reliability $R(t)$	0.62706
Hazard rate $h(t)$	0.59474
Availability $A(t)$	0.99000

From the table (7), it has shown that:

- The probability fault of the machine no(4) is $f(t = 100) = 0.37294$ during (100) hours, this indicate that the probability fault of this machine (4) is low during this period.
- The reliability for machine no(4) is $R(t = 100) = 0.62706$ it is high reliability, this mean that the probability for machine to work for (100) hours without fault is (0.63), the reliability is very high.
- The rate of randomly fault occurred for machine no(4) $h(t = 100) = 0.59474$, this indicate that the rate that occurred fault randomly during (100) hours is middle .
- The probability of available time to repair machine no(1) when it fault is (0.99), this indicate that the machine has high availability.

Table (8): Life Tables (Times) for machine no(4)

Time	Reliability $R(t)$	Cum.Hazard $h(t)$
0	1.00000	0.0000
100	0.62706	0.5947
200	0.44404	0.8118
300	0.32552	1.1223
400	0.24360	1.4122
500	0.18494	1.6877
600	0.14195	1.9523

From the table (8), it has shown that the reliability decreases whenever the working time of the machine increase. When the time (t=100) hours the reliability is about (63%), at time (t=200) hours the reliability is about (44%), at time (t=300) hours the reliability (33%), at time (t=400) hours the reliability is about (24%), at time (t=500) hours the reliability is about (18%), at time (t=600) hours the reliability is about (14%). The hazard function increases whenever the working time increases too.

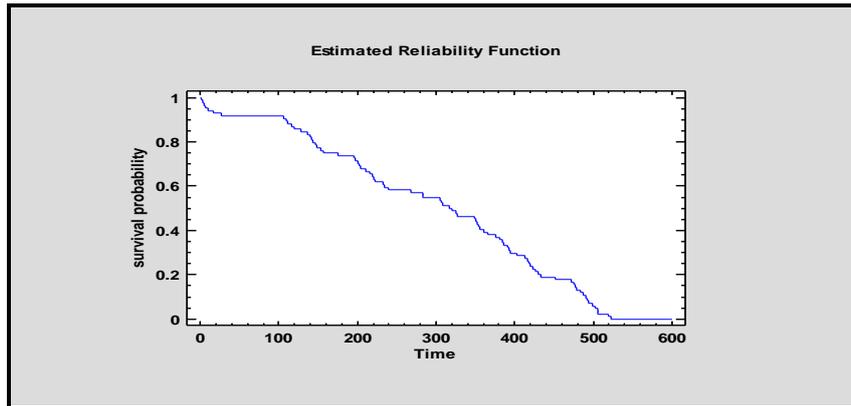


Figure (6): Reliability function vs time for machine no(4)

From the figure (6), it has shown that the reliability decreases whenever the working time of the machine increase.

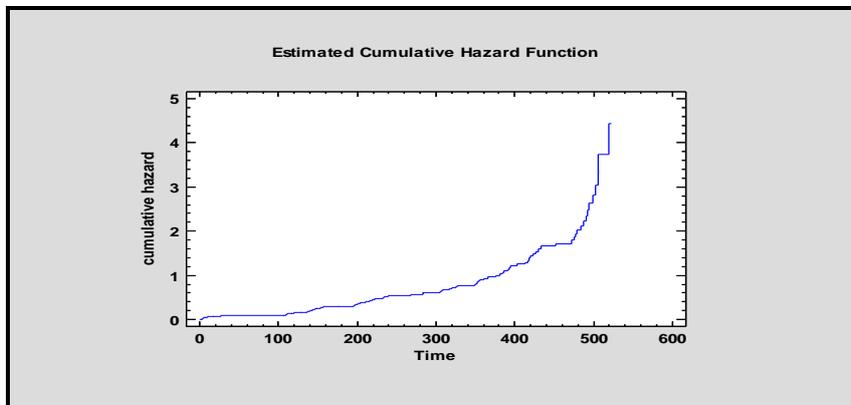


Figure (7): hazard function vs time for machine no(4)

From the figure (7), the hazard function increases whenever the working time increases too.

3.6: Lifetime Model for Machine no(5):

The lifetime model has been conducted for machine no(5) for a period of time (100) hours and the following measure has been calculated:

Table (9): Measures of lifetime model for machine no(5)

Measure	Value
Distribution of fault $f(t)$	0.52511
Reliability $R(t)$	0.47489
Hazard rate $h(t)$	1.10575
Availability $A(t)$	0.99

From the table (9), it has shown that:

- The probability fault of the machine no(5) is $f(t = 100) = 0.52511$ during (100) hours, this indicate that the probability fault of this machine is low during this period.
- The reliability for machine no(5) is $R(t = 100) = 0.47489$ it is high reliability, this mean that the probability for machine to work for (100) hours without fault is (0.47), the reliability is weak.
- The rate of randomly fault occurred for machine no(5) $h(t = 100) = 1.10575$, this indicate that the rate that occurred fault randomly during (100) hours is very high.
- The probability of available time to repair machine no(5) when it fault is (0.99), this indicate that this machine has high availability.

Table (10): Lifetable (times) for machine no(5)

Time	Reliability $R(t)$	Cum.Hazard $h(t)$
0	1.00000	0.0000
100	0.47489	1.1058
200	0.24683	1.3991
300	0.13223	2.0232
400	0.07219	2.6285
500	0.03995	3.2202
600	0.02235	3.8011

From the table (10), it has shown that the reliability decreases whenever the working time of the machine increase. When the time (t=100) hours the reliability is about (47%), at time (t=200) hours the reliability is about (25%), at time (t=300) hours the reliability (13%), at time (t=400) hours the reliability is about (7%), at time (t=500) hours the reliability is about (4%), at time (t=600) hours the reliability is about (2%). The hazard function increases whenever the working time increases too.

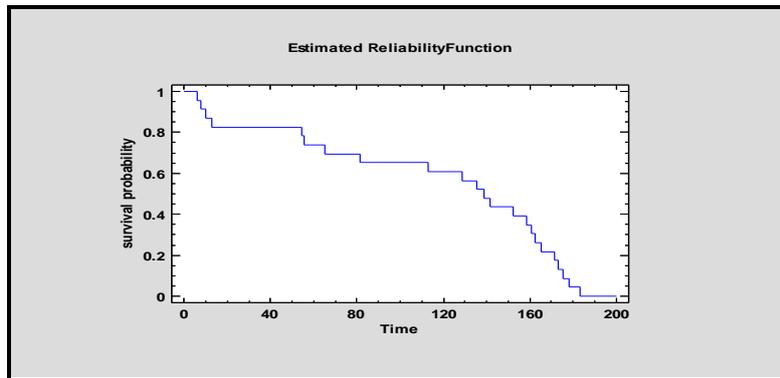


Figure (8): Reliability function vs time for machine no(5)

From the figure (8), it has shown that the reliability decreases whenever the working time of the machine increase.

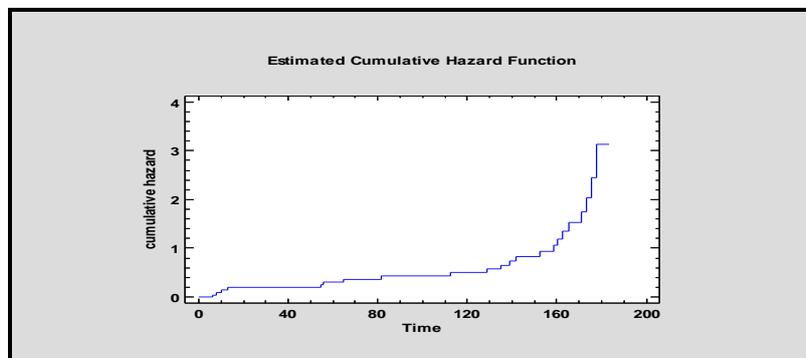


Figure (9): Hazard function vs time for machine no(5)

From the figure (9), the hazard function increases whenever the working time increases too.

3.7: Lifetime Model for Machine no(6):

The lifetime model has been conducted for machine no(6) for a period of time (100 hours) and the following measure has been calculated:

Table (11): Measures of lifetime Model for Machine no(6)

Measure	Value
Distribution of fault $f(t)$	0.25874
Reliability $R(t)$	0.74126
Hazard rate $h(t)$	0.34905

Availability $A(t)$	0.97000
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From the table (11), it has shown that:

- The probability fault of the machine no(6) is $f(t = 100) = 0.25874$ during (100) hours, this indicate that the probability fault of this machine is high during this period.
- The reliability for machine no(4) is $R(t = 100) = 0.74126$ it is weak reliability, this mean that the probability for machine to work for (100) hours without fault is (0.74), the reliability is very high.
- The rate of randomly fault occurred for machine no(6) $h(t = 100) = 0.349054$, this indicate that the rate that occurred fault randomly during (100) hours is very weak.
- The probability of available time to repair machine no(6) when it fault is (0.97), this indicate that this machine has high availability.

Table (12): Lifetable (times) for machine no(6)

Time	Reliability $R(t)$	Cum.Hazard $h(t)$
0	1.00000	0.0000
100	0.74126	1.1058
200	0.48040	0.7331
300	0.28999	1.2379
400	0.16610	1.7952
500	0.09117	2.3951
600	0.04826	3.0312

From the table (12), it has shown that the reliability decreases whenever the working time of the machine increase. When the time (t=100) hours the reliability is about (74%), at time (t=200) hours the reliability is about (48%), at time (t=300) hours the reliability (29%), at time (t=400) hours the reliability is about (17%), at time (t=500) hours the reliability is about (9%), at time (t=600) hours the reliability is about (5%). The hazard function increases whenever the working time increases too.

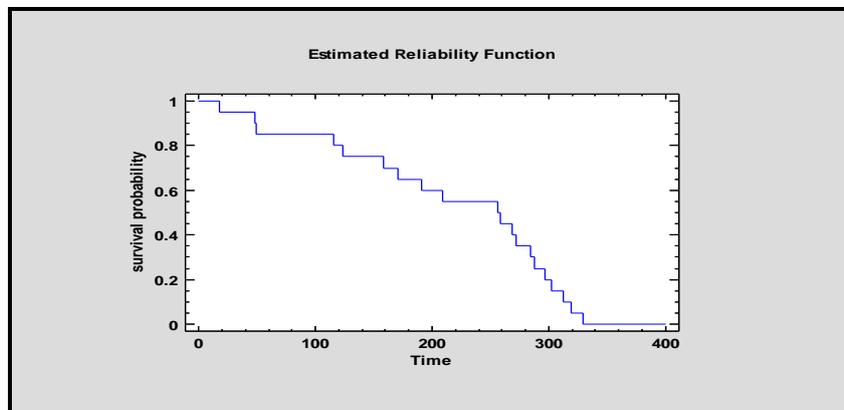


Figure (10): Reliability function vs time for machine no(6)

From the figure (10), it has shown that the reliability decreases whenever the working time of the machine increase.

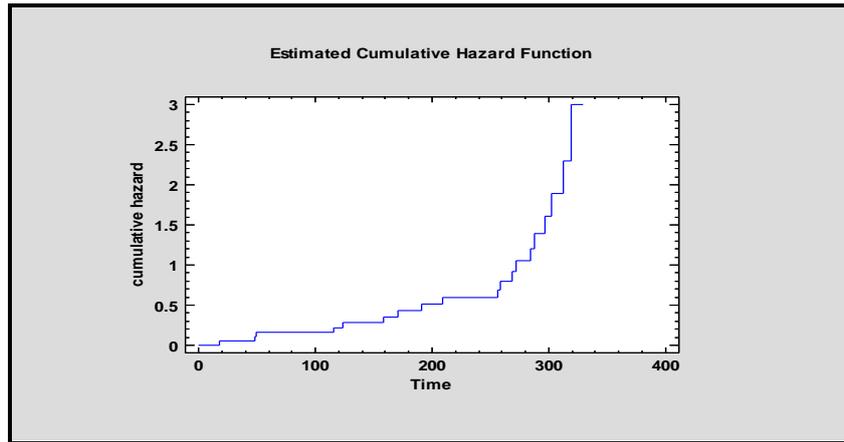


Figure (11): Hazard function vs time for machine no(6)

From the figure (11), the hazard function increases whenever the working time increases too.

3.8: Comparison between Machines:

We compare the five machines according to lifetime model, the comparison was among the following measures probability of fault, reliability, hazard rate and availability:

Table (13): Lifetime models comparison

machine	$f(t)$	$R(t)$	$h(t)$	$A(t)$
Machine no(1)	0.51858	0.48142	1.07719	0.98
Machine no(3)	0.28492	0.71508	0.39844	0.97
Machine no(4)	0.37294	0.62706	0.59474	0.99
Machine no(5)	0.52511	0.47489	1.10575	0.99
Machine no(6)	0.25874	0.74126	0.34905	0.97

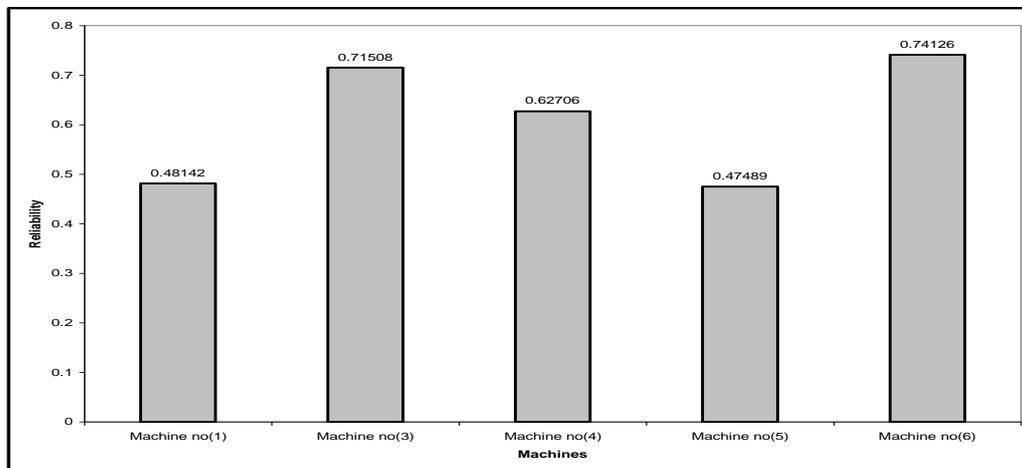


Figure (12): Reliability vs machines

Form above table and figure we note that ,the machines no (3,4 and 6) have high reliability and the machines no (1 and 5) have low reliability, the machines with high reliability have a low faults probability and hazard rate but the machines with low reliability have high faults probability and hazard rate.

Table (14): MTBF and reliability comparison

Machine	MTBF	$R(t)$
Machine no(1)	6.70036	0.48142
Machine no(3)	8.37763	0.71508
Machine no(4)	6.18406	0.62706
Machine no(5)	7.95355	0.47489
Machine no(6)	16.5139	0.74126

From above table , we note that whenever mean time between renewals (repairable) increased the reliability increased too and that appear clearly in the machinen no(6) result which its mean between renewals is approximately (17 hours) and the reliability (0.74).

IV. Conclusions:

The main findings of this paper are:

1. Fault time of machines follows Weibull distribution with 2-parameters which means the underlying distribution of the lifetime model is Weibull
2. When operation time of machines increase the performance decreased or the machine got fault.
3. The machines no (3, 4 and 6) have high reliability and the machines no (1 and 5) have low reliability.
4. The hazard rate of machines increase according to the time.
5. The machines with high reliability have a low faults probability and hazard rate but the machines with low reliability have high faults probability and hazard rate.
6. 6. Whenever mean time between failures (MTBF) for machines increase that indicate the machine has high reliability.
7. All the machines have high availability.

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APPENDIX

Failure data for five machines during period (2011-2015)

Failure time				
Machine no(1)	Machine no(3)	Machine no(4)	Machine no(5)	Machine no(6)
0	768	0.95	5.98	24
48	48	24	528	288
24	192	24	48	1.73
1704	1008	648	1608	65.87
504	240	120	5.98	7.77
48	10.98	240	528	35.43
14.38	72	120	48	11.85
2.20	2496	264	1608	20.78

Failure time				
Machine no(1)	Machine no(3)	Machine no(4)	Machine no(5)	Machine no(6)
8.92	336	24	1.32	18.25
1.42	192	4	10.53	27.43
30.18	2.45	120	2.07	2.17
30.53	3.40	168	5.53	10.03
1.42	0.00	312	28.80	3.50
2.75	3.43	144	13.68	11.95
4.50	0.82	24	0.95	3.00
7.77	2.3	96	21.25	9.22
36.47	3.25	144	1.95	5.42
1.05	14.90	288	1.77	10.33
1.80	5.33	96	7.75	6.47
7.77	2.02	48	2.15	10.13
2.85	4.08	168	4.82	
1.58	42.27	456	1.92	
3.62	2.77	576	22.55	
1.70	4.17	96	41.58	
1.97	29.65	168	1.22	
4.03	1.77	96	9.35	
1.21	0.98	264	16.65	
0.88	3.88	192	31.17	
2.77	23.53	48	16.27	
3.50	1.67	672	6.30	
	1.17	288	3.83	
	2.77	1.30	2.78	
	4.67	0.95	10.67	
	1.30	2.12	5.98	
	1.08	1.73	2.02	
	17.22	0.95	2.07	
	0.78	4.73	2.65	
	7.18	6.27	1.53	
	12.77	10.33	1.93	
	10.22	73.83	5.47	
	1.77	3.93	2.83	
	14.37	1.38	2.85	
	0.88	4.82	0.8	
	7.32	1.40	13.73	
	0.83	2.97	3.62	
		8.45	5.30	
		8.92		
		3.95		
		1.68		
		1.67		
		3.23		
		2.73		
		4.28		
		3.52		
		18.07		
		20.67		
		3.25		
		0.92		
		2.70		
		0.0		
		5.53		
		2.67		
		3.07		

Source: Bahri Thermal Station, Efficiency Department, 201